Settlement and Load Distribution Analysis of Pile Groups

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Summary.—Analyses are made of the settlement interaction between two compressible floating piles in a semi-infinite mass, and between two compressible piles resting on a rigid bearing stratum. The results of these analyses are expressed in terms of interaction factors relating the increase in settlement due to the second pile to the settlement of a single pile. It is shown that, as previously found with incompressible floating pile groups, the interaction factors for two piles may be superposed to analyse the settlement and load distribution within any compressible pile group.

The influence on group behaviour of the compressibility of the piles relative to the soil is examined for square groups of piles. It is found that, for a floating pile group, the settlement interaction decreases as the piles become relatively less compressible, and for a group with a rigid pile cap, the load distribution within the group becomes more uniform. In contrast, for a group of end-bearing piles, the settlement interaction increases as the piles become relatively less compressible while the load distribution within a group having a rigid cap becomes less uniform.

Comparisons between theoretical group behaviour and that observed from model and field tests generally show good agreement.

LIST OF SYMBOLS

A_p	Cross-sectional area of pile.
D_{ij}	Pile displacement influence factor (end-bearing pile).
$[D]$	Matrix of factors of type D_{ij}.
E_p	Young's modulus of pile.
E_s	Modulus of elasticity of soil.
E'_s	Modulus of elasticity of soil skeleton.
${}_sI_{ij}$	Displacement influence factor for soil displacement at element i due to uniform stress on element j on pile i.
$[I]$	Unit matrix or order $(n+1)$ by $(n+1)$.
I	Pile displacement factor.
$[{}_sI]$	Matrix of soil displacement influence factors of type $({}_1I_{ij} + {}_2I_{ij})$.
K	Pile stiffness factor $= (E_p/E_s) \, . \, R_A$.
L	Length of pile.
P	Load on pile.
P_{av}	Average pile load in group.
P_G	Total load on pile group.
P_i	Load on pile i in group.
R_A	Area ratio of pile $= A_p/(\pi \, d^2/4)$.
R_G	Group reduction factor.
R_s	Settlement ratio.
S_d	Maximum differential settlement between centre and corner piles of a group with equally-loaded piles.
S_{max}	Maximum settlement within a group with equally loaded piles.
S_R	Settlement of a group of piles with a rigid pile cap.
S_{TF}	Total final settlement.
S_u	Immediate settlement.
$[Y]$	Matrix of constants.
d	Diameter of pile.
f	Constant $= (L/d)/(n \, . \, R_A)$.
h_i	Distance between centre of element i and base of pile.
$\{h\}$	Vector of values of h_i.
i, j, i', j'	Integers.
m	Number of piles in group.
n	Number of pile elements.
p_b	Uniform normal stress on base of pile.
p_j	Uniform shear stress around periphery of pile element j.
$[p]$	Vector of stresses p_j and p_b.
s	Centre-to-centre spacing of adjacent piles in group.
α	Pile interaction factor.
α_{kj}	Pile interaction factor between piles k and j.
δ	Length of pile element $= L/n$.
ρ	Group displacement.
ρ_1	Displacement of a single pile under unit load.
$[{}_p\rho]$	Vector of pile displacements.
$[{}_s\rho]$	Vector of soil displacements.
ν_s	Poisson's ratio of soil.
ν'_s	Poisson's ratio of soil skeleton.

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1.—INTRODUCTION

Theoretical analyses of the behaviour of pile groups have recently been reported by Pichumani and D'Appolonia (Ref. 8) and Poulos (Ref. 9). In the former paper, solutions have been presented for the distribution of load and the displacement of square pile groups, both floating and end-bearing, in a soil which exhibits perfectly elastic-plastic behaviour. In the latter paper, the behaviour of groups of incompressible floating piles at working loads has been analysed and it has been shown that the principle of superposition may be applied to analyse the settlement and load distribution in any general pile group. Solutions have been presented for square pile groups for a wide range of cases, and the effects of group size, pile spacing and length-to-diameter ratio on the behaviour of the groups have been considered. In both papers, use is made of elastic theory, and comparisons between theoretical and observed group behaviour show reasonable agreement.

Neither of the above papers however examines the influence of the pile compressibility on group behaviour, although the method of analysis of Pichumani and D'Appolonia (Ref. 8) is capable of taking this into account. In this paper, a method for analysing the behaviour of groups of compressible piles, both floating and end-bearing, is presented. The method of analysis presented here for general pile groups has some advantage over the general computer method presented by Pichumani and D'Appolonia in that it is amenable to hand calculation. Also, in the present analysis, the shear stresses acting on the pile surface are approximated by a series of uniformly distributed loads acting around the surface of the pile whereas the analysis of Pichumani and D'Appolonia assumes a series of point loads acting along the pile axis. The former approximation has been shown to be more accurate by Poulos and Davis (Ref. 10).

Theoretical solutions for the settlement and load distribution in compressible pile groups are presented, with emphasis placed on examining the influence on group behaviour of the relative compressibility of the piles. Comparisons are then made between theoretical and observed settlement characteristics to determine the applicability of the theoretical approach to real problems. The use of the theory in practical problems is subsequently discussed.

The work in this paper is an extension of the work described by Poulos (Ref. 9) for incompressible floating groups, Mattes and Poulos (Ref. 7) for a single compressible floating pile, and by Poulos and Mattes (Ref. 11) for a single end-bearing pile. Attention is confined to the case of floating piles in a semi-infinite elastic mass and end-bearing piles resting on a rigid bearing stratum. All groups are assumed to be free-standing, i.e., there is assumed to be no contact between the soil and the pile cap. No account is taken of the effects of yield within the soil, although the analysis could be modified to take this aspect of soil behaviour into account (see for example, Pichumani and D'Appolonia, Ref. 8). However, from the analysis of a single compressible pile by Mattes and Poulos (Ref. 7), it would appear likely that the effects of local yield within the soil at normal working loads are unlikely to be significant unless the piles are very compressible.

2.—ANALYSIS OF A GROUP OF TWO PILES

2.1 Floating Piles:

Two equally-loaded, identical, cylindrical piles in an ideal semi-infinite mass having constant elastic parameters E_s and ν_s are considered, as shown in Fig. 1. The piles are of length L, diameter d and cross-sectional area A_p, and are spaced at a centre-to-centre distance s. The elastic modulus of the pile material is E_p, and each pile is divided into n equal cylindrical elements, any element j being loaded by a uniform vertical

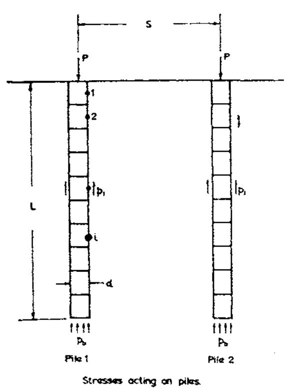

Fig. 1.—Group of Two Floating Piles.

shear stress p_j acting around the periphery of the element, and a circular base acted upon by a uniform vertical stress p_b. Since elastic conditions are assumed at the pile-soil interface, the displacements of the pile and the soil at each element are equated in order to solve for the n unknown stresses p_j, the base stress p_b, and the corresponding displacement distribution along the pile.

It has been shown by Poulos (Ref. 9) that the soil displacements at the element centres along the pile may be written as

$$[{}_s\rho] = \frac{d}{E_s}[{}_sI] \, . \, [p] \qquad \ldots\ldots(1)$$

where $[{}_sI]$ is the $n+1$ by $n+1$ matrix of soil displacement influence factors, any element ${}_sI_{ij}$ representing the displacement influence at point i of stress on element j of pile 1 and pile 2, and $[p]$ and $[{}_s\rho]$ are $(n+1)$ stress and soil displacement vectors. All elements of $[{}_sI]$ are obtained by double integration of the Mindlin equation as described by Poulos and Davis (Ref. 10).

Because the condition of no slip between pile and soil has been imposed, Eq. (1) can be combined with the equation obtained by Mattes and Poulos (Ref. 7) relating pile deformations to the stresses acting over the surface of the pile to give the equation

$$[p] = \left[[I] - \frac{n^2}{4(L/d)^2} \, . \, K \, . \, [{}_pI] \, . \, [{}_sI] \right]^{-1} . \, [Y] \qquad \ldots\ldots(2)$$

where $[I]$ is the unit matrix of order $n+1$ by $n+1$,

$K = (E_p/E_s) \, . \, R_A$, the pile stiffness factor, which is a measure of the compressibility of the pile relative to the soil (the smaller the value of K, the more compressible is the pile),

$R_A = A_p/(\pi \, d^2/4)$ = ratio area (for solid piles, $R_A = 1$),

$[{}_pI]$ is an $(n+1)$ by $(n+1)$ matrix of coefficients of pile action,

$[Y]$ is an $(n+1)$ column vector.

Eq. (2) can be solved with the equilibrium equation,

$$P = \frac{\pi \, dL}{n} \sum_{j=1}^{n} p_j + \frac{\pi \, d^2}{4} \, . \, p_b \qquad \ldots\ldots(3)$$

to obtain the unknown stresses acting on the pile, whence the displacement distribution may be determined from Eq. (1). The range of values of the pile stiffness factor K likely to occur in practical problems is discussed in Section 7 of this paper.

2.2 End-Bearing Piles:

As shown in Fig. 2, two equally-loaded identical cylindrical piles of diameter d, length L, cross-sectional area A_p and material elastic modulus E_p, resting on a perfectly rigid base, are considered. Each pile is divided into n equal cylindrical elements while the soil layer is again assumed to be an ideal elastic material with constant elastic parameters E_s and ν_s.

The soil displacement at each element may be calculated as for the floating piles, except that an allowance must be made for the influence of the rigid base on the soil displacements. An approximate method whereby this may conveniently be done has been suggested by D'Appolonia and Romualdi (Ref. 2) in which a mirror-image element j' of element j, loaded by an equal and opposite shear stress, is introduced. This method has been used by Poulos and Mattes (Ref. 11) in dealing with the behaviour of single end-bearing piles. Taking downwards displacements as positive,

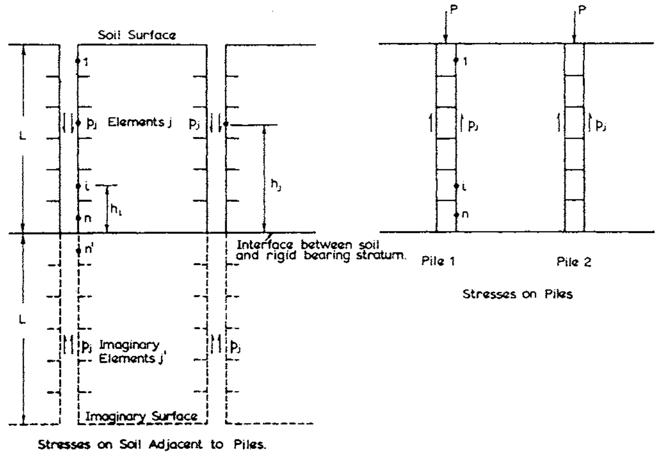

Fig. 2.—Group of Two End-Bearing Piles.

and referring to Fig. 2, the soil displacements along either pile may again be expressed by Eq. (1), where $[{}_sI]$ is now the n by n matrix of soil displacement influence factors, any element ${}_sI_{ij}$ being the influence factor for displacement at i due to the uniform shear stress on elements j of pile 1 and pile 2, and on elements j' of the imaginary mirror-image piles 1 and 2. The elements of $[{}_sI]$ are again obtained by double integration of the Mindlin equation.

Since no slip is allowed between piles and soil, the soil displacement equation may be combined with the pile action equation obtained by Poulos and Mattes (Ref. 11) to give the following equation:

$$\left[[{}_sI] + \frac{1}{Kd} \, . \, [D] \right] . \, [p] = \frac{4P}{\pi \, d^3 \, K}[h] \qquad \ldots\ldots(4)$$

where K is the pile stiffness factor, defined in Section 2.1,

$[D]$ is an n by n matrix of coefficients related to the pile geometry,

$[h]$ is the vector of distances from the rigid base to the centre of pile elements.

Eq. (4) may be solved for the unknown stresses p on each pile whereby the displacements can be calculated. The load transferred to the pile tip is calculated as the difference between the applied load P and the sum of the shear forces acting along the pile surface.

3.—SOLUTIONS FOR TWO-PILE GROUPS

It is convenient to define the additional displacement at the top of a pile due to an equally loaded identical adjacent pile in terms of an interaction factor α (Ref. 9) where

$$\alpha = \frac{\text{additional displacement due to adjacent pile}}{\text{displacement of pile under its own load}} \qquad \ldots\ldots(5)$$

TABLE I
Interaction Factors α for 2-Pile Groups

L/d		10				25				100			
	K	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞
	s/d												
Floating Group	0	0.667	0.794	0.968	1.000	0.722	0.618	0.880	1.000	0.903	0.669	0.590	1.000
	2	0.313	0.454	0.548	0.572	0.368	0.407	0.579	0.638	0.557	0.471	0.450	0.739
	5	0.152	0.263	0.323	0.333	0.177	0.264	0.396	0.439	0.324	0.320	0.350	0.568
	10	0.078	0.144	0.180	0.188	0.092	0.164	0.265	0.293	0.176	0.204	0.268	0.440
	25	0.030	0.055	0.069	0.071	0.036	0.071	0.123	0.134	0.069	0.093	0.162	0.283
End-Bearing Group	0	0.554	0.158	0.020	0	0.702	0.388	0.079	0	0.901	0.656	0.397	0
	2	0.212	0.084	0.011	0	0.336	0.245	0.056	0	0.548	0.454	0.304	0
	5	0.062	0.030	0.004	0	0.140	0.134	0.035	0	0.312	0.299	0.230	0
	10	0.009	0.004	0.001	0	0.054	0.060	0.017	0	0.161	0.181	0.165	0
	25	0	0	0	0	0.006	0.006	0.002	0	0.052	0.068	0.080	0

Interaction curves showing the relationship between α and the dimensionless centre-to-centre spacing s/d between the piles may be obtained by solving Eqs. (2) and (3) and Eq. (5) for various values of L/d and K. Typical interaction curves are presented in Appendix A: in all cases the Poisson's ratio of the soil, ν_s, has been taken as 0.5, and the equations have been solved using $n = 10$ elements, this number having been found to give results of sufficient accuracy for practical purposes.

The interaction curves are summarised in Table I, and an inspection of this table reveals several trends in group behaviour. For both floating and end-bearing groups interaction decreases as spacing increases, but for relatively compressible floating piles interaction is still quite significant at relatively large spacings.

For floating piles, the general trend is for interaction to increase as pile stiffness increases; only for slender, very compressible piles (e.g., $K = 10$) at close spacings is the trend reversed.

For end-bearing piles, interaction decreases as L/d decreases since more load is then transmitted to the bearing stratum. In contrast to the floating piles, interaction generally decreases as stiffness increases. In most cases, the interaction between end-bearing piles is much less than for the corresponding floating piles.

In comparison with the effects of K and L/d the effect of ν_s on interaction is found to be relatively slight, and thus attention is confined to the case $\nu_s = 0.5$.

Although the solutions for floating piles are only for the case of piles in a semi-infinite mass, the solutions obtained for a single pile in a finite layer by Mattes and Poulos (Ref. 7) suggest that the finite layer will only significantly influence group behaviour for relatively incompressible piles; in such cases, use may be made of the interaction factors obtained by Poulos (Ref. 9) for incompressible piles in a finite layer.

For end-bearing piles, the influence of a bearing stratum of finite compressibility may be inferred from the solutions for a single end-bearing pile obtained by Poulos and Mattes (Ref. 11). For ratios of modulus of bearing stratum E_b to soil modulus E_s greater than 100, the bearing stratum may be considered as rigid. For smaller values of E_b/E_s, only relatively incompressible short piles are markedly influenced by the base compressibility; in such cases, an approximate, but conservative, estimate of the interaction factor may be obtained by considering the value of α for the corresponding floating pile. In all other cases involving relatively slender or relatively compressible piles, the values of α for an end-bearing pile on a rigid base may be used with sufficient accuracy.

4.—ANALYSIS OF GENERAL PILE GROUPS

As described by Poulos (Ref. 9) the analysis for two piles may be extended to any number of piles, provided that all piles in the group behave identically, i.e., that the piles are symmetrically spaced around the circumference of a circle. It has also been found that for incompressible pile groups, the displacement of the group from such an analysis is almost identical with that obtained by applying the principle of superposition, i.e., by calculating the displacement increase of a pile in the group as the sum of the displacement increases due to all the adjacent piles considered in turn. For example, for a square group of four piles at a spacing of s diameters, the displacement is given, by superposition, as

$$\rho = P_1 \rho_1 (1 + 2\alpha_1 + \alpha_2) \qquad \ldots\ldots(6)$$

where P_1 is the load in each pile,

ρ_1 is the displacement of a single pile under unit load,

α_1 is the value of the interaction factor α for two piles at a spacing of s diameters,

α_2 is the value of α for two piles at a spacing of $s\sqrt{2}$ diameters.

Calculations have shown that the principle of superposition also applies closely to the top displacement of compressible floating and end-bearing piles. For example, the correct and approximate displacements for a three pile floating group generally agree to within 2%, while for the corresponding end-bearing group, the correct and approximate displacements agree to within 10%, the larger errors being associated with high values of K and small spacings. In view of the generally small amount of interaction between piles in the latter case, the accuracy of the method of superposition appears to be quite satisfactory for practical purposes.

As with the incompressible groups considered previously by Poulos (Ref. 9), the applicability of the principle of superposition to symmetrically-spaced groups suggests that it may be employed to analyse general pile groups. Thus, for a group of m piles, the displacement of any pile k in the group is

$$\rho_k = \rho_1 \sum_{j \neq k}^{m} P_j \,.\, \alpha_{kj} + \rho_1 P_k \qquad \ldots\ldots(7)$$

where α_{kj} is the value of α for two piles corresponding to the spacing between pile k and pile j,

P_j is the load in pile j,

ρ_1 is the displacement of a single pile under unit load.

TABLE II

Floating Pile Groups, Rigid Cap. Group Reduction Factor, R_G

$\nu_s = 0.5$

Group Size		2^2				3^2				4^2				5^2			
L/d	K	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞
	s/d																
10	2	0.457	0.562	0.636	0.654	0.309	0.422	0.491	0.498	0.235	0.343	0.400	0.408	0.190	0.288	0.339	0.347
	5	0.351	0.432	0.471	0.476	0.203	0.277	0.313	0.317	0.141	0.203	0.234	0.239	0.107	0.159	0.188	0.190
	10	0.302	0.347	0.370	0.374	0.158	0.196	0.219	0.221	0.102	0.134	0.154	0.154	0.074	0.101	0.118	0.118
	20	0.276	0.295	0.310	0.310	0.135	0.152	0.166	0.166	0.083	0.097	0.109	0.109	0.056	0.069	0.079	0.079
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040
25	2	0.497	0.534	0.662	0.718	0.334	0.404	0.538	0.588	0.264	0.336	0.465	0.506	0.216	0.290	0.411	0.450
	5	0.367	0.436	0.532	0.547	0.220	0.290	0.387	0.415	0.154	0.221	0.310	0.334	0.118	0.179	0.260	0.281
	10	0.313	0.365	0.435	0.445	0.166	0.217	0.286	0.303	0.109	0.154	0.214	0.227	0.079	0.119	0.171	0.180
	20	0.278	0.313	0.358	0.366	0.137	0.167	0.209	0.214	0.084	0.110	0.147	0.148	0.059	0.080	0.112	0.112
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040
100	2	0.640	0.578	0.565	0.789	0.492	0.450	0.457	0.683	0.401	0.384	0.406	0.620	0.339	0.336	0.370	0.574
	5	0.471	0.470	0.502	0.661	0.311	0.327	0.375	0.541	0.234	0.253	0.311	0.471	0.187	0.207	0.270	0.422
	10	0.368	0.391	0.441	0.569	0.217	0.241	0.303	0.437	0.153	0.175	0.238	0.364	0.118	0.139	0.200	0.315
	20	0.310	0.327	0.380	0.481	0.165	0.183	0.239	0.338	0.109	0.125	0.179	0.270	0.080	0.094	0.144	0.228
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040

Also, from equilibrium, if the total load on the pile group is P_G, then

$$P_G = \sum_{j=1}^{m} P_j \quad \text{.................................(8)}$$

Eqs. (7) and (8) may be solved for two limiting conditions:

(*a*) equal load in all piles,

(*b*) equal displacement of all piles, corresponding to a perfectly rigid pile cap.

The displacement of the group so calculated may be expressed either in terms of the settlement ratio R_s of the group, where R_s is the ratio of the settlement of the group to the settlement of a single pile carrying the same *average* load as a pile in the group, or the group reduction factor R_G, where R_G is the ratio of the group settlement to the settlement of a single pile carrying the same *total* load as the group. For a group of m piles, R_s is related to R_G as follows:

$$R_s = m \,.\, R_G \quad \text{.............................(9)}$$

It should be noted that for a group in which all piles carry equal load, different values of R_s and R_G may be appropriate for different piles in the group. In practical problems, R_s is likely to be the more useful quantity, but in examining the theoretical behaviour of pile groups under elastic conditions R_G has some advantage as it always lies within the range 1 to $1/m$. Consequently in the solutions described in the following section, displacements are expressed in terms of R_G. It may be helpful to note that R_G in fact represents the settlement of the group if the settlement of the corresponding single pile is unity—thus R_G is a direct measure of the settlement of a group under a given load.

To illustrate the use of the method of superposition for calculating the settlement of a pile group, an example is given in Appendix C. Solutions obtained for square groups of piles are described below.

5.—TYPICAL SOLUTIONS FOR SQUARE PILE GROUPS

Solutions have been obtained for square groups of 4, 9, 16 and 25 piles, denoted subsequently as 2^2, 3^2, 4^2 and 5^2 groups. Both floating and end-bearing piles have been considered and in each case the settlement distribution within a group in which all the piles are equally loaded, and the load distribution and settlement of a group with a rigid cap, have been investigated.

5.1 Floating Pile Groups:

Group reduction factors R_G for typical floating pile groups with rigid caps are given in Table II. For all groups, R_G decreases as the pile spacing increases, and in almost every case, R_G decreases as pile stiffness K decreases, the only exception being groups of slender, very compressible piles at very close spacings.

For a given spacing, R_G decreases as the number of piles in the group increases. If however R_G is plotted against total group breadth, as in Fig. 3, the influence of the number of piles in the group is much smaller, and for larger groups R_G does not vary greatly with the number of piles in the group. For groups containing more than 25 piles it appears that, for a practical range of group breadths, a common limiting curve of R_G versus breadth, almost coincident with the curve for the 5^2 group, is obtained. The dependence of R_G on group breadth rather than on the actual number of piles has also been noted by Poulos (Ref. 9) for incompressible groups.

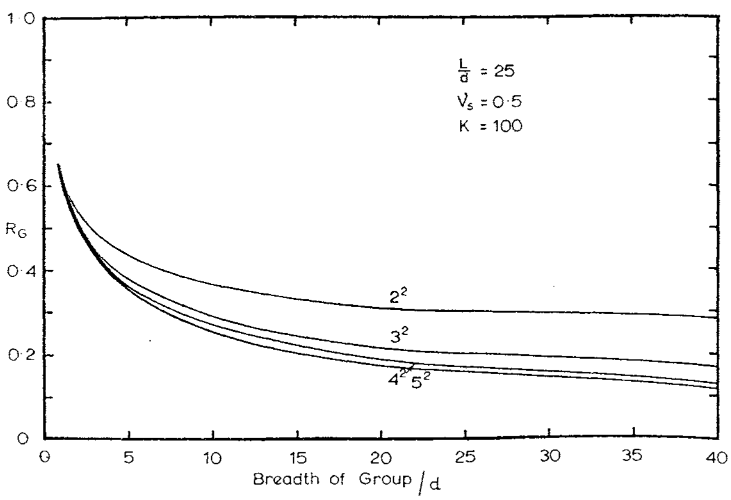

Fig. 3.—R_G versus Group Breadth—Floating Groups.

For a pile group in an ideal two-phase elastic soil mass, it is possible to calculate the ratio of immediate to total final settlement. S_u/S_{TF}, as described by Poulos and Davis (Ref. 10). It is found that for compressible groups the major portion of total final settlement occurs as immediate settlement and that pile compressibility has little influence on the value of S_u/S_{TF}.

Typical load distributions within a floating pile group with a rigid cap are plotted against pile spacing in Fig. 4 for a 3^2 group, for $K = 100$, 1000 and ∞, the pile load being expressed as a fraction of the average pile load in the group. The outer piles take the greatest load while the inner pile takes the least and as with incompressible groups, the non-uniformity of the load distribution becomes more pronounced as L/d increases and the number of piles in the group increases. Table B1 in Appendix B gives load distributions within a floating pile group for a wide range of L/d, s/d, K and group size.

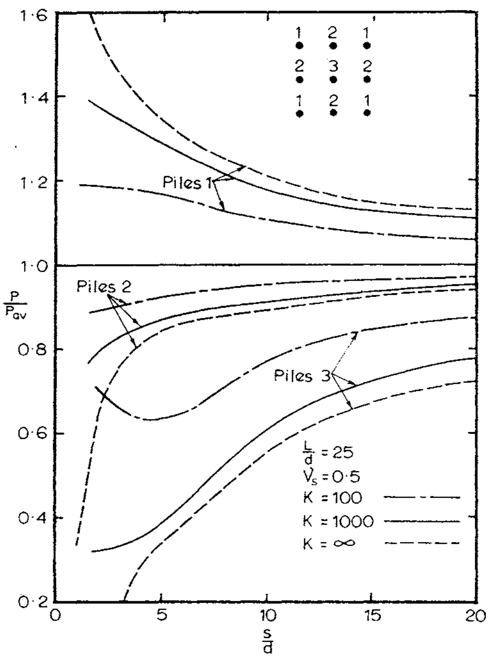

Fig. 4.—Load Distributions in 3^2 Floating Group.

For groups in which all piles are equally loaded, the maximum settlement occurs at the centre pile or piles, and the minimum at the corner piles. It is found that the ratio of the maximum settlement S_{max} of a group to the uniform settlement S_R of the corresponding group with a rigid pile cap is almost independent of pile compressibility, so that values of this ratio tabulated by Poulos (Ref. 9) for incompressible groups may be used for all values of K.

The ratio of the maximum differential settlement S_d between the centre and corner piles of the group, to the settlement S_R of the corresponding group with a rigid cap, is tabulated in Table III, for 3^2 and 5^2 groups with L/d 10, 25 and 100. Although the values in Table III are for a pile stiffness factor K of 1,000, it has been found that pile compressibility has little effect on S_d/S_R, so that the factors in Table III can be used with sufficient accuracy for all values of K.

TABLE III

Differential Settlement of Floating Pile Group

Values of S_d/S_R

L/d	10		25		100	
Group size	3^2	5^2	3^2	5^2	3^2	5^2
s/d						
2	0.155	0.310	0.110	0.208	0.045	0.117
5	0.204	0.340	0.163	0.286	0.108	0.206
10	0.170	0.280	0.170	0.312	0.130	0.226
20	0.111	0.230	0.153	0.277	0.121	0.225

5.2 End-Bearing Groups :

Group reduction factors for end-bearing pile groups with rigid caps are given in Table IV for a wide range of L/d, s/d, group size and K. In general, R_G decreases as pile stiffness increases. The smaller the value of L/d the closer the spacing at which R_G reaches its limiting value of $1/m$ (i.e., no interaction between the piles). As with floating groups, R_G decreases as the number of piles in the group increases, but if R_G is plotted against total group breadth, as in Fig. 5, it will be seen that again there appears to be a limiting curve as the number of piles in the group increases.

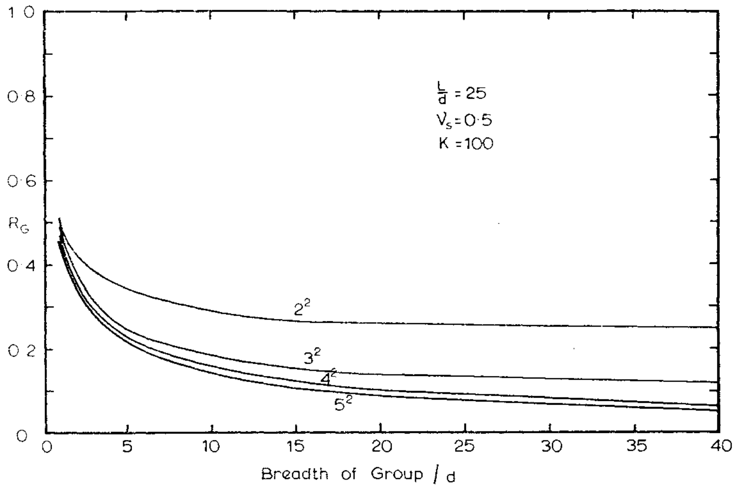

Fig. 5.—R_G versus Group Breadth—End-Bearing Groups.

As similarly found by Poulos and Mattes (Ref. 11) for a single end-bearing pile, the theoretical ratio S_u/S_{TF} of the immediate to total final settlement for groups of end-bearing piles is close to unity; for almost all cases likely to be met in practice, the time-dependent movement is less than 10% of the total final movement.

Typical load distributions within a 3^2 end-bearing group are shown in Fig. 6. As with the floating group, the non-uniformity in the load distribution increases as L/d increases, but in contrast to the corresponding floating groups, the load distribution generally becomes more uniform as pile stiffness increases. A wide range of load distributions is tabulated in Table B2 in Appendix B.

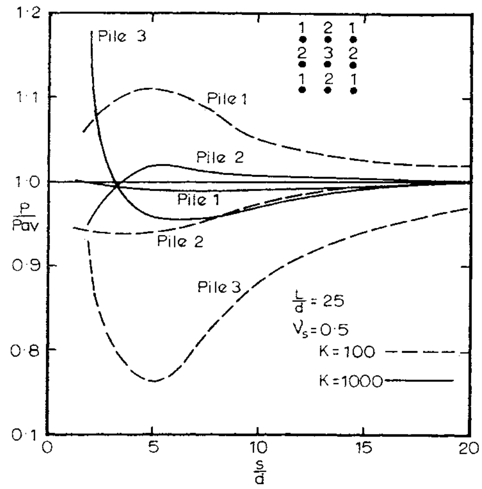

Fig. 6.—Load Distributions in 3^2 End-Bearing Group.

For end-bearing groups in which all piles are equally loaded, the maximum displacement generally occurs at the centre and the minimum at the corner. The ratios S_{max}/S_R and S_a/S_R both decrease as pile stiffness increases, and approach their limiting values of 1 and 0 respectively at relatively close pile spacings. Values of S_a/S_R are given in Table V for relatively compressible piles ($K = 100$).

6.—COMPARISONS BETWEEN THEORETICAL AND OBSERVED GROUP BEHAVIOUR

In order to assess the applicability of the theoretical approach to field cases, it is desirable to compare the theoretical solutions with the published results of observations on model and full-scale pile groups. Such a comparison was made by Poulos (Ref. 9), but no distinction was made between piles having different values of K, all piles being assumed to be incompressible. However, it is possible in some cases to estimate values of K

TABLE IV

End-Bearing Pile Groups, Rigid Cap. Group Reduction Factor, R_G

$\nu_s = 0.5$

Group Size		2^2				3^2				4^2				5^2			
L/d	K / s/d	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞
10	2	0.379	0.285	0.250	0.250	0.224	0.146	0.111	0.111	0.149	0.093	0.062	0.062	0.108	0.065	0.040	0.040
	5	0.287	0.270	0.252	0.250	0.137	0.124	0.113	0.111	0.081	0.071	0.063	0.062	0.053	0.046	0.041	0.040
	10	0.256	0.252	0.250	0.250	0.115	0.113	0.111	0.111	0.065	0.063	0.062	0.062	0.041	0.041	0.040	0.040
	20	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040
25	2	0.471	0.405	0.263	0.250	0.316	0.286	0.129	0.111	0.231	0.205	0.083	0.062	0.179	0.165	0.060	0.040
	5	0.341	0.341	0.271	0.250	0.186	0.189	0.129	0.111	0.121	0.125	0.077	0.062	0.086	0.089	0.051	0.040
	10	0.284	0.288	0.259	0.250	0.137	0.140	0.118	0.111	0.081	0.083	0.067	0.062	0.053	0.055	0.043	0.040
	20	0.259	0.259	0.251	0.250	0.116	0.117	0.112	0.111	0.066	0.066	0.063	0.062	0.042	0.042	0.040	0.040
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040
100	2	0.636	0.565	0.452	0.250	0.489	0.439	0.338	0.111	0.390	0.368	0.288	0.062	0.327	0.317	0.256	0.040
	5	0.463	0.459	0.418	0.250	0.301	0.308	0.280	0.111	0.221	0.234	0.216	0.062	0.173	0.187	0.178	0.040
	10	0.359	0.373	0.365	0.250	0.204	0.221	0.220	0.111	0.138	0.155	0.158	0.062	0.101	0.119	0.124	0.040
	20	0.296	0.310	0.317	0.250	0.149	0.165	0.173	0.111	0.093	0.107	0.115	0.062	0.066	0.078	0.085	0.040
	∞	0.250	0.250	0.250	0.250	0.111	0.111	0.111	0.111	0.062	0.062	0.062	0.062	0.040	0.040	0.040	0.040

TABLE V
Differential Settlement of End-Bearing Pile Group
Values of S_d/S_R
$K = 100$

L/d	10		25		100	
Group size	3^2	5^2	3^2	5^2	3^2	5^2
s/d						
2	0.103	0.197	0.059	0.254	0.096	0.236
5	0.082	0.150	0.181	0.370	0.179	0.336
10	0.001	0.001	0.129	0.240	0.181	0.296
20	0	0	0.041	0.043	0.113	0.236

from the published data, and a summary of these cases is given in Table VI. In all cases, floating pile groups are considered. Comparisons between observed settlement ratios for square pile groups and those calculated theoretically are shown in Fig. 7. In all cases, the load level corresponds to a factor of safety of at least 2 against overall failure. It will be seen that with the exception of the tests by Hanna (Ref. 5) in loose sand, the general agreement is quite satisfactory for both large and small values of K.

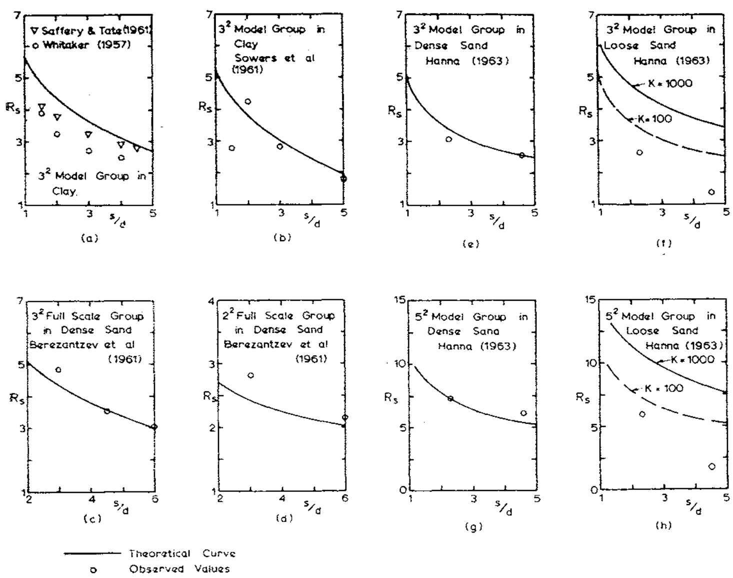

Fig. 7.—Comparison between Theoretical and Observed Settlement Ratios.

The comparisons between theoretical and observed values of R_S for the tests carried out by Hanna (Ref. 5) on wooden piles in initially loose sand (Figs. 7 (*f*) and 7 (*h*)), were made using a value of K estimated from the value of E_s for loose sand. It was found that the observed values were much less than the theoretical and this discrepancy is probably due to a greater degree of densification occurring near the group than near the single piles. However, if the value of K corresponding to dense sand is used for the theoretical values of R_S, the agreement is closer, although R_S is still over-estimated, especially at larger spacings. The less satisfactory agreement for groups in loose sand is not surprising since the local densification around the pile groups leads to inhomogeneity of the sand mass, the extent of this inhomogeneity varying as the pile spacing varies. Because of this inhomogeneity, the sand departs considerably from the ideal homogeneous elastic soil assumed in the analysis.

Comparisons between observed and theoretical load distributions within pile groups have been made by Poulos (Ref. 9). In the cases considered, the piles were effectively incompressible and the agreement between observed and theoretical distributions was found to be close. To the authors' knowledge, no further cases involving relatively low values of K are readily available.

The comparisons shown in Fig. 7, and those obtained previously by Poulos (Ref. 9) suggest that the theoretical approach described in this paper is capable of predicting with reasonable accuracy the settlement ratio and pile load distribution at working loads in practical problems. Some caution however appears desirable in applying the theory to pile groups in initially loose sand.

While no data is available on groups of end-bearing piles, the above comparisons, together with those made by Poulos and Mattes (Ref. 11) for a single end-bearing pile, suggest that a similar measure of agreement between theory and observation may be expected for end-bearing groups. A more reliable assessment of the applicability of the theory must however await results of further field measurements.

7.—APPLICATION OF THEORETICAL SOLUTIONS TO PRACTICAL PROBLEMS

In order to use the preceding theoretical solutions to estimate the immediate and total final settlement of a pile group, it is necessary to determine representative values of the 'elastic' parameters of the soil. The immediate settlement is given by putting E_s equal to the undrained Young's modulus E_u of the soil and using the displacement influence factors and interaction factors for the undrained value of Poisson's ratio ν_u, which is 0.5 for saturated clays. The total final settlement is given by putting E_s equal E'_s, the Young's modulus of the soil skeleton, and using the displacement influence factors and interaction factors for $\nu_s = \nu'_s$, Poisson's ratio of the soil skeleton. Although it is possible to carry out triaxial tests in order to determine representative values of E_u, E'_s and ν'_s (Refs. 3, 4 and 6), the initial and final stress states in the soil near a loaded pile are difficult to estimate, and consequently, the reliability of soil parameters determined in laboratory tests must at present be considered dubious. It would appear that a more satisfactory means of obtaining values of E_u, E'_s and ν'_s is to carry out an actual loading test on a single pile, the results of which may be fitted to the theoretical solutions for the settlement of a single pile to determine the required parameters. These parameters may then be used for calculating the immediate and total final settlement of the pile group.

In order to estimate the settlement of the group on a theoretical basis, it is necessary to use the influence factors for the top displacement of a single compressible pile. These have been presented by Poulos and Mattes (Ref. 11) for end-bearing piles and Mattes and Poulos (Ref. 7) for floating piles.

TABLE VI
Summary of Data on Floating Pile Group Tests

Test	Pile Material	Soil Type	Assumed Parameters for Comparison			Remarks
			L/d	K	Layer Depth/L	
Whitaker (Ref. 14)	Brass	Remoulded London Clay	24	∞	2	Model tests
Saffery and Tate (Ref. 12)	Stainless Steel	Remoulded Clay	20	∞	2	Model tests
Sowers et al (Ref. 13)	Aluminium Tube	Remoulded Bentonite	24	2000	2	Model tests
Berezantzev et al (Ref. 1)	Concrete	Dense Sand	20	1000	∞	Field tests. K estimated from quoted values of E_s.
Hanna (Ref. 5)	Wood	Dense Sand	33	100	2	Model tests. K estimated from Table IX.
Hanna (Ref. 5)	Wood	Loose Sand	33	1000	2	„ „ „ „

For convenience, these influence factors are summarized in Tables VII and VIII for $\nu_s = 0.5$. In both cases, the variation of the influence factors with ν_s is relatively small so that, for practical purposes, the values in Tables II and III may be used for all values of ν_s.

TABLE VII
Displacement Influence Factors for a Single Floating Pile in a Semi-Infinite Mass
$\nu_s = 0.5$
Top Displacement $\rho = I \, . \, (P/LE_s)$
Values of I

L/d / K	10	25	100
10	3.26	6.78	14.45
50	2.05	4.46	12.00
100	1.80	3.58	10.61
500	1.49	2.34	6.60
1000	1.44	2.10	5.18
5000	1.41	1.90	3.26
∞	1.40	1.89	2.70

TABLE VIII
Displacement Influence Factors for a Single End-Bearing Pile Resting on a Rigid Stratum
$\nu_s = 0.5$
Top Displacement $\rho = I \, . \, (PL/E_p A_p)$

L/d / K	10	25	100
10	0.24	0.08	0.01
50	0.57	0.26	0.05
100	0.72	0.38	0.08
500	0.92	0.71	0.26
1000	0.96	0.81	0.38
5000	0.99	0.92	0.72
∞	1.00	1.00	1.00

It is also necessary to estimate a value of the pile stiffness factors K in order to use the theoretical solution. Since the value of E_s of the soil will change from E_u to E'_s as dissipation of excess pore pressures occurs around the pile, the value of K will also change during the consolidation process. However, this change in K is likely to be relatively small and, in view of the uncertainties involved in obtaining values of E_s, may be ignored. As a guide for practical problems, typical values of K for solid steel, concrete and timber piles in various types of soil were suggested by Poulos and Mattes (Ref. 11) and are reproduced in Table IX. It will be seen that, for solid piles, low values of K occur only with concrete or timber piles in stiff clays and dense sands. However, low values of K can also occur with steel pipe piles which typically have an area ratio R_A of 0.1 to 0.2, and for compacted sand piles, for which K may be as low as about 10. For such cases, the compressibility of the pile may significantly influence the behaviour of the pile group, especially if the piles are slender.

It must be borne in mind that the theoretical solutions presented in this paper are strictly only applicable to uniform soil deposits, and that various practical aspects are not taken into account by the present theory, for example, the influence of the method of pile installation, non-homogeneity and anisotropy of the soil, and layering of the soil profile. The application of the theoretical solutions to such field situations therefore requires a certain amount of engineering judgment in order to successfully reduce the actual problem to an idealized one for the purposes of analysis. It should also be reiterated that the piles are assumed free-standing and no

TABLE IX
Average Values of K for Solid Piles*
(after Poulos and Mattes, Ref. 11)

Soil Type	Pile Material		
	Steel	Concrete	Timber
Soft Clay	60,000	6,000	3,000
Medium Clay	20,000	2,000	1,000
Stiff Clay	3,000	300	150
Loose Sand	15,000	1,500	750
Dense Sand	1,500	150	75

*For piles which are not solid, the above K values should be multiplied by the area ratio R_A.

account is taken of raft action of the pile cap. However, preliminary analyses have revealed that, at normal pile spacings, the influence of the pile cap resting on the surface on the behaviour of the group at working loads is relatively minor, although at loads approaching the ultimate, the pile cap may largely dictate the behaviour of the group. Another important factor not taken into account in the present analysis is the possibility of increased settlements due to deep-seated compressible layers situated beneath a floating pile group. A method for calculating such settlements will be presented in a subsequent paper.

8.—CONCLUSIONS

The displacement and load distribution within groups of compressible floating end-bearing piles has been analysed by employing the method of superposition as previously applied to incompressible floating pile groups. The influence of the relative compressibility of the piles on the behaviour of groups has been examined in relation to square groups of piles, and it has been found that for floating pile groups, the group reduction factor R_G decreases as the pile stiffness factor K decreases. For a group with a rigid pile cap, the load distribution within the group becomes more uniform as K decreases.

For end-bearing groups, the interaction between the piles in the group increases as the piles become more compressible, and consequently, the group reduction factor R_G increases as K decreases, while the load distribution within a group with a rigid pile cap becomes less uniform.

As with incompressible floating pile groups, it is found that the group settlement is dependent more on the total breadth of the group than on the number of piles in the group, and that the major proportion of the total final settlement occurs immediately for both floating and end-bearing groups.

Comparisons between theoretical values of settlement ratios and values observed in model and field tests on floating pile groups show generally good agreement and suggest that, with the possible exception of pile groups in loose sand, the theory is capable of predicting with reasonable accuracy the settlement ratio and pile load distribution at working loads in practical problems.

ACKNOWLEDGMENTS

The work described in this paper forms part of a general programme of research into the settlement of all types of foundations, being carried out at the University of Sydney under the general direction of Professor E. H. Davis, Professor of Civil Engineering (Soil Mechanics). The work is supported by a research grant from the Australian Research Grants Committee, and by the award of a Commonwealth Research Scholarship to the second author. The computational work was carried out using the facilities of the Basser Computing Department of the School of Physics at Sydney University.

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APPENDIX A

Interaction Curves for Floating and End-Bearing Two-Pile Groups

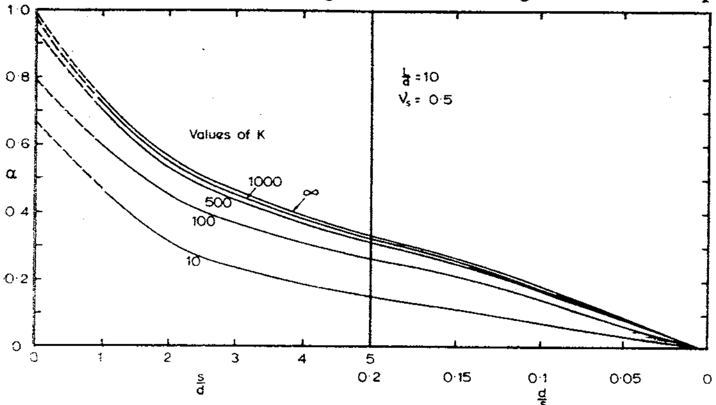

FIG.A1 INTERACTION FACTORS FOR FLOATING PILES L/d = 10.

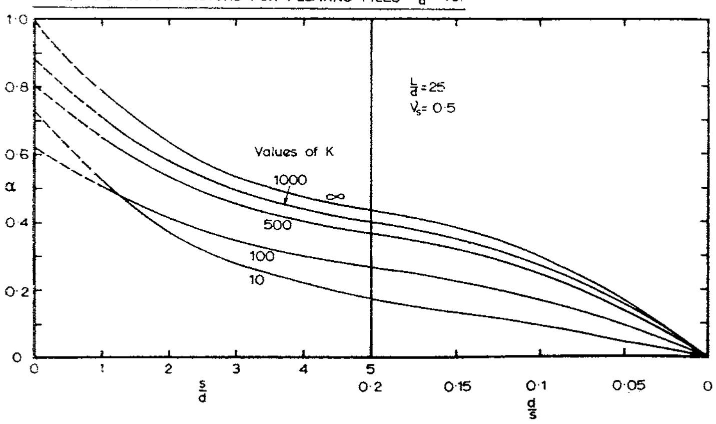

FIG.A2 INTERACTION FACTORS FOR FLOATING PILES L/d = 25

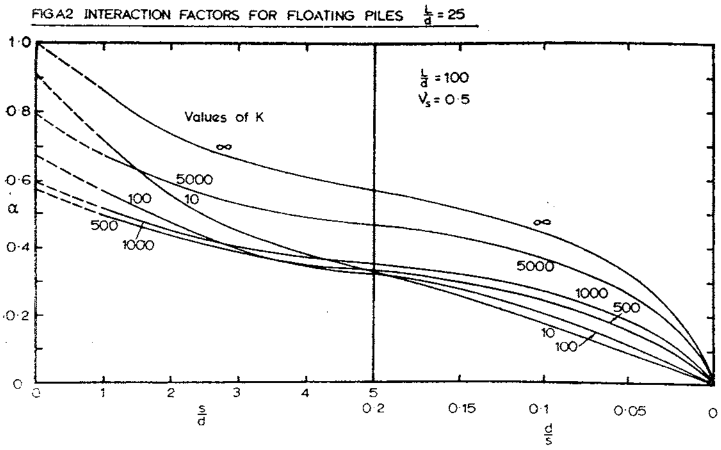

FIG.A3 INTERACTION FACTORS FOR FLOATING PILES L/d = 100

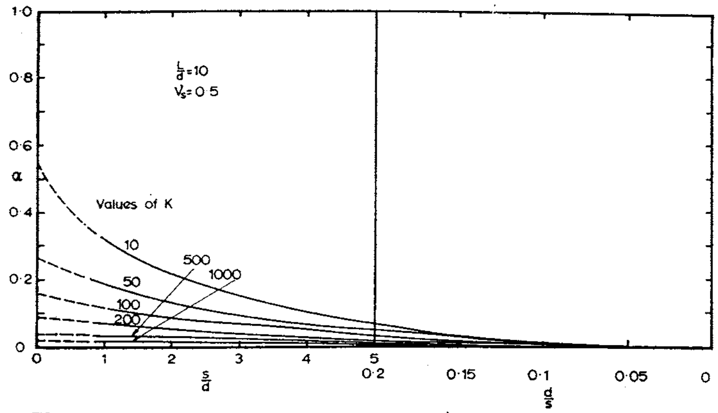

FIG.A4 INTERACTION FACTORS FOR END-BEARING PILES L/d = 10

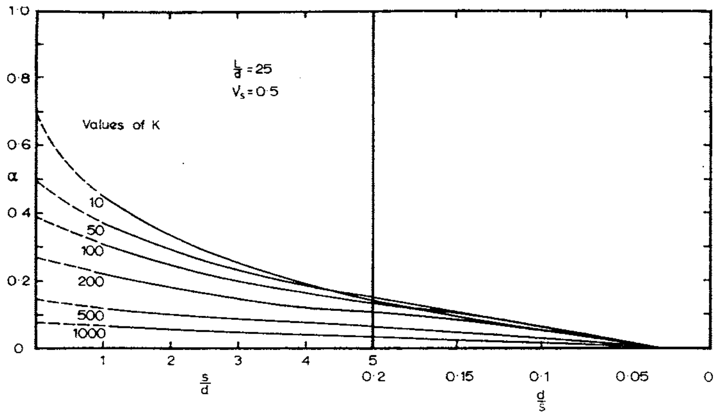

FIG.A5 INTERACTION FACTORS FOR END-BEARING PILES L/d = 25

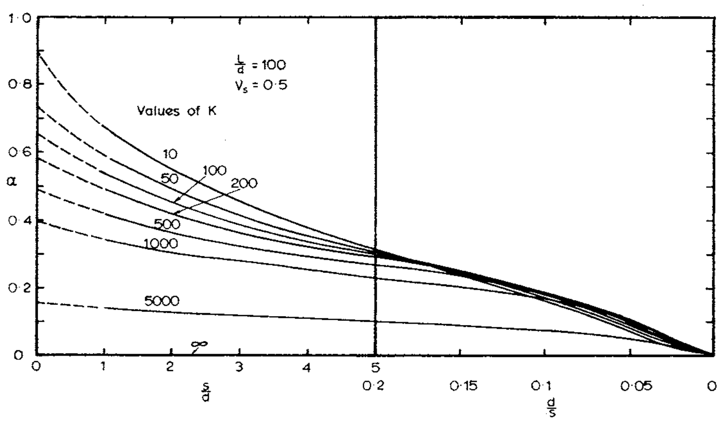

FIG.A6 INTERACTION FACTORS FOR END-BEARING PILES L/d = 100

APPENDIX B

Load Distributions within Floating and End-Bearing Groups with Rigid Pile Caps

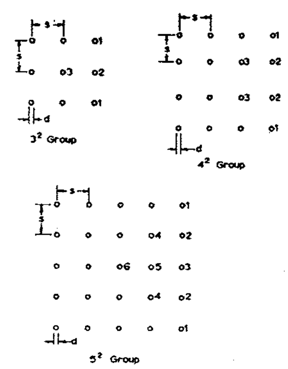

Fig. B1.—Identification of Piles in Square Groups for Tables B1 and B2.

TABLE B1

Load Distributions within Floating Pile Groups

Values of P/P_{av}

	L/d	K / s/d	Pile 1			Pile 2			Pile 3		
			100	1000	∞	100	1000	∞	100	1000	∞
3^2 Group	10	2	1.28	1.47	1.56	0.84	0.75	0.72	0.52	0.16	−0.15
		5	1.20	1.25	1.26	0.91	0.88	0.88	0.57	0.47	0.45
		10	1.10	1.13	1.14	0.95	0.94	0.94	0.78	0.73	0.70
		20	1.04	1.05	1.06	0.98	0.97	0.97	0.91	0.88	0.88
	25	2	1.18	1.38	1.50	0.89	0.79	0.65	0.71	0.32	−0.35
		5	1.17	1.29	1.32	0.92	0.87	0.84	0.63	0.38	0.34
		10	1.11	1.18	1.21	0.95	0.91	0.89	0.77	0.61	0.55
		20	1.06	1.11	1.12	0.97	0.95	0.94	0.87	0.77	0.73
	100	2	1.24	1.11	1.70	0.86	0.93	0.66	0.58	0.84	−0.45
		5	1.22	1.17	1.37	0.90	0.92	0.81	0.53	0.61	0.24
		10	1.14	1.15	1.28	0.94	0.93	0.86	0.70	0.68	0.42
		20	1.07	1.10	1.21	0.97	0.95	0.90	0.86	0.79	0.55
4^2 Group	10	2	1.68	2.00	2.14	0.97	0.95	0.95	0.38	0.09	−0.04
		5	1.42	1.51	1.52	1.01	1.00	1.00	0.56	0.48	0.47
		10	1.21	1.25	1.28	1.01	1.00	1.00	0.77	0.73	0.70
		20	1.10	1.13	1.12	1.00	1.00	1.00	0.89	0.86	0.86
	25	2	1.50	1.87	2.25	0.97	0.95	0.89	0.54	0.23	−0.05
		5	1.40	1.62	1.70	1.01	1.01	0.99	0.59	0.36	0.30
		10	1.25	1.41	1.48	1.00	1.01	1.00	0.74	0.57	0.50
		20	1.14	1.23	1.26	1.00	1.00	1.00	0.85	0.76	0.72
	100	2	1.56	.135	2.30	0.96	0.97	1.01	0.52	0.70	−0.15
		5	1.50	1.45	1.84	1.02	1.01	0.98	0.47	0.52	0.18
		10	1.29	1.35	1.65	1.00	1.00	1.00	0.70	0.63	0.34
		20	1.15	1.24	1.42	1.00	1.01	1.00	0.83	0.75	0.56

	L/d	K / s/d	Pile 1			Pile 2			Pile 3			Pile 4			Pile 5			Pile 6		
			100	1000	∞	100	1000	∞	100	1000	∞	100	1000	∞	100	1000	∞	100	1000	∞
5^2 Group	10	2	2.12	2.48	2.65	1.18	1.19	1.12	1.05	1.07	1.20	0.42	0.21	− 0.15	0.26	0.10	0.16	0.12	0.01	0.45
		5	1.64	1.75	1.79	1.13	1.14	1.15	1.07	1.09	1.10	0.60	0.53	0.49	0.55	0.49	0.46	0.49	0.45	0.42
		10	1.31	1.39	1.41	1.07	1.09	1.09	1.03	1.04	1.05	0.81	0.75	0.74	0.76	0.71	0.69	0.73	0.68	0.64
		20	1.18	1.22	1.19	1.05	1.06	1.05	1.01	1.02	1.02	0.90	0.87	0.89	0.86	0.83	0.85	0.82	0.79	0.82
	25	2	1.90	2.46	2.90	1.17	1.19	1.13	1.01	1.09	1.20	0.58	0.20	− 0.20	0.38	0.12	0.09	0.24	0.04	0.25
		5	1.62	1.98	2.11	1.14	1.18	1.19	1.05	1.09	1.07	0.63	0.40	0.35	0.54	0.34	0.27	0.46	0.29	0.22
		10	1.39	1.63	1.73	1.10	1.14	1.16	1.04	1.05	1.07	0.77	0.64	0.56	0.70	0.55	0.47	0.64	0.45	0.37
		20	1.22	1.37	1.40	1.06	1.09	1.09	1.02	1.04	1.05	0.87	0.77	0.74	0.82	0.72	0.71	0.78	0.68	0.67
	100	2	2.06	1.75	3.00	1.15	1.14	1.10	1.08	1.00	1.20	0.41	0.65	− 0.30	0.33	0.48	0.05	0.25	0.33	0.40
		5	1.77	1.78	2.34	1.18	1.18	1.22	1.07	1.06	1.09	0.54	0.55	0.21	0.48	0.42	0.14	0.33	0.30	0.07
		10	1.45	1.58	2.05	1.10	1.13	1.21	1.05	1.04	1.08	0.72	0.66	0.38	0.68	0.58	0.26	0.63	0.49	0.17
		20	1.25	1.41	1.78	1.17	1.10	1.13	1.02	1.02	1.02	0.85	0.77	0.55	0.80	0.70	0.50	0.75	0.62	0.52

TABLE B2
Load Distributions within End-Bearing Pile Groups

			Pile 1		Pile 2		Pile 3	
	L/d	K / s/d	100	1000	100	1000	100	1000
3^2 Group	10	2	0.98	0.92	0.99	1.01	1.11	1.20
		5	1.02	1.00	0.99	1.00	0.94	1.00
		10	1.00	1.00	1.00	1.00	1.00	1.00
		20	1.00	1.00	1.00	1.00	1.00	1.00
	25	2	1.07	0.95	0.94	1.00	0.93	1.17
		5	1.11	1.02	0.94	0.99	0.76	0.96
		10	1.05	1.01	0.98	0.99	0.88	0.97
		20	1.02	1.00	1.00	1.00	0.97	1.00
	100	2	1.22	1.02	0.87	0.97	0.65	1.04
		5	1.21	1.13	0.90	0.94	0.53	0.73
		10	1.13	1.10	0.94	0.95	0.71	0.78
		20	1.06	1.06	0.97	0.97	0.88	0.87
4^2 Group	10	2	1.04	0.88	0.98	0.98	1.00	1.17
		5	1.05	1.00	1.00	1.00	0.94	1.00
		10	1.00	1.00	1.00	1.00	0.99	1.00
		20	1.00	1.00	1.00	1.00	1.00	1.00
	25	2	1.26	0.95	0.98	0.98	0.77	1.10
		5	1.23	1.05	1.01	1.00	0.75	0.94
		10	1.10	1.02	1.00	1.00	0.88	0.98
		20	1.02	1.00	1.00	1.00	0.98	1.00
	100	2	1.61	1.19	0.97	0.98	0.44	0.86
		5	1.48	1.33	1.00	1.00	0.51	0.65
		10	1.27	1.23	1.00	1.00	0.72	0.75
		20	1.14	1.13	1.00	1.00	0.85	0.84

			Pile 1		Pile 2		Pile 3		Pile 4		Pile 5		Pile 6	
	L/d	K / s/d	100	1000	100	1000	100	1000	100	1000	100	1000	100	1000
5^2 Group	10	2	1.11	0.86	1.02	0.94	0.95	0.95	0.99	1.13	0.90	1.14	0.81	1.18
		5	1.06	1.01	1.02	1.00	1.01	1.00	0.96	1.00	0.95	1.00	0.94	1.00
		10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	25	2	1.55	0.99	1.12	0.99	0.99	0.96	0.77	1.07	0.59	1.03	0.44	0.99
		5	1.37	1.09	1.09	1.03	1.04	1.00	0.77	0.95	0.72	0.92	0.67	0.87
		10	1.15	1.03	1.04	1.01	1.02	1.01	0.90	0.98	0.88	0.97	0.86	0.97
		20	1.02	1.00	1.00	1.00	1.00	1.00	0.98	1.00	0.98	1.00	0.98	1.00
	100	2	2.08	1.48	1.20	1.11	1.01	0.96	0.48	0.83	0.25	0.63	0.36	0.44
		5	1.75	1.55	1.16	1.13	1.08	1.04	0.53	0.68	0.46	0.58	0.39	0 47
		10	1.42	1.39	1.08	1.09	1.04	1.04	0.73	0.77	0.70	0.71	0.67	0.65
		20	1.21	1.22	1.05	1.05	1.01	1.01	0.88	0.87	0.83	0.82	0.79	0.78

APPENDIX C

Illustrative Example:

A free-standing group of six 12 in. dia. concrete piles is driven into a deep layer of medium clay, and is to be subjected to a load of 300 tons (see Fig. C1). A test on a single pile gives a final settlement of 0.60 in. under a load of 50 tons. It is required to determine the final settlement of the 6-pile group.

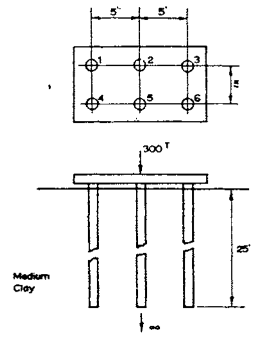

Fig. C1.

From Table IX, the value of K is about 2,000. Piles 1, 3, 4 and 6 behave identically and will be called piles A while piles 2 and 5 will be called piles B. The loads on Piles A and B are P_A and P_B respectively. From Fig. A2 for $L/d = 25$, the interaction factors may be interpolated for $K = 2{,}000$. The factors are tabulated in Table C1.

The settlement of pile 1 (and all piles A) is given by Eq. (7), as

$$\rho_A = \rho_1\{P_A(0.27 + 0.42 + 0.25) + P_B(0.42 + 0.35) + P_A\}$$

i.e.,

$$\rho_A/\rho_1 = 1.94\,P_A + 0.77\,P_B \qquad \text{.....................(C1)}$$

where ρ_1 is the settlement of a single pile under unit load. Similarly for pile 2 (and all piles B),

$$\rho_B = \rho_1\{P_A(0.42 + 0.42 + 0.35 + 0.35) + P_B(0.42) + P_B\}$$

i.e.,

$$\rho_B/\rho_1 = 1.54\,P_A + 1.42\,P_B \qquad \text{.....................(C2)}$$

TABLE C1

	Pile 1 (Type A)		Pile 2 (Type B)	
Pile j	s/d	α_{1j}	s/d	α_{2j}
1	0	—	5	0.42
2	5	0.42	0	—
3	10	0.27	5	0.42
4	5	0.42	7.07	0.35
5	7.07	0.35	5	0.42
6	11.2	0.25	7.07	0.35

Also, from equilibrium,

$$4P_A + 2P_B = 300 \qquad \text{..........................(C3)}$$

For a rigid cap, $\rho_A = \rho_B$.

Solution of Eqs. (C1), (C2) and (C3) for this case yields the following solutions:

$$P_A = 57.4 \text{ tons}$$
$$P_B = 35.2 \text{ tons}$$
$$\rho_A/\rho_1 = \rho_B/\rho_1 = 138.4$$

From the pile load test,

$$\rho_1 = 0.60/50 = 0.012 \text{ in./ton.}$$
$$\therefore \quad \rho_A = \rho_B = 1.66 \text{ in.}$$

(The settlement ratio $R_S = \dfrac{1.66}{0.60} = 2.77$.)

If the pile cap is sufficiently flexible and the load is uniformly distributed so that all piles are equally loaded, then

$$P_A = P_B = 50 \text{ tons}$$
$$\rho_A = 50 \times 2.71 \times \rho_1 = 1.62 \text{ in.}$$
$$\rho_B = 50 \times 2.96 \times \rho_1 = 1.77 \text{ in.}$$