A Mathematical Method for Determining the Anisotropic Velocity of P-Wave Propagation and for the Seismic Location of Rockbursts

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Summary.—A mathematical method is presented for determining the velocities of P-wave propagation when the anisotropy of the rock mass is characterised by an ellipsoidal type velocity law. The analysis is developed using measured seismic data obtained by firing charges of explosives at known locations in the rock and registered by a network of seismometers distributed around the mine workings. In the second part of the paper the already determined in-situ seismic velocities of P-wave propagation are utilised in the derivation of a set of simultaneous linear equations necessary for the location of the focus of an actual rockburst.

INTRODUCTION

One of the major hazards generally associated with deep level mining is the phenomenon of rockburst which is a sudden and violent failure of the rock in the vicinity of mine openings. For several years, intensive research has been carried out into the causes and the nature of rockbursts. The statistical analysis of empirical data, the study of rock properties and the seismic location of the foci of rockbursts have been used to enhance the basic understanding of the mechanism of rockbursts (Ref. 1).

In this search, the application of seismic techniques to locate the foci of rockbursts with respect to the underground workings is particularly important. Although several research centres and large mining companies (Refs. 1, 2, 3 and 4) built recording equipment and carried out field observations, most of the publications on this subject is due to Cook. Calibration measurements conducted underground in homogeneous massive quartzites by Cook (Ref. 1) showed that 90% of the seismic paths had velocities between 18.1 and 18.9 ft./millisec. Under these circumstances the assumption of isotropic propagation of P-wave velocity was justified. However, preliminary analysis of the field observations recorded by Bhattacharyya (Ref. 5) at Kolar Gold Field in India indicates that the velocity of P-wave propagation is directionally dependent. Anisotropic behaviour with an ellipsoidal distribution of moduli was also found by Douglass and Voight (Ref. 6) and Peres Rodrigues (Ref. 7) to be a characteristic of several granites. When such anisotropy is significant, it may be necessary to apply a method of analysis to the location of the origin of a rockburst which takes into account the anisotropic velocity of a seismic wave propagation. It is the purpose of this paper to describe a mathematical method which can be used to determine this inherent rock property and to locate the focus of a rockburst in an idealised anisotropic rock.

In the context of this paper the network of seismometers distributed in that region of a rock mass where a rockburst is to be expected to occur can serve a dual purpose. First they can be used to register the times of arrival of a seismic event initiated by firing a charge of explosive at a point of known location. If the initiation time of the blast is also obtained from the recording of a seismometer placed very near the blast location, then the in-situ velocity of the P-wave propagation can be determined by the general analysis to be described. The second purpose of the seismometers is to register the times of arrival of the P-waves in case of an actual rockburst. From the recorded data, using the already determined seismic velocities, the co-ordinates of the rockburst focus can be easily calculated. It is assumed in what follows that the variation of the velocities of P-wave propagation around a point are governed by an ellipsoidal type of law and that these velocities are uniform in their respective directions. Because of the law accuracy of recording the time of arrival of the S-wave on short distances involved, the time of arrival of the P-wave is used only in the analysis presented here.

DETERMINATION OF THE ELLIPSOID OF ANISOTROPIC VELOCITY

In order to obtain the necessary information from an experimental investigation it is required to record the initiation time of a blast and the corresponding times of arrival of the P-wave at the individual seismometers in the network. From these data the time interval for the transmission of the seismic event through the rock to each of the seismometers are determined. Then, knowing the co-ordinates of the blast point and those of the seismometers, the P-wave velocities along respective lines from the centre of the explosion to the individual seismometers are directly calculated. If the network contains a sufficient number of seismometers both the magnitudes and orientations of the principal axes of the ellipsoid of anisotropic velocity of P-wave propagation can be determined.

Using the mine co-ordinate system as a basic frame of reference, let the co-ordinates of the blast point be denoted by

$$(x_0\,,\,y_0\,,\,z_0)$$

and those of the seismometers by

$$(x_1\,,y_1\,,z_1);\ (x_2\,,y_2\,,z_2);\ \ldots\ (x_i\,,y_i\,,z_i);\ \ldots\ (x_n\,,y_n\,,z_n).$$

The distance d_i between the blast point and the i-th seismometer is given by

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \qquad \text{...........(1)}$$

Denoting the time of arrival of the wave by t_i and the initiation time by t_0 the velocity v_i to the i-th seismometer along the assumed straight seismic path is

$$v_i = \frac{d_i}{t_i - t_0} \qquad \text{................................(2)}$$

If $\alpha_i\,,\,\beta_i\,,\,\gamma_i$ are the angles between a directed line and the x, y, z axes of the co-ordinate system, the direction cosines of the line from the blast point to the corresponding seismometer are expressed by:

$$\cos\alpha_i = \frac{x_i - x_0}{d_i}\,,\quad \cos\beta_i = \frac{y_i - y_0}{d_i}\,,\quad \cos\gamma_i = \frac{z_i - z_0}{d_i} \qquad \text{......(3)}$$

From Eqs. (2) and (3) the equations for the velocity components of v_i along the x, y, z axes respectively can be deduced:

$$v_{xi} = v_i\cos\alpha_i\,,\ v_{yi} = v_i\cos\beta_i\,,\ v_{zi} = v_i\cos\gamma_i \qquad \text{...........(4)}$$

If a new co-ordinate system, with its axes $v_x\,,\,v_y\,,\,v_z$ directed parallel to the x, y, z co-ordinate axes respectively, is introduced so that its origin coincides with the blast location, then the general equation of the central velocity ellipsoid in this co-ordinate system may be written as:

$$a\,v_x^2 + b\,v_y^2 + c\,v_z^2 + 2f\,v_y\,v_z + 2g\,v_z\,v_x + 2h\,v_x\,v_y - 1 = 0 \qquad \text{...(5)}$$

The unknown constants a, b, c, f, g and h can be determined by substituting the velocity components, calculated from the actual measurements as shown above, into Eq. (5). The observations at each seismometer yields one such equation and for n seismometers the following set of equations is obtained:

$$\begin{aligned}
&a\,v_{x1}^2 + b\,v_{y1}^2 + c\,v_{z1}^2 + 2f\,v_{y1}\,v_{z1} + 2g\,v_{z1}\,v_{x1} + 2h\,v_{x1}\,v_{y1} - 1 = 0\\
&a\,v_{x2}^2 + b\,v_{y2}^2 + c\,v_{z2}^2 + 2f\,v_{y2}\,v_{z2} + 2g\,v_{z2}\,v_{x2} + 2h\,v_{x2}\,v_{y2} - 1 = 0\\
&\qquad\vdots\\
&a\,v_{xi}^2 + b\,v_{yi}^2 + c\,v_{zi}^2 + 2f\,v_{yi}\,v_{zi} + 2g\,v_{zi}\,v_{xi} + 2h\,v_{xi}\,v_{yi} - 1 = 0\\
&\qquad\vdots\\
&a\,v_{xn}^2 + b\,v_{yn}^2 + c\,v_{zn}^2 + 2f\,v_{yn}\,v_{zn} + 2g\,v_{zn}\,v_{xn} + 2h\,v_{xn}\,v_{yn} - 1 = 0
\end{aligned} \qquad \text{...(6)}$$

Since the resulting set of linear equations contain six independent constants it is necessary to have six independent equations ($n = 6$), in order to determine the unknown constants exactly. The network must, therefore, contain at least six seismometers which do not all lie in a common plane.

*Paper No. 2945, submitted by the author on 22nd July, 1970.
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If more than six seismometers yield equally weighted observations, it can be shown that using the principle of least squares, the most probable values of the constants can be calculated from the normal equations:

$$\begin{aligned}
&(\Sigma v_{xi}^4)a + (\Sigma v_{xi}^2 v_{yi}^2)b + (\Sigma v_{xi}^2 v_{zi}^2)c + \\
&\quad + 2(\Sigma v_{xi}^2 v_{yi} v_{zi})f + 2(\Sigma v_{xi}^3 v_{zi})g + \\
&\quad\quad + 2(\Sigma v_{xi}^3 v_{yi})h - \Sigma v_{xi}^2 = 0 \\
&(\Sigma v_{xi}^2 v_{yi}^2)a + (\Sigma v_{yi}^4)b + (\Sigma v_{yi}^2 v_{zi}^2)c + \\
&\quad + 2(\Sigma v_{yi}^3 v_{zi})f + 2(\Sigma v_{xi} v_{yi}^2 v_{zi})g + \\
&\quad\quad + 2(\Sigma v_{xi} v_{yi}^3)h - \Sigma v_{yi}^2 = 0 \\
&(\Sigma v_{xi}^2 v_{zi}^2)a + (\Sigma v_{yi}^2 v_{zi}^2)b + (\Sigma v_{zi}^4)c + \\
&\quad + 2(\Sigma v_{yi} v_{zi}^3)f + 2(\Sigma v_{xi} v_{zi}^3)g + \\
&\quad\quad + 2(\Sigma v_{xi} v_{yi} v_{zi}^2)h - \Sigma v_{zi}^2 = 0 \\
&(\Sigma v_{xi}^2 v_{yi} v_{zi})a + (\Sigma v_{yi}^3 v_{zi})b + (\Sigma v_{yi} v_{zi}^3)c + \\
&\quad + 2(\Sigma v_{yi}^2 v_{zi}^2)f + 2(\Sigma v_{xi} v_{yi} v_{zi}^2)g + \\
&\quad\quad + 2(\Sigma v_{xi} v_{yi} v_{zi})h - \Sigma v_{yi} v_{zi} = 0 \\
&(\Sigma v_{xi}^3 v_{zi})a + (\Sigma v_{xi} v_{yi}^2 v_{zi})b + (\Sigma v_{xi} v_{zi}^3)c + \\
&\quad + 2(\Sigma v_{xi} v_{yi} v_{zi}^2)f + 2(\Sigma v_{xi}^2 v_{zi}^2)g + \\
&\quad\quad + 2(\Sigma v_{xi}^2 v_{yi} v_{zi})h - \Sigma v_{xi} v_{zi} = 0 \\
&(\Sigma v_{xi}^3 v_{yi})a + (\Sigma v_{xi} v_{yi}^3)b + (\Sigma v_{xi} v_{yi} v_{zi}^2)c + \\
&\quad + 2(\Sigma v_{xi} v_{yi}^2 v_{zi})f + 2(\Sigma v_{xi}^2 v_{yi} v_{zi})g + \\
&\quad\quad + 2(\Sigma v_{xi}^2 v_{yi}^2)h - \Sigma v_{xi} v_{yi} = 0
\end{aligned} \qquad \ldots(7)$$

In Eqs. (7) the summations are carried out over the range from 1 to k, where k is the number of observations. As stated above, it is necessary that k should be greater than 6.

When it is possible to assign weights to the individual readings (w_i), the normal equations become

$$\begin{aligned}
&(\Sigma w_i v_{xi}^4)a + (\Sigma w_i v_{xi}^2 v_{yi}^2)b + (\Sigma w_i v_{xi}^2 v_{zi}^2)c + \\
&\quad + 2(\Sigma w_i v_{xi}^2 v_{yi} v_{zi})f + 2(\Sigma w_i v_{xi}^3 v_{zi})g + \\
&\quad\quad + 2(\Sigma w_i v_{xi}^3 v_{yi})h - \Sigma w_i v_{xi}^2 = 0 \\
&(\Sigma w_i v_{xi}^2 v_{yi}^2)a + (\Sigma w_i v_{yi}^4)b + (\Sigma w_i v_{yi}^2 v_{zi}^2)c + \\
&\quad + 2(\Sigma w_i v_{yi}^3 v_{zi})f + 2(\Sigma w_i v_{xi} v_{yi}^2 v_{zi})g + \\
&\quad\quad + 2(\Sigma w_i v_{xi} v_{yi}^3)h - \Sigma w_i v_{yi}^2 = 0 \\
&(\Sigma w_i v_{xi}^2 v_{zi}^2)a + (\Sigma w_i v_{yi}^2 v_{zi}^2)b + (\Sigma w_i v_{zi}^4)c + \\
&\quad + 2(\Sigma w_i v_{yi} v_{zi}^3)f + 2(\Sigma w_i v_{xi} v_{zi}^3)g + \\
&\quad\quad + 2(\Sigma w_i v_{xi} v_{yi} v_{zi}^2)h - \Sigma w_i v_{zi}^2 = 0 \\
&(\Sigma w_i v_{xi}^2 v_{yi} v_{zi})a + (\Sigma w_i v_{yi}^3 v_{zi})b + (\Sigma w_i v_{yi} v_{zi}^3)c + \\
&\quad + 2(\Sigma w_i v_{yi}^2 v_{zi}^2)f + 2(\Sigma w_i v_{xi} v_{yi} v_{zi}^2)g + \\
&\quad\quad + 2(\Sigma w_i v_{xi} v_{yi}^2 v_{zi})h - \Sigma w_i v_{yi} v_{zi} = 0 \\
&(\Sigma w_i v_{xi}^3 v_{zi})a + (\Sigma w_i v_{xi} v_{yi}^2 v_{zi})b + (\Sigma w_i v_{xi} v_{zi}^3)c + \\
&\quad + 2(\Sigma w_i v_{xi} v_{zi}^2)f + 2(\Sigma w_i v_{xi}^2 v_{zi}^2)g + \\
&\quad\quad + 2(\Sigma w_i v_{xi}^2 v_{yi} v_{zi})h - \Sigma w_i v_{xi} v_{zi} = 0 \\
&(\Sigma w_i v_{xi}^3 v_{yi})a + (\Sigma w_i v_{xi} v_{yi}^3)b + (\Sigma w_i v_{xi} v_{yi} v_{zi}^2)c + \\
&\quad + 2(\Sigma w_i v_{xi} v_{yi}^2 v_{zi})f + 2(\Sigma w_i v_{xi}^2 v_{yi} v_{zi})g + \\
&\quad\quad + 2(\Sigma w_i v_{xi}^2 v_{yi}^2)h - \Sigma w_i v_{xi} v_{yi} = 0
\end{aligned} \qquad \ldots(8)$$

Having obtained the values of constants a, b, c, f, g and h the general equation of the ellipsoid of anisotropic velocity of P-wave propagation is given by Eq. (5).

ORIENTATION OF THE ELLIPSOID OF ANISOTROPIC VELOCITY

It is known from co-ordinate geometry (Ref. 8), that in order to calculate the magnitudes of the principal axes of the velocity ellipsoid, it is necessary to determine the roots of the discriminating cubic:

$$\lambda^3 - \lambda^2 (a + b + c) + \lambda(bc + ca + ab - f^2 - g^2 - h^2) - e = 0, \quad \ldots(9)$$

where $e = a(bc - f^2) - h(hc - fg) + g(hf - bg)$. It is shown in Ref. 8, that all the roots of this discriminating cubic are real.

It can also be shown, that when the principal axes are taken as co-ordinate axes Eq. (5) transforms into

$$\lambda_{\text{I}}\, \xi^2 + \lambda_{\text{II}}\, \eta^2 + \lambda_{\text{III}}\, \zeta^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

In Eq. (10) λ_{I}, λ_{II} and λ_{III} denote the roots of the cubic equation and ξ, η and ζ are the co-ordinates measured along the principal axes of the velocity ellipsoid. The index notation I, II and III is introduced here to distinguish between the reference number of seismometers and the principal axes.

The comparison of Eq. (10) with the normal equation of an ellipsoid yields the following relations for the magnitudes of the principal semi-axes of the velocity ellipsoid:

$$v_{\text{I}} = \sqrt{1/\lambda_{\text{I}}}\,, \quad v_{\text{II}} = \sqrt{1/\lambda_{\text{II}}}\,, \quad v_{\text{III}} = \sqrt{1/\lambda_{\text{III}}} \quad \ldots\ldots\ldots(11)$$

The direction cosines of the principal axes, relative to the mine co-ordinate system, can be determined by solving the following three sets of simultaneous linear equations for the corresponding values of l_j, m_j and n_j the direction cosines:

$$\begin{aligned}
l_j (a - \lambda_j) + m_j h + n_j g &= 0 \\
l_j h + m_j (b - \lambda_j) + n_j f &= 0 \\
l_j g + m_j f + n_j (c - \lambda_j) &= 0
\end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

where j represents indices I, II and III in turn.

LOCATION OF THE FOCUS OF A ROCKBURST

From the recording of the seismic signals initiated by an actual rockburst, the times of arrival of the P-waves and consequently the time intervals can be determined. In this part of the paper the time intervals Δt_i denote the time lag between the seismometer which first registers the seismic event and each of the other seismometers in the network. The Δt_i's are measured quantities and include Δt_1 which is introduced for mathematical convenience, but its value will be taken to be zero. Using this notation the relative arrival time of the seismic event at the i-th seismometer can be written as

$$t_i = (t_1 + \Delta t_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

where t_1 is an undetermined quantity and denotes the transmission time of the P-wave from the focus to the first seismometer. It should be noted that in what follows the reference numbers of the seismometers must coincide with the order in which they register the seismic signals. Therefore, the seismometer being the first, second, third and i-th in registering the arrival of the P-wave are referred to by the suffixes 1, 2, 3 and i respectively. Although the analysis includes the determination of the numerical value of t_1, it is the ultimate aim to derive expressions from which x_c, y_c and z_c, the co-ordinates of the focus of a rockburst can be calculated.

The equation of an ellipsoid referred to the principal axes of the velocity ellipsoid is given by Eq. (10). Replacing λ_{I}, λ_{II} and λ_{III} by $1/v_{\text{I}}^2$, $1/v_{\text{II}}^2$ and $1/v_{\text{III}}^2$ respectively, Eq. (10) takes the form:

$$\frac{\xi^2}{v_{\text{I}}^2} + \frac{\eta^2}{v_{\text{II}}^2} + \frac{\zeta^2}{v_{\text{III}}^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots(14)$$

If it is required to represent the ellipsoidal travelling wave surface, Eq. (14) must be made to be a function of time. This is achieved by multiplying the semi-axes of the ellipsoid by t, the time variable. Eq. (14) then becomes:

$$\frac{\xi^2}{v_{\text{I}}^2} + \frac{\eta^2}{v_{\text{II}}^2} + \frac{\zeta^2}{v_{\text{III}}^2} = t^2 \quad \ldots\ldots\ldots\ldots\ldots(15)$$

Since the initial data are referred to the mine co-ordinate system, it is convenient to carry out the calculations using this co-ordinate system as the basic frame of reference. Consequently Eq. (15) has to be expressed with respect to the mine co-ordinate axes. This transformation is effected by the following equations:

$$\begin{aligned}
\xi &= l_{\text{I}}\ (x_i - x_c) + m_{\text{I}}\ (y_i - y_c) + n_{\text{I}}\ (z_i - z_c) \\
\eta &= l_{\text{II}}\ (x_i - x_c) + m_{\text{II}}\ (y_i - y_c) + n_{\text{II}}\ (z_i - z_c) \\
\zeta &= l_{\text{III}} (x_i - x_c) + m_{\text{III}} (y_i - y_c) + n_{\text{III}} (z_i - z_c)
\end{aligned} \quad \ldots\ldots(16)$$

Insertion of Eqs. (13) and (16) into Eq. (15) and using the relations given by Eqs. (11) yields

$$\begin{aligned}
&\lambda_{\text{I}} \{l_{\text{I}} (x_i - x_c) + m_{\text{I}} (y_i - y_c) + n_{\text{I}} (z_i - z_c)\}^2 + \\
&\quad + \lambda_{\text{II}} \{l_{\text{II}} (x_i - x_c) + m_{\text{II}} (y_i - y_c) + n_{\text{II}} (z_i - z_c)\}^2 + \\
&\quad + \lambda_{\text{III}} \{l_{\text{III}} (x_i - x_c) + m_{\text{III}} (y_i - y_c) + \\
&\qquad + n_{\text{III}} (z_i - z_c)\}^2 = (t_1 + \Delta t_i)^2
\end{aligned} \quad \ldots\ldots(17)$$

When $i = 1$, $\Delta t_1 = 0$ and

$$\lambda_{I}\{l_{I}(x_1 - x_c) + m_{I}(y_1 - y_c) + n_{I}(z_1 - z_c)\}^2 + \lambda_{II}\{l_{II}(x_1 - x_c) + m_{II}(y_1 - y_c) + n_{II}(z_1 - z_c)\}^2 + \lambda_{III}\{l_{III}(x_1 - x_c) + m_{III}(y_1 - y_c) + n_{III}(z_1 - z_c)\}^2 = t_1^2 \quad \ldots\ldots(18)$$

Eq. (17) retains its original form when $i = 2, 3, \ldots, n$. Taking the difference of Eqs. (17) and (18) yields

$$\lambda_{I}\{-2l_{I}A_i x_c - 2m_{I}A_i y_c - 2n_{I}A_i z_c + D_i + 2G_i\} + \lambda_{II}\{-2l_{II}B_i x_c - 2m_{II}B_i y_c - 2n_{II}B_i z_c + E_i + 2H_i\} + \lambda_{III}\{-2l_{III}C_i x_c - 2m_{III}C_i y_c - 2n_{III}C_i z_c + F_i + 2I_i\} = (t_1 + \Delta t_i)^2 - t_1^2 \quad \ldots(19)$$

where

$$\begin{aligned}
A_i &= l_{I}(x_i - x_1) + m_{I}(y_i - y_1) + n_{I}(z_i - z_1)\\
B_i &= l_{II}(x_i - x_1) + m_{II}(y_i - y_1) + n_{II}(z_i - z_1)\\
C_i &= l_{III}(x_i - x_1) + m_{III}(y_i - y_1) + n_{III}(z_i - z_1)\\
D_i &= l_{I}^2(x_i^2 - x_1^2) + m_{I}^2(y_i^2 - y_1^2) + n_{I}^2(z_i^2 - z_1^2)\\
E_i &= l_{II}^2(x_i^2 - x_1^2) + m_{II}^2(y_i^2 - y_1^2) + n_{II}^2(z_i^2 - z_1^2)\\
F_i &= l_{III}^2(x_i^2 - x_1^2) + m_{III}^2(y_i^2 - y_1^2) + n_{III}^2(z_i^2 - z_1^2)\\
G_i &= l_{I}m_{I}(x_i y_i - x_1 y_1) + l_{I}n_{I}(x_i z_i - x_1 z_1) + m_{I}n_{I}(y_i z_i - y_1 z_1)\\
H_i &= l_{II}m_{II}(x_i y_i - x_1 y_1) + l_{II}n_{II}(x_i z_i - x_1 z_1) + m_{II}n_{II}(y_i z_i - y_1 z_1)\\
I_i &= l_{III}m_{III}(x_i y_i - x_1 y_1) + l_{III}n_{III}(x_i z_i - x_1 z_1) + m_{III}n_{III}(y_i z_i - y_1 z_1)
\end{aligned}$$

After further simplification and rearrangement Eq. (19) takes the form:

$$2L_i x_c + 2M_i y_c + 2N_i z_c + 2\Delta t_i t_1 = K_i - (\Delta t_i)^2 \quad \ldots\ldots(20)$$

where

$$\begin{aligned}
L_i &= \lambda_{I}l_{I}A_i + \lambda_{II}l_{II}B_i + \lambda_{III}l_{III}C_i\\
M_i &= \lambda_{I}m_{I}A_i + \lambda_{II}m_{II}B_i + \lambda_{III}m_{III}C_i\\
N_i &= \lambda_{I}n_{I}A_i + \lambda_{II}n_{II}B_i + \lambda_{III}n_{III}C_i\\
K_i &= \lambda_{I}(D_i + 2G_i) + \lambda_{II}(E_i + 2H_i) + \lambda_{III}(F_i + 2I_i)
\end{aligned}$$

When $i = 2$, 3, 4 and 5 Eq. (20) yields four linear equations with four unknowns

$$\begin{aligned}
2L_2 x_c + 2M_2 y_c + 2N_2 z_c + 2\Delta t_2 t_1 &= K_2 - (\Delta t_2)^2\\
2L_3 x_c + 2M_3 y_c + 2N_3 z_c + 2\Delta t_3 t_1 &= K_3 - (\Delta t_3)^2\\
2L_4 x_c + 2M_4 y_c + 2N_4 z_c + 2\Delta t_4 t_1 &= K_4 - (\Delta t_4)^2\\
2L_5 x_c + 2M_5 y_c + 2N_5 z_c + 2\Delta t_5 t_1 &= K_5 - (\Delta t_5)^2
\end{aligned} \quad \ldots\ldots(21)$$

Since the three-dimensional network has been assumed to contain at least six seismometers, it is possible to obtain a least squares solution for the location of the focus of a rockburst. In the case of equally weighted observations the normal equations replacing Eqs. (21) are

$$\begin{aligned}
&(\Sigma L_i^2)x_c + (\Sigma M_i L_i)y_c + (\Sigma N_i L_i)z_c + (\Sigma\Delta t_i L_i)t_1 - \tfrac{1}{2}\Sigma L_i\{K_i - (\Delta t_i)^2\} = 0\\
&(\Sigma L_i M_i)x_c + (\Sigma M_i^2)y_c + (\Sigma N_i M_i)z_c + (\Sigma\Delta t_i M_i)t_1 - \tfrac{1}{2}\Sigma M_i\{K_i - (\Delta t_i)^2\} = 0\\
&(\Sigma L_i N_i)x_c + (\Sigma M_i N_i)y_c + (\Sigma N_i^2)z_c + (\Sigma\Delta t_i N_i)t_1 - \tfrac{1}{2}\Sigma N_i\{K_i - (\Delta t_i)^2\} = 0\\
&(\Sigma L_i \Delta t_i)x_c + (\Sigma M_i \Delta t_i)y_c + (\Sigma N_i \Delta t_i)z_c + \{\Sigma(\Delta t_i)^2\}t_1 - \tfrac{1}{2}\Sigma\Delta t_i\{K_i - (\Delta t_i)^2\} = 0
\end{aligned} \quad \ldots(22)$$

For unequally weighted observations, the appropriate normal equations can easily be written down by following the procedure used for deriving Eqs. (8) from Eqs. (7).

CONCLUDING REMARKS

As was stated earlier, the foregoing analysis is based on the assumption that the homogeneous rock mass is characterised by an anisotropic P-wave propagation with an ellipsoidal velocity distribution. It is obvious that a reliable location of a rockburst focus can only be obtained if this anisotropic behaviour is constant throughout the regions of the rock in which the velocity ellipsoid is determined and the rockburst phenomenon is observed. If the stipulated condition is satisfied then the application of the above analytical approach, due to the acceptance of the ellipsoidal velocity distribution, gives a flexible and a useful description of this anisotropic properties of rocks. In order to investigate the accuracy of the location of a rockburst a further application of the analysis presented is under consideration to the study of a number of important factors regarding the spatial distribution of seismometers and the effects of fractured rock around mine excavations on the velocity of P-wave propagation.

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