MEASURING K_o IN THE TRIAXIAL TEST

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1 INTRODUCTION

The sophistication of modern testing methods enables many types of laboratory and field tests to be carried out. However, it is sometimes the case that some types of tests are specified without questioning whether they have any meaning in a particular situation. Because of this somewhat uncritical attitude, some myths have grown up about just what it is that we can and do measure in various tests.

One such myth is that the *in situ* value of K_o, the coefficient of earth pressure "at rest", can be measured by reconsolidating an undisturbed sample of the soil under zero lateral strain conditions, and measuring the horizontal stress required to maintain zero lateral strain. A natural consequence of this myth is that it is often assumed that *in situ* strength and stiffness properties of soil can be best examined in triaxial testing if a sample of the soil is reconsolidated to the *in situ* vertical effective stress while maintaining zero lateral strain conditions. This myth is demonstrably incorrect for practically all realistic situations.

2 K_o IN AN ELASTIC SOLID

Consider the case of a sample of elastic material under some anisotropic stresses σ_v and σ_h (assuming axisymmetric conditions). This stress state results from external forces, and therefore in general the ratio of σ_v to σ_h can have any value. Define this ratio as K_o.

If the applied stresses are removed, the question is whether there is there any way of determining what the initial stress state was. In the "sampling" operation, with the removal of all stresses, the sample will undergo elastic deformations given by:

$$\begin{Bmatrix} \varepsilon_v \\ \varepsilon_h \\ \varepsilon_h \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \Delta\sigma_v \\ \Delta\sigma_h \\ \Delta\sigma_h \end{Bmatrix} \quad \text{.......................} (1)$$

so that:

$$\varepsilon_v = \frac{\Delta\sigma_v}{E}\left(1 - 2K_o\nu\right) \quad \text{....................................} (2)$$

and:

$$\varepsilon_h = \frac{\Delta\sigma_v}{E}\left[K_o - \nu\left(1 + K_o\right)\right] \quad \text{..............................} (3)$$

If the sample is recompressed in such a way that the lateral strain during compression is maintained at zero (as in an oedometer test), then the same elastic equations above can be used to show that the ratio of radial to axial stresses is:

$$\frac{\Delta\sigma_r}{\Delta\sigma_a} = \left(\frac{\nu}{1-\nu}\right) \quad \text{...} (4)$$

Thus, irrespective of the initial stress condition in the ground (and hence of initial K_o) the K_o measured in the confined compression test is:

$$K_o = \left(\frac{\nu}{1-\nu}\right) \quad \text{..} (5)$$

and there is no way in this case that such a test procedure can identify the previous stress state.

3 MEASUREMENT OF K_o IN SOIL

3.1 In situ stress state

The state of stress of soil *in situ* depends primarily on whether it is normally consolidated or overconsolidated. For normally consolidated soil, while the current value of σ_v' is the maximum value ever experienced by the soil (i.e. $(\sigma_v')_{\max}$), there appears to be a good correlation between K_o and effective friction angle ϕ':

$$K_{onc} \approx 1 + \sin\phi' \quad \text{...} (6)$$

where the additional subscript "*nc*" refers to normally consolidated conditions. This equation is a simplified version of that suggested originally by Jáky (1944).

However, when the vertical stress is reduced below its maximum value, this relationship no longer holds. This is illustrated in Figure 1, which shows schematically the vertical and horizontal stresses in confined compression for initial loading (OA), unloading (AB) and reloading (BD). In the early stages of unloading, the horizontal stress changes at a slower rate than the vertical stress, so that the value of K_o increases. The limit to this behaviour corresponds to the initiation of passive failure (at C in this case). If reloading occurs at B, the curve eventually rejoins the virgin loading curve.

It can be seen from this Figure that for any given vertical effective stress $(\sigma_v')_i$, the horizontal stress could be anywhere between the values corresponding to points E and F, depending on how many cycles of loading and unloading below the maximum past stress occurred, so that K_o can have a wide range of values.

An investigation of data from 170 soils was undertaken by Mayne and Kulhawy (1982), who found that K_o for the first unloading stage (K_{ou}) depends on overconsolidation ratio (OCR):

$$K_{ou} = K_o \mathrm{OCR}^{\sin\phi'} \quad \text{.......................................} (7)$$

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with K_{onc} being given satisfactorily by Equation 1. They went on to derive a general expression for K_o which applies to initial loading, first unloading and first reloading:

$$K_o = K_{onc}\left[\frac{\mathrm{OCR}}{\mathrm{OCR}_{\max}^{(1-\sin\phi')}} + \frac{3}{4}\left(1 - \frac{\mathrm{OCR}}{\mathrm{OCR}_{\max}}\right)\right] \qquad \ldots\ldots\ldots\ldots\ldots (8)$$

in which $\mathrm{OCR}_{\max}$ is the value of OCR at the point where the vertical stress is a minimum in the cycle. This expression gives a picture similar to that shown in Figure 1.

It is clear therefore that whereas in an elastic solid, the relationship between vertical and horizontal stresses at all stages "zero lateral strain" loading, unloading and reloading is constant (Equation 4), this is not the case for soil.

3.2 Sampling and testing

When considering the stress changes which occur during sampling and subsequent recompression, an important distinction must be made between fine grained saturated soils and coarse grained or partially saturated soils. Essentially, the difference in behaviour is due to the ability of the fine grained soils to remain at the same mean effective stress due to suctions in the sample, whereas in the coarser grained soils, which are incapable of maintaining significant suctions, the mean effective stress state must change. Consider each type of soil in turn:

3.2.1 Fine grained saturated soils

For soils with sufficiently low air-entry value, the process of removing a sample from the ground can be assumed to be undrained. Consider a sample in the ground at a depth 10 m below the ground surface, and a water table at the ground surface. Assume $\gamma = 20$ kN/m^3, and $\gamma_w = 10$ kN/m^3, and that K_o is 2.5 (say). Thus, $\sigma_v = 200$ kPa, $\sigma_v' = 100$ kPa, $\sigma_h' = 250$ kPa, and $\sigma_h = 350$ kPa. This give a mean effective stress of 200 kPa, a mean total stress of 300 kPa.

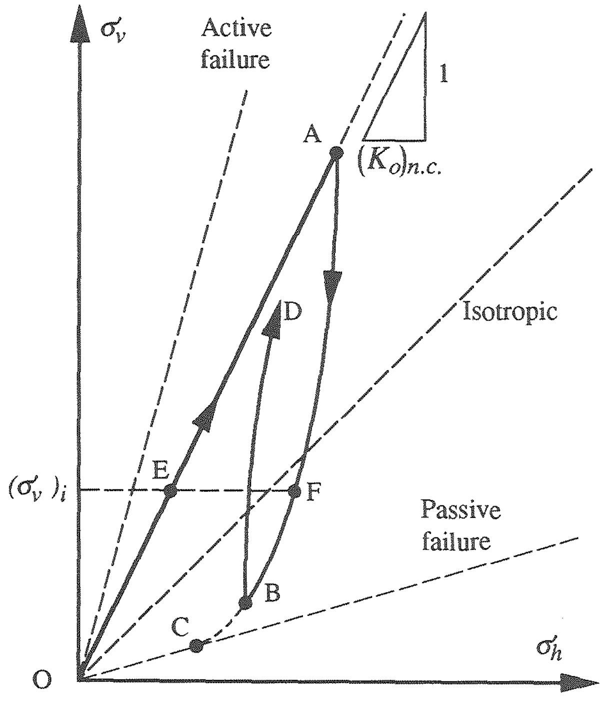

Figure 1. Stress changes during initial loading, unloading and reloading.

Removing this sample from the ground involves reducing the mean total stress to zero, a change of – 300 kPa. If the mean effective stress is to stay the same, the pore pressure must change by the same amount as the total stress. Thus the new value of pore pressure is 100 – 300 kPa, or – 200 kPa. At this stage, though the mean effective stress is unchanged, the vertical and horizontal stresses must be equal (and hence are equal to 200 kPa). Thus, the sample has undergone an increase of 100 kPa in σ_v' and a reduction of 50 kPa in σ_h'. The sample will then have changed shape, even though the volume will theoretically be the same. In reality, some expansion of the sample is inevitable, due to the fact that the pore water goes from compression to suction, with consequent appearance of some air bubbles.

In the re-consolidation stage, assume that we know what the *in situ* values of σ_v' and pore pressure were. There are then a number of possible approaches to discovering what the horizontal effective stress is:

- Measure the suctions in the sample when in the unconfined state. The difference between the suction and the *in situ* pore pressure should be equal to the change in mean total stress in the sampling operation, which is equal to the *in situ* mean total stress. From this, and a knowledge of the *in situ* vertical effective stress, the *in situ* horizontal stress can be calculated.
- Apply total stresses to the sample in a completely undrained mode, and measure the pore pressures. The correct stress state will have been reached when the total vertical stress is equal to the *in situ* total vertical stress and the horizontal stress has been adjusted such that the pore pressure is equal to the *in situ* pore pressure. This would be valid only if it was certain that the *in situ* pore pressure was hydrostatic, or if it had been measured in some way.

Just as in the sampling operation, the undrained recompression should in theory result in zero volumetric strain, but again the water may be compressible in going from suction to compression. Thus, the second option should produce a better estimate of the *in situ* stress state, as it relies on re-establishing the *in situ* pore pressure rather than relying on equality between total stress and pore pressure change in the sampling operation.

3.2.2 Free-draining soils

In a free draining soil, some suctions may develop in the sample during the sampling operation, but these suctions will not be sufficient to maintain the sample in the same mean effective stress state. If the sample is overconsolidated, *there is no direct way of finding out what was the value of the in situ horizontal stress from the sample.*

This is especially the case where, as is common in triaxial testing, the sample is allowed to saturate for some time before commencing the re-compression phase of the test. This process will wipe out all suctions in the sample, and hence remove any evidence of past stress state. At the end of the re-saturation, the sample will be under zero effective stress, and will have undergone some vertical and horizontal strain during the whole sampling and re-saturation process. Thus, a subsequent "zero lateral strain" re-compression will do no more than provide a measure of Poisson's ratio (ν), as for an elastic solid (Equation 4). The correct horizontal stress to apply would be that which reverses the horizontal strain occurring during sampling and re-saturation. Only if that is zero will the zero lateral strain test provide the correct value of *in situ* horizontal stress.

4 CONCLUSION

Measurement of horizontal effective stress remains one of the most difficult problems in geotechnical engineering. Under ideal circumstances (with fine-grained soils of very low permeability), it might be possible to get some idea of the *in situ* stress state by measuring the suctions in the sample, or by carrying out undrained re-compression with pore pressure measurement.

However, it must be concluded for overconsolidated free-draining soils that there is no hope of determining the horizontal stress from a "zero lateral strain" test. In fact, this type of test can provide only a measure of Poisson's ratio, since the K_o measured in this test is a unique function of Poisson's ratio (Equation 5), and irrespective of the true degree of overconsolidation and the true *in situ* value of K_o, must always give a K_o value less than 1. Hence, as an indicator of the *in situ* value of K_o in overconsolidated soils, the test is worthless.

5 REFERENCES

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