ANALYTICAL SOLUTION TO 1D LARGE STRAIN CONSOLIDATION FOR PREDICTION OF FINAL DENSITY

Gareth Swarbrick
PSM

https://doi.org/10.56295/AGJ6048

ABSTRACT

This paper provides closed form solutions to 1D consolidation incorporating two different forms of compressibility relationships (*e* vs log *p* and power curves), single and two-way drainage and with and without the effects of overburden. These solutions can be used in a variety of ways, such as determining final density of tailings profiles based on compressibility parameters, specific gravity and stored mass, determining the increase in tailings settlement due the placement of overburden and back analysis of tailings compressibility parameters based on the initial mass and settled height of column testing.

The paper shows the derivation of the closed form expressions for the different compressibility relationships and drainage conditions. Examples provide the applications based on publicly sourced data and data available to the authors. It can be shown that the closed form solutions closely match results derived from numerical analysis of consolidation including large strain effects. Steps to derive a final density profile are also provided with examples.

1 INTRODUCTION

Consolidation is an important consideration in tailings storage design, operation and closure. Consolidation theory provides insights into the following:

- rate of settlement (which typically reduces over time),
- amount of water released through a combination of:
    - surface expression,
    - seepage through the base or sidewalls,
- density profile which affects:
    - permeability profile (which may be interacting with connected groundwater)
    - strength profile (which is a controlling factor on the ability to support a cap),
- time to complete settlement
- overall capacity of the storage.

The reader is directed to Fourie *et al.* (2022) for a comprehensive summary of the state of art in geotechnical research in tailings including measurement and predication of consolidation.

The well-established (conventional) theory to describe rate of consolidation is that proposed by Terzaghi (1943) who assumed that the total settlement compared to the layer thickness was relatively small and rate of change could be defined by a single constant called the consolidation coefficient (Holtz and Kovacs (1981). These assumptions meant that a closed-form solution of rate of consolidation could be established. The Terzaghi approach is often termed small-strain theory reflecting these underlying assumptions. The parameter that governs total settlement due to change in stress (i.e. primary consolidation) is termed compressibility and most commonly expressed as a relationship between void ratio and vertical effective stress. This relationship is usually assumed non-linear and often taken to be log-linear.

Total settlement due to primary consolidation in soils using Terzaghi theory is usually estimated by assuming average parameters within a soil layer and using the compressibility relationship to calculate the change in void ratio based on the stress change at the centre of that layer (Holtz and Kovacs (1981). When the variation in parameters with depth is significant then the soil profile can be divided into a greater number of layers. The author is not aware of an analytical solution for the final consolidated height due to non-linear compressibility.

Tailings cover a stress range from very low stresses due to self-weight alone and therefore varying from effectively zero effective or net stress at the surface to increasing stress with depth. Tailings typically undergo a large variation in deformation as they transition from no effective stress to a final consolidated state under self-weight loading (plus overburden stress if a cap is applied). These large variations in deformation require what is termed large strain theory. Further discussion of the differences in tailings consolidation theories is provided in in Section 1.1 below.

The objective of this paper is to derive a simple, accurate solution to the final height of a column of tailings incorporating large strain effects, overburden pressure and variable drainage. The relationships developed can be used to predict behaviour without and with overburden (such as the effects of capping upon closure) and under one or two-way drainage. The relationships can also be inverted to back-calculate material properties from laboratory column settling tests and provide accurate profiles of final density and other parameters.

1.1 LARGE STRAIN FORMULATION

The most appropriate (i.e. widely accepted) theory for consolidation of tailings is the large strain formulation developed by Gibson *et al.* (1967) as opposed to the traditional small strain theory developed by Terzaghi (1943). Large strain theory incorporates the effects of the reduction in height and therefore changes in void ratio and drainage path. In addition, the large strain formulation decouples compressibility and permeability parameters. This is in contrast to the Terzaghi formulation where compressibility and permeability are combined into one single constant, the consolidation coefficient, c_V.

A good example of the difference between Terzaghi and Gibson theory was demonstrated in Carrier *et al.* (1983). The key graph from this reference is reproduced in Figure 1 where it is clear that under the same set of parameters the rate of settlement under the more appropriate large strain (i.e. Gibson) theory can be significantly faster than under the small strain (i.e. Terzaghi) theory.

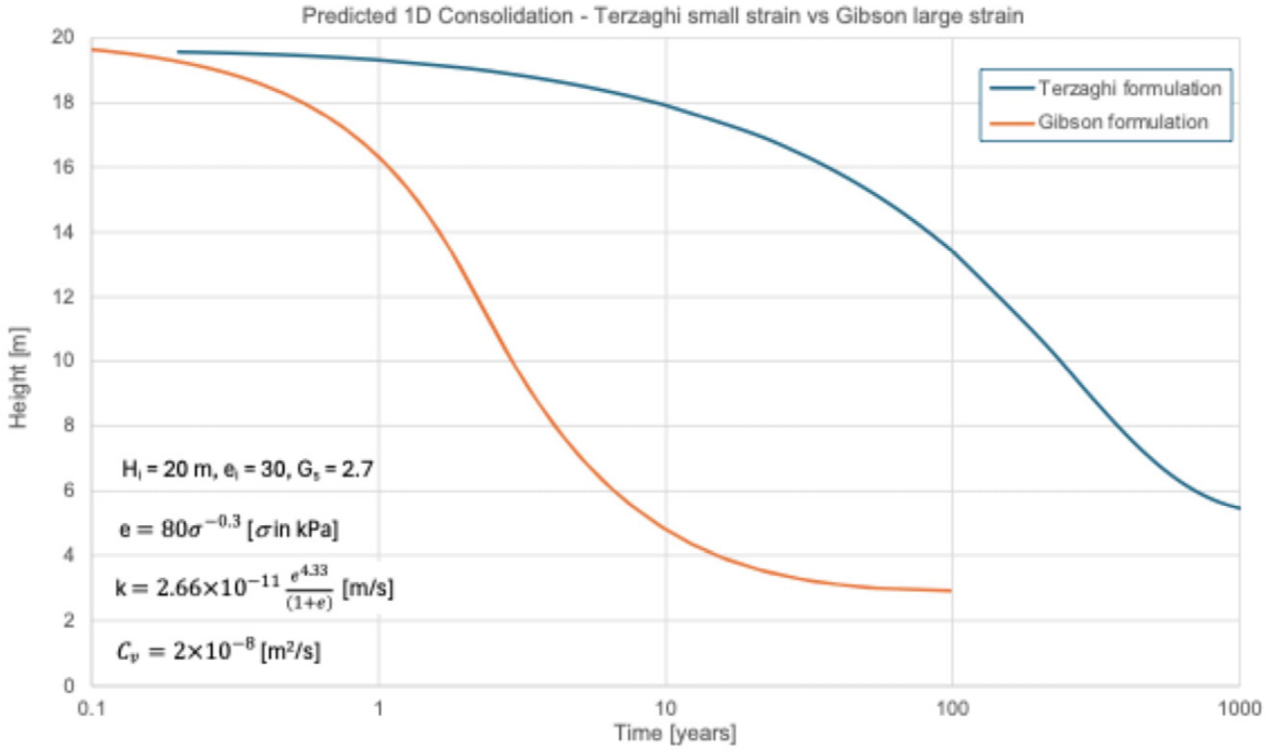

Figure 1: Comparison between small strain and large strain theory (recreated from Carrier *et al.* (1983))

The main reason that large strain theory predicts an increase in the time to consolidation is that the shortening of the draining path due to consolidation is accounted for in the theory thus properly capturing the decrease in the time to consolidate. In in small strain theory the drainage path length is assumed to remain constant.

Other key differences between consolidation analysis of tailings compared to traditional soil mechanics are:

- traditional consolidation is usually focussed on volume change driven by applied stress (e.g. foundations) whereas tailings consolidation is usually focussed on volume change driven by self-weight,
- pore pressure within tailings profiles are typically initially above hydrostatic and transition to hydrostatic or even zero if fully under-drained compared to traditional consolidation where pore pressure typically trends towards hydrostatic alone,

The solution to tailings consolidation using large strain is usually achieved using numerical integration of the non-linear differential equations that govern the theory. Numerical integration typically uses the finite difference or element method where the profile is discretised into a series of discrete 'elements' or 'nodes'. Solutions typically require computerised code as described in Pane and Schiffman (1981), Tito (2015) and GWP (2021) to capture the changes in compressibility, permeability and drainage path.. This approach affords great flexibility in the solution of the highly non-linear large strain theory allowing incorporation of variable material properties and boundary conditions.

More discussion on the use of large strain theory can be found in Schiffman and Carrier (1990) and examples numerical solutions to the theory include Priestley (2011), Ito and Azam (2013) and Zhou *et al.* (2019).

Analytical solution to the fully consolidated state of a column of tailings (such as a single mathematical expression) has advantages. The development of such an expression is described in Section 2.

1.2 CONSOLIDATION TESTING

Irrespective of whether traditional or large strain theory is used, predicting consolidation involves estimation or measurement of compressibility and permeability. These parameters can be used to predict the final settled density profile. Test methods are designed to replicate conditions within the tailings, such as stress and pore pressure, and measure volume change in response to changes to these conditions. In some tests only stress is controlled, such as oedometers (Holtz and Kovacs (1981), while some tests allow the control of stress and pore pressure, such as Rowe cells (Rowe and Barden (1966). In some cases, tests control volume change, such as the constant rate of deformation testing (Znidarcic *et al.* (1986), Pane and Schiffman (1997). The change in response within the tailings is measured through pressure change, sample displacement and in some cases sub-sampling. There are advantages and disadvantages in all test methods and some are not well suited to capturing the highly deformable nature of tailings or mimicking low stress conditions. Testing in a conventional oedometer, for example:

- ignores self-weight affects,
- assumes homogeneous conditions within the sample,
- assumes a relatively small change in void ratio during the test (simulating the application of overburden stress),
- ignores stresses below around10 kPa,
- can exhibit errors due to seepage induced consolidation if the seepage force is higher than the applied pressure,
- usually assumes that the rate of consolidation is a constant (i.e. combing compressibility and permeability into a single c_v value however these can be differentiated),

Further discussion on the limitations of traditional oedometer testing for assessing tailings consolidation properties are provided in Ahmed *et al.* (2023).

Consequently, conventional consolidation tests are generally not suitable for determining the consolidation parameters of tailings and procedures and analyses consistent with the nonlinear consolidation theory must be used for characterisation (Fourie *et al.* (2022), Ahmed *et al.* (2023). More specialised testing, such as Rowe cells, allow compressibility and permeability to be measured independently.

Devices often termed 'slurry consolidometer' or 'slurrometer' are designed to measure low stresses and accommodate larger changes in void ratio (McBride and Baumgartner (1992), Islam *et al.* (2021), Ahmed *et al.* (2023). Slurry consolidometers are typically:

- an oedometer with an extension to facilitate a greater change in void ratio - sometimes referred to as a 'slurryometer' – for example Swarbrick (1992); or
- a purpose-built device, often mainly for research purposes such as the devices described in McBride and Baumgartner (1992), Islam *et al.* (2021)

Other more specialised testing includes derivation of consolidation properties from inverse modelling of boundary condition controlled lab testing, such as constant rate of deformation (CRD) testing (Znidarcic *et al.* (1986), Pane and Schiffman (1997) and under seepage-induced consolidation testing (SICT), see Fourie *et al.* (2022). These methods overcome some of the deficiencies of traditional testing, such as the ability to apply low stress (as in the CRD and SICT) and variation in conditions within the sample (such as CRD).

The initial void ratio generally does not affect the compressibility relationship. If the initial void ratio is relatively high samples will simply settle and not significantly affect the compressibility relationship, unless there is significant segregation. However, if the initial void ratio is relatively low due to, for example, paste thickening, or the tailings particles are highly active (highly negatively charged colloids), then the compressibility may be non-unique under low stress conditions as discussed by Imai (1981). Examples of this are described in Li *et al.* (2009) and Suthaker (1995). Under either condition it is important to test tailings based on samples at their expected void ratio upon discharge.

Simple column settling tests have some advantages over these specialist tests as they accommodate greater sample thicknesses. They also test consolidation effectively under low stress conditions similar to the upper portion of tailings in the field, simulating non-homogeneous density, stress and pore pressure conditions. This makes them an attractive supplement to the more traditional testing described above given they can be undertaken cost effectively with inexpensive equipment. They also allow sedimentation to be studied in detail.

2 ANALYSIS OF COLUMN SETTLING BEHAVIOUR

Column settling tests are commonly used by researchers and practitioners to estimate key tailings properties, see Seddon (2021), Li and van Zyl (2024) including:

- rates of settling under varies conditions such as initial solids content, the effects of additives or different pore water chemistry,
- initially settled density at the cessation of flocculation.

Examples of column testing is provided in Fourie *et al.* (2022) and Ahmed *et al.* (2023).

Column tests can be conducted in a variety of vessels but most commonly conducted in relatively tall cylinders (typically ranging from, but not limited to, 0.5 to 2 m in height)) with diameters that are large enough to reduce drag effects to an acceptable level (typically greater than 100 mm), see) Swarbrick and Fell (1992), Seddon (2021) and Li and van Zyl (2024). Columns can either be operated as sealed at the base (one way drainage or often termed 'undrained') or with additional drainage from the base (two-way drainage or often termed 'drained'). The terms *drained* and *undrained* have been used in this paper to describe column tests with two-way drainage (drained base) and one-way drainage (sealed base) respectively.

The column test method simply requires the sample to be placed in the column in an initial, mixed state (suspension) and allowed to settle over time. The suspension progresses though several stages beginning with flocculation (aggregation of particles within the suspension), followed by settling (particles coming into contact), the process continues with consolidation (particles in contact increasing in density as excess pore pressure is squeezed out due to self-weight). This process is well presented by the conceptual diagram of Imai (1981) which is reproduced in Figure 2.

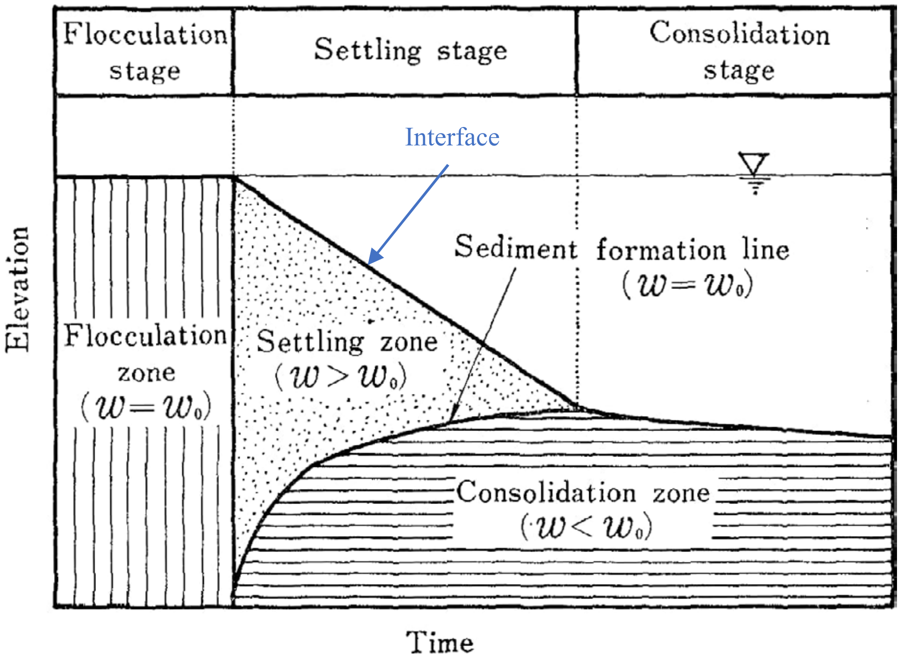

Figure 2: Conceptual diagram of a tailings suspension progressing from flocculation to consolidation (Modified from Imai, (1981): Conceptual diagram of a tailings suspension progressing from flocculation to consolidation (Imai (1981))

The height of the solid-water interface is measured over time, initially at high frequency when the change in solids height is rapid, reducing as the rate of settlement decreases following the interface line as indicated on Figure 2. Once settling is completed the sample continue to consolidate over time within the column. An example of column settling testing including how samples can be prepared and tested is provided in Yang *et al.* (2011).

An example of column settling is shown in Figure 3.

Figure 3: Simple column settling test

Once all downward vertical movement has ceased then the material in the column can be assumed to be fully settled. This represents a state to the far right of the interface line on Figure 2 when there is no further reduction in height due to self-weight. Under these conditions:

- the final average void ratio is known (assuming the initial average solids content and particle density are known),
- pore pressure can be estimated as either being hydrostatic in the case of undrained or effectively zero in the case of a drained test,
- the distribution of void ratio within the column will be related to the effective stress and the relationship between void ratio and effective stress.

These conditions can be harnessed to derive the parameters that describe the compressibility relationship between void ratio and effective stress by integrating void ratio over the length of the sample and comparing this to the average void ratio.

To undertake this integration, it is required to know the form of the void ratio and effective stress relationship as only the parameters of this relationship can be derived and not the relationship itself. The most common form is the well-known logarithmic relationship between void ratio, e, and effective stress, σ':

$$e = e_o - C_c \log(\sigma') \tag{1}$$

However, the common consensus in tailings consolidation research is that this relationship does not properly capture the relationship between e and σ' over the full range of effective stress experienced by tailings. There have been a number of alternate expressions suggested by researchers such as Krizek *et al.* (1977), Gibson *et al.* (1981), Carrier *et al.* (1983). One of the most common expressions is this simple power relationship found by Somogyi (1980), Carrier and Beckman (1984), Swarbrick (1995), Murphy (1997) and others:

$$e = A\sigma'^{b} + e_{\min} \tag{2}$$

The term $e_{\min}$ is the minimum void ratio representing a lower bound that compressibility may asymptote to and taken as zero in many cases. The value of $e_{\min}$ can be derived simply by curve fitting or based on the maximum density derived by other means, such as air drying. Examples of equations 1 and 2 are shown in Figure 4.

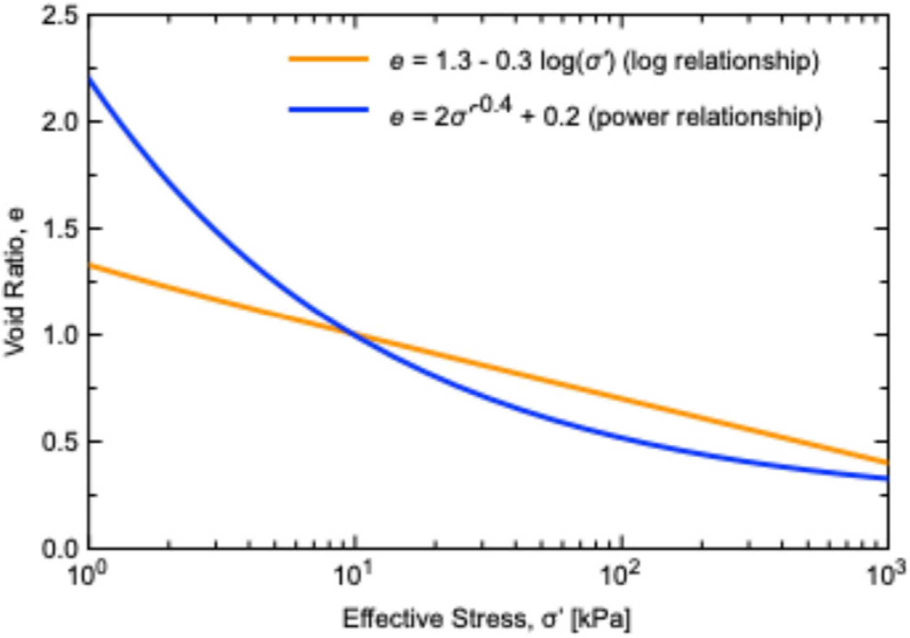

Figure 4: Examples of the logarithmic and power compressibility relationships

Other forms of the compressibility relationship are occasionally used however there is general agreement that tailings compressibility is well represented by the stated power function as discussed in Fourie *et al.* (2022).

Solutions to the derivation of the parameters of both equations (1) and (2) from column tests by integration, these being e_o, C_c, A , b and $e_{\min}$, are provided below.

It is not possible to directly integrate the conditions within a column settlement test using Cartesian coordinates, given the dependence on void ratio on effective stress and vice versa. Consequently, most solutions utilise a coordinate transform to perform the integration. The most common transform for consolidation is the use of Lagrangian or material coordinates, see Gibson *et al.* (1981), Schiffman *et al.* (1988). Material coordinates represent a reference system that is fixed to the solid skeleton and not to Cartesian coordinates thereby taking into account deformation of the soil skeleton.

Use of material coordinates allows the height of a column to be represented as an equivalent height of solid, H_s, which can be derived from the initial height, H_i, and initial void ratio, e_i, of the test:

$$H_s = \frac{H_i}{1 + e_i} \tag{3}$$

Because the amount of solid is fixed, H_s is a constant with only the volume (or height for a 1D column) of fluid changing. If the void ratio is expressed in material coordinates, $e = f(\zeta)$, the final height of the tailings, H_f, can be derived from:

$$H_f = \int_o^{H_s} \left[1 + e(\zeta)\right] \partial\zeta \tag{4}$$

Firstly, it is assumed that we can express void ratio as a unique function of effective stress at the cessation of primary consolidation such that $e = f(\sigma')$, noting the potential effects of high initial solids content as discussed in Section 1.2. We can then derive an expression for effective stress as a function of the material coordinate, $\sigma' = f(\zeta)$ by combining stress due to the solid particles alone based on particle unit weight, γ_s, plus the weight of water (integrated over void ratio), less hydrostatic pore pressure (assuming a drained column) plus any addition uniform surcharge pressure, p, such as a capping material. The resulting relationship in material coordinates is:

$$\sigma'(\zeta) = \gamma_s \zeta + \gamma_w \left(\int_0^{\zeta} [e] d\zeta \right) - \gamma_w h + p \tag{5}$$

Where h is the depth below the surface. We can also express h in terms of the material coordinates:

$$h(\zeta) = \int_{0}^{\zeta} \left[1 + e\right] \mathrm{d}\zeta \tag{6}$$

Substituting Eqn 3 into Eqn 2 results in a simple expression for effective stress under fully consolidated, hydrostatic conditions in terms of material coordinates:

$$\sigma'(\zeta) = \left[\gamma_s - \gamma_w\right]\zeta + p \tag{7}$$

Under drained conditions the expression simplifies to:

$$\sigma'(\zeta) = \gamma_s \zeta + p \tag{8}$$

This means that effective stress can be written entirely in terms of material coordinates.

$$H_f = \int_{o}^{H_s} \left[1 + e(\sigma'(\zeta))\right] \partial\zeta \tag{9}$$

In the case of an assumed $e(\sigma')$ relationship based on Eqn 1 this integral becomes:

$$H_f = \int_{o}^{H_s} \left[1 + e_o - C_c \log\left(\left[\gamma_s - \gamma_w\right]\zeta + p\right)\right] \partial\zeta \tag{10}$$

In the case of an assumed $e(\sigma')$ relationship based on Eqn (2) this integral becomes:

$$H_f = \int_{0}^{H_s} \left[A\left(\left[\gamma_s - \gamma_w\right]\} + p\right)^{b} + e_{\min}\right] \partial\zeta \tag{11}$$

Expanding and simplifying Eqn 10 gives this expression for a e log p relationship:

$$H_f = H_s\left(1 + \left[e_o - C_c \log\left(\frac{\left[\gamma_s - \gamma_w\right]H_s + p}{\exp(1)}\right) + \frac{C_c p}{\left[\gamma_s - \gamma_w\right]H_s} \log\left(\frac{p}{\left[\gamma_s - \gamma_w\right]H_s + p}\right)\right]\right) \tag{12}$$

While expanding and simplifying Eqn 11 gives this expression for a power relationship:

$$H_f = H_s\left(1 + e_{\min}\right) + \frac{A\left(\left[\gamma_s - \gamma_w\right]H_s + p\right)^{b+1} - Ap^{b+1}}{\left[\gamma_s - \gamma_w\right](b+1)} \tag{13}$$

Equations 12 and 13 are closed-form solutions to the consolidated height and can be used in a number of applications as discussed in subsequent sections. Verification of these relationships is also provided.

If the column is drained or there is no surcharge loading, then Eqns 12 and 13 can be simplified. For example, Eqn 13 without pore pressure becomes:

$$H_f = H_s\left(1 + e_{\min}\right) + \frac{A\left(\left[\gamma_s\right]H_s + p\right)^{b+1} - Ap^{b+1}}{\left[\gamma_s\right](b+1)} \tag{14}$$

Eqn 13 without surcharge loading reduces to:

$$H_f = H_s\left(1 + e_{\min}\right) + \frac{A\left(\left[\gamma_s - \gamma_w\right]H_s\right)^{b+1}}{\left[\gamma_s - \gamma_w\right](b+1)} \tag{15}$$

While Eqn 13 without pore pressure or surcharge loading reduces to:

$$H_f = H_s\left(1 + e_{\min}\right) + \frac{A\left(\left[\gamma_s\right]H_s\right)^{b+1}}{\left[\gamma_s\right](b+1)} \tag{16}$$

Eqn 12 can be simplified in the same manner. The average final void ratio, e_f^{avg}, is given by:

$$e_f^{\mathrm{avg}} = \frac{H_i}{H_f} - 1 \tag{17}$$

3 VERIFICATION

The validity of equations 12 and 13 were checked in a variety of ways. If parameter constants such as C_c and b are set to zero, then void ratio becomes a constant and these equations reduce to $H_f = H_s(1 + e)$ as expected. These relationships were also checked by comparing H_f derived by numerical integration, equivalent to finite difference integration used by programs such as Pane and Schiffman (1981), Tito (2015) and GWP (2021).

3.1 VERIFICATION USING LAYER APPROXIMATION

An approximate solution to a fully consolidated tailings profile was derived using a discrete layered approach similar to that described in Holtz and Kovacs (1981). The process is as follows:

1. Divide the profile into a number of layers assuming each can be approximated by a constant void ratio.
2. Calculate the effective stress at the centroid of each layer based on the density derived from the constant void ratio, the weight of overlying layers and pore pressure based on depth (hydrostatic or none for undrained or drained, respectively).
3. Estimate the void ratio from the effective stress using Eqn 1 or 2.
4. Recalculate the void ratio and iterate over these steps until a converged solution is found.

Figure 5 shows a comparison of settled heights calculated by the approximate, layered approach described above and Eqn 12. The example is for a column of tailings initially 10 m high with an initial, uniform void ratio of 1.5. The compressibility is given by the expression $e = 1.378 - 0.321\,\log(\sigma')$.

Multiple solutions are provided for the column divided up into 1 to 20 layers and with uniform surcharge loading ranging from 1 to 50 kPa. The closed-form solution given by Eqn 12 is plotted as a constant, dashed line.

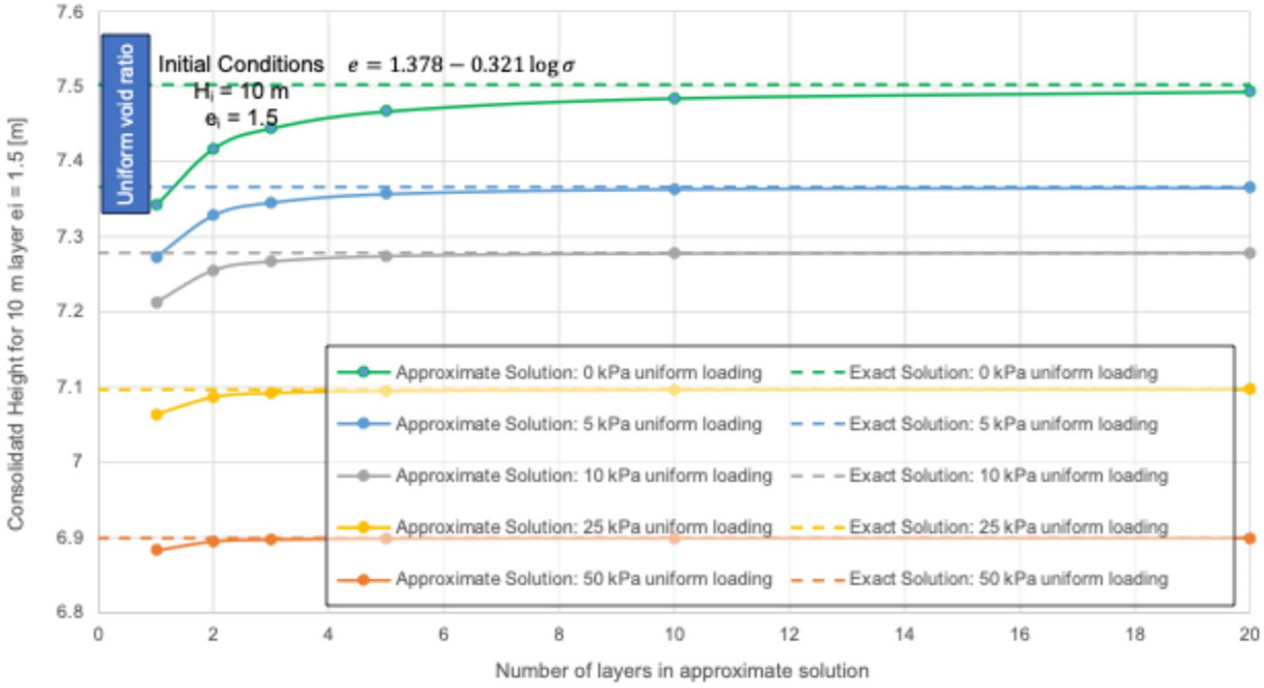

Figure 5: Comparison of the approximate solution based on a number of layers and applied surcharge load and the closed-form solution in Eqn (12)

From Figure 5 it is evident that:

- the approximate solution approaches the closed-form solution as the number of layers increases,
- the approximate solution predicts higher consolidation in all cases,
- the difference between approximate and closed-form solutions is less with increasing layers and increasing surcharge loading.

These results are aligned with expectations given that the determination of consolidated height in traditional soil mechanics assumes that compression is due to external loading (such as overburden) alone and has no self-weight component. This means the error from using the approximate solution is generally small. Figure 5 shows that this error is always conservative because the approximate solution always over predicts the amount of settlement.

Figure 6 reinterprets the results shown in Figure 5 by plotting the height against the applied load rather than number of layers. The results show that the error reduces with increasing surcharge load and the approximate solution predicts greater consolidation in all cases.

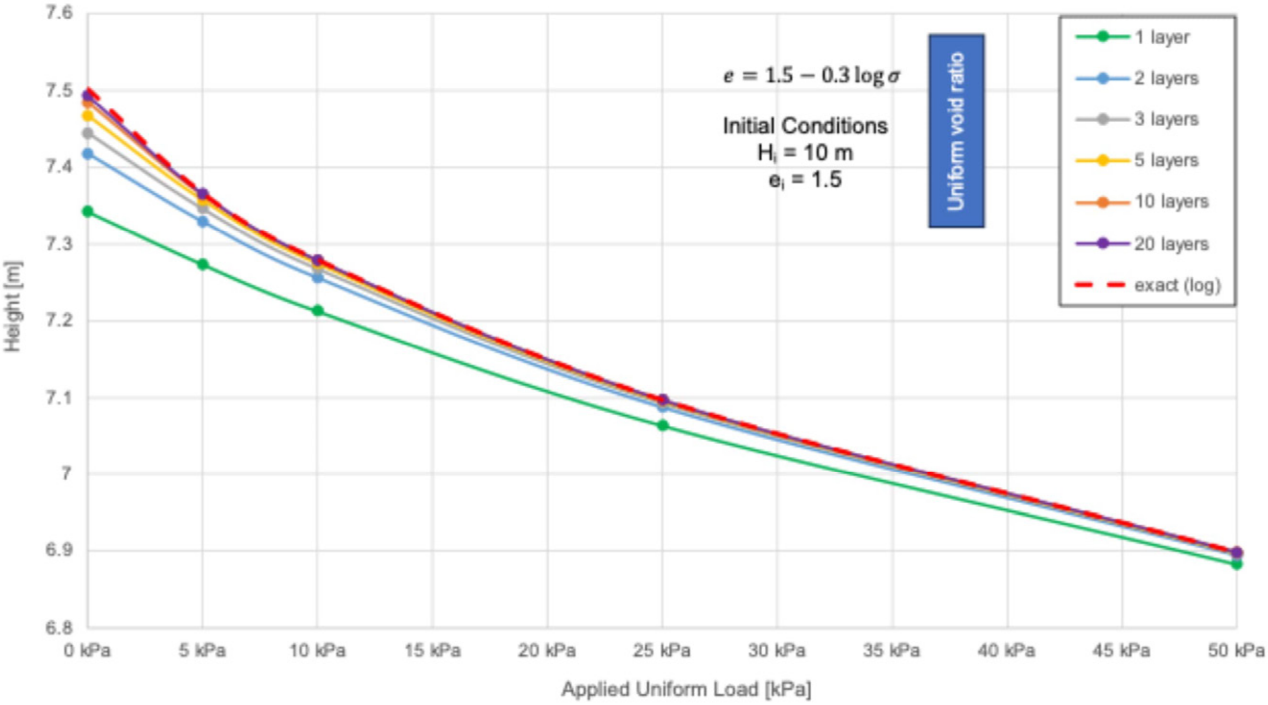

Figure 6: Comparison of the approximate solution based on a number of layers and applied surcharge load and the closed-form solution in Eqn (12)

3.2 VERIFICATION USING NUMERICAL ANALYSIS

Equations 13 and 14 have been checked against the commercial software package FSConsol developed by GWP Geo Software (GWP (2021). FSConsol provides approximate solutions to 1D large strain consolidation programs using numerical integration. Solutions are approximate due to the solution of the Gibson *et al.* (1967) theory using the finite difference method which assumes void ratio and pore pressure is constant within each segmental volume.

Equations 15 and 16 and FSConsol have been used to predict void ratio and density profiles for an example consolidation relationship as shown in Figure 7. Equations 15 and 16 can be used to derive void ratio and density profiles using the following approach:

1. Calculate the height of solids in the profile, H_s, from the initial height, H_i, and the initial void ratio, e_i, (assumed to be uniform) and Equation 3.
2. Divide the profile into a number of layers based on increments of H_s up to and including the total H_s.
3. Calculate the final height from the height of solid using either Equation 12 or 13 (depending on available compressibility parameters) modified for the drainage condition (drained or undrained),
4. Calculate the effective stress under fully consolidated conditions using the relevant undrained or drained relationship given in Equations 7 or Equation 8 respectively,
5. Calculate the corresponding void ratio from the effective stress (and thus dry density or any other required parameters).
6. Calculate any other required parameters, such as dry density, density, and moisture content, using the appropriate phase relationships and particle density.

In this case 12 layers have been used to derive the void ratio profile and density profiles for both one-way drainage (using Eqn 15) and two-way drainage (using Eqn 16) as shown in Figure 7. The same approach can be used to determine profiles based on the solution for log-linear compressibility (Eqn 12) with or without overburden and either one or two-way drainage.

The accuracy of the solution predicted by FSConsol is dependent upon the number of layers and therefore, a relatively large number is required to ensure an accurate prediction. The material parameters and initial sample height used for both analyses are included in Figure 7.

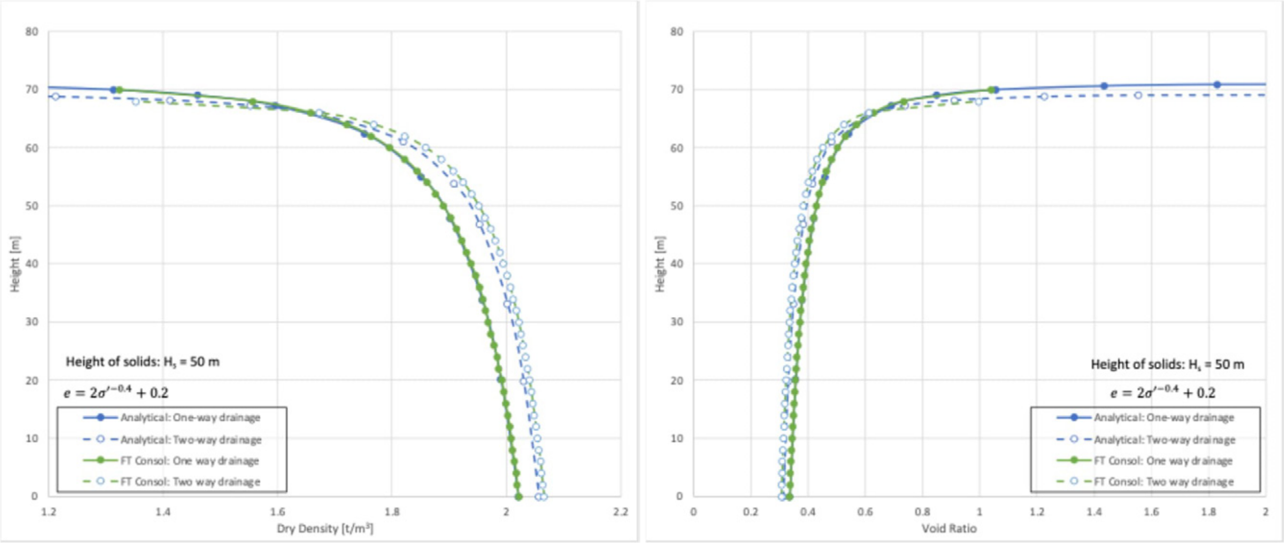

Figure 7: Comparison between Equation (13) and approximate (FSConsol) density and void ratio profiles

Figure 7 shows close agreement between analytical and numerical solutions, particularly one-way drainage cases, which plot exactly over one another, indicating the validity of Equation 13 against a well-accepted numerical solution. A paired t-test found there was no significant difference between the analytical (Equation 13) and numerical (FSConsol data sets.

4 APPLICATION

4.1 ALTERNATE APPLICATIONS

The derived solutions are analytical solutions, which can be solved quickly. The solutions are useful for reliability assessments because they allow the application of, for example, Monte Carlo techniques. In addition, analytical solutions lend themselves to being inverted or rearranged such that they can be used to solve other unknowns, such as the underlying material properties (e.g. compressibility).

Examples of how equations 12 and 13 may be used are provided below:

1. A closed form solution to the consolidated height under self-weight and surcharge loading.
2. Back-calculation of compressibility parameters from column settling tests, assuming either a logarithmic or power compressibility relationship.
3. To determine the void ratio and effective stress profiles within a tailings mass after consolidation.
4. Estimate the final settled density from the knowledge of the deposited mass of tailings and compressibility parameters.
5. Facilitate compressibility parameter determination by the best fit of compression (Rowe cell and/or oedometer) and column test data combined into one set of data.
6. Back-calculation of compressibility parameters from a soil density profile.

Application 1 has already been demonstrated in Section 2. Examples of applications 2, 3, 4, 5 and 6 are provided below.

4.2 APPLICATION 2 – ANALYSIS OF COLUMN SETTLING DATA

Figure 8**Error! Reference source not found.** shows an example of predictions for column setting data. All samples were tailings from the same gold mine, tested using drained and undrained cylindrical columns. The initial heights were similar; however, the initial solids content varied from 25 to 60% by mass, such that the final height varied significantly. As suggested by Equations 12 and 13, it is the height of solids that dictates the final settled height, and this can be varied between tests by either varying the initial solids content, initial column height, or both. Usually, off-the-shelf equipment comes in standard heights, such that changing the solids content is easier than trying to run tests on shorter samples.

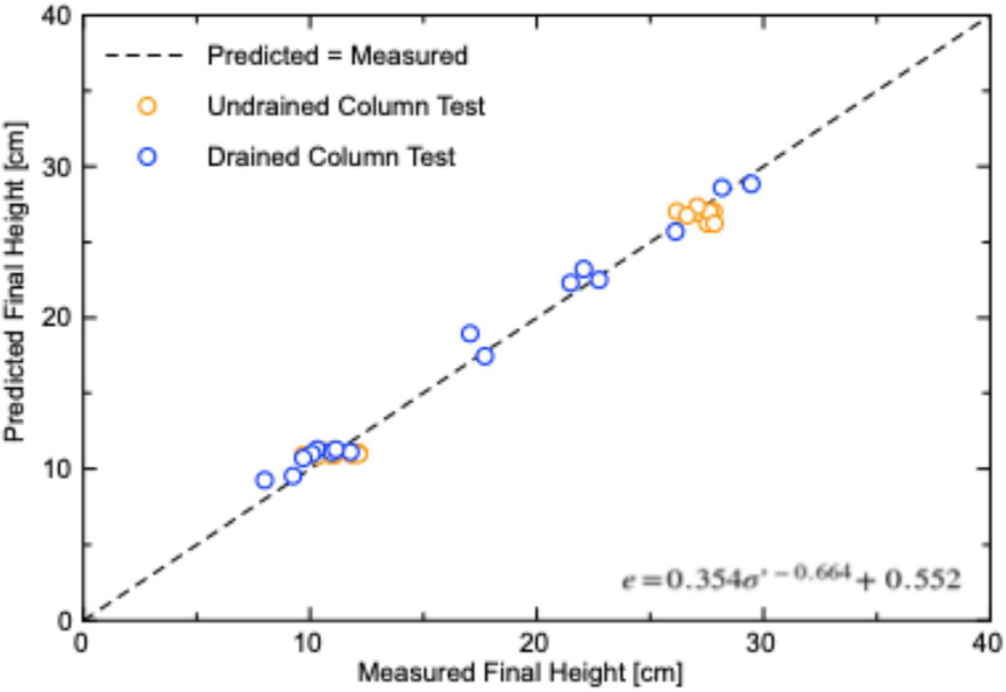

Figure 8: Estimated settled height of column tests

Figure 8 was prepared by the following steps:

1. The particle density, γ_s, final height, H_f, and height of solids, H_s, were determined for each column test while the height of solids was derived from the initial height and initial void ratio following Equation 3.
2. Initial estimates of compressibility parameters were set assuming a power relationship (Equation 2), i.e. initial estimates for A, b and $e_{\min}$.
3. The predicted final height was calculated for each column test based on these initial estimates of compressibility parameters and Equation 13, noting that there was no surcharge in any tests (p = 0 Equation 15) and γ_s = 0 (Equation 16) for column tests that were allowed to drain from the base (two-way drainage).
4. The difference between the measured and predicted final heights was calculated and summed to determine the overall error. Optimising software (in this case, Solver in Excel) was used to adjust the compressibility parameters to determine the best fit by iteration and error minimisation.
5. Predicted final heights were then plotted against measured final heights to create Figure 8.

The line of best fit for the data in this example is included in Figure 8. The fit for the power relationship (Equation 13) was found to be slightly better than a logarithmic relationship (Equation 12). From Figure 8 it is evident that:

- Predicted heights are a reasonable match to measured heights across a range of final heights (100 to 400 mm); and
- Both undrained and drained tests show acceptable predictions, with different relationships to capture different drainage conditions (i.e., Equations 15 and 16, respectively).

The parameters A, b, and $e_{\min}$ derived from this fitting process describe the tailings compressibility and can be used for any tailings thickness.. However, care should be taken not to extrapolate too far beyond the range used in laboratory testing. This approach can be used to supplement testing under higher stress, such as Rowe cell testing, as described in Section 4.5..

4.3 APPLICATION 3 – DENSITY PROFILES

The analytical solutions provided here can be used to derive a fully consolidated profile, such as void ratio, density or water content. The steps to derive such a profile are outlined in Section 3.2.

Figure 9 and Figure 10 depict density and void ratio profiles under fully consolidated conditions respectively (primary consolidation) using this approach.

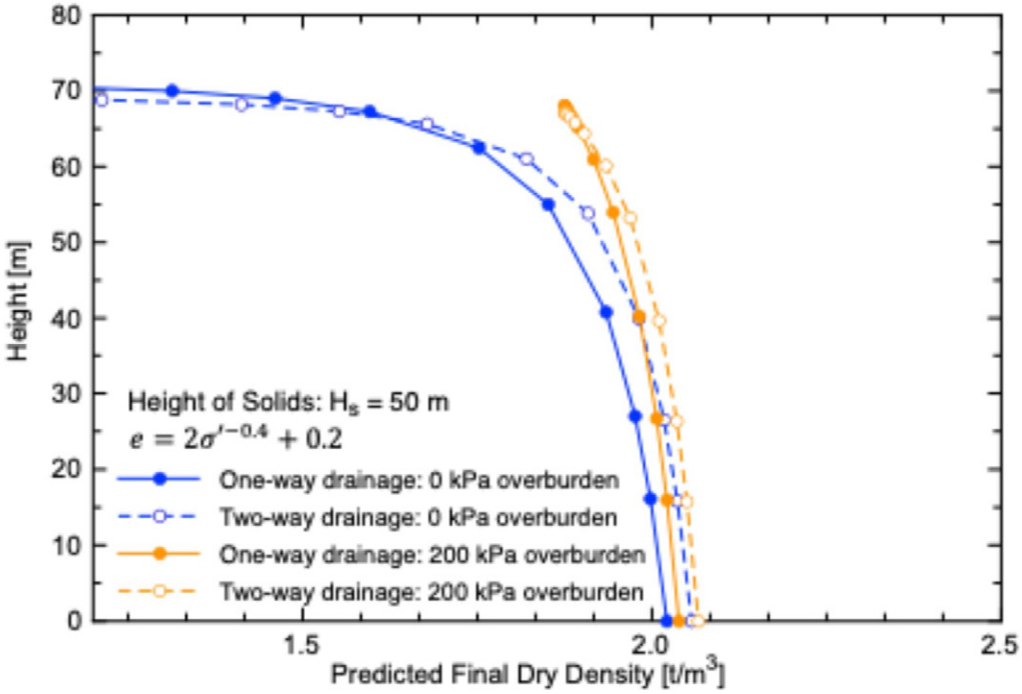

Figure 9: Estimated density profiles using Equation 15

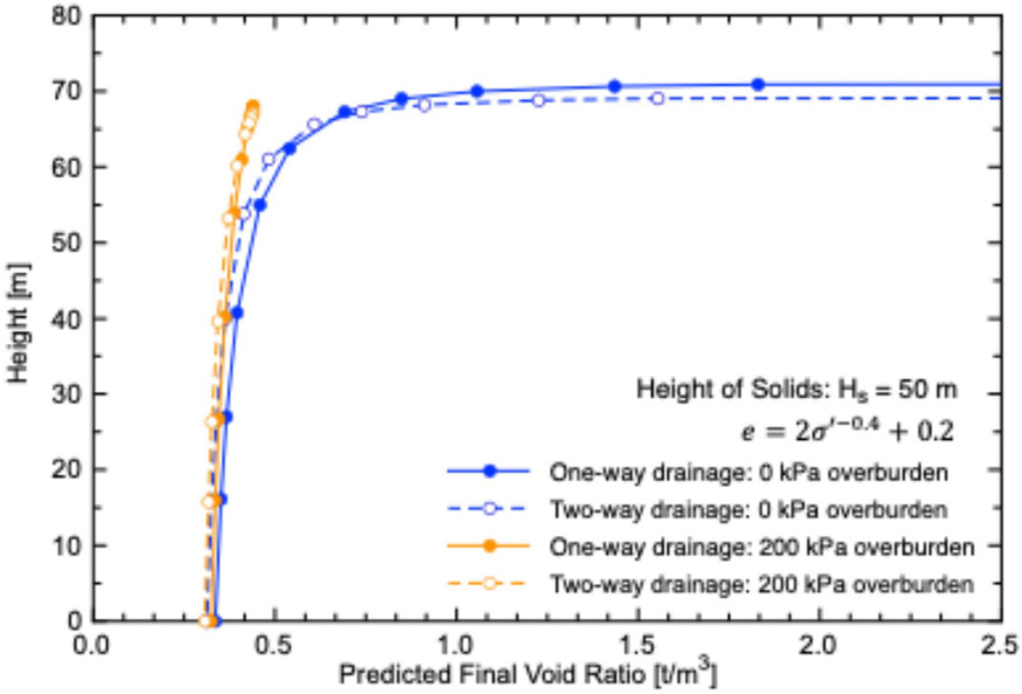

Figure 10: Estimated void ratio using Equation 15

Knowledge of the consolidated profiles allow the derivation or estimation of other parameters such as strength and permeability through correlations or established relationships.

4.4 APPLICATION 4 – ESTIMATE OF FINAL DENSITY FROM STORED MASS

Another example is the derivation of final density based on the stored mass of tailings in combination with compressibility parameters. Examination of Equations 12 and 13 show that the final height of a depth of tailings is related to the compressibility parameters, soil particle density and height of solids. The particle density is often known or can be estimated. This means that if the compressibility is known in terms of Equations 1 or 2, and the mass of solids is known or can be estimated, then the average density can be estimated from the settled height using equations 12 and 13.

Examples of this application are shown in Table 1 whereby total stored dry mass, material parameters being particle density and compressibility (as defined by Eqn 2) and geometric parameters such as average area and height of solids (derived from stored mass, particle density and area) were derived for six sites from public literature or technical reports. Particle density, compressibility and height of solids were then used to derive the final height using Eqn 13, including any overburden pressure. The final dry density was derived from the final height and compared to the actual measured dry densities provided in the same public publications and reports.

The six examples provided in Table 1 demonstrate that it is possible to get an accurate estimate of final dry density without the need for any detailed numerical analysis. This approach may also be helpful in demonstrating the benefits of accurately recording the mass of solids deposited into a storage facility.

Table 1: Predicted and measured final dry density based on storage mass and compressibility

Parameter	Uranium[1]	Zinc[2]	Gold[3]	Gold[4]	Gold[5]	Rare Earth[6]
Particle density [t/m^3]	2.7	3.16	2.81	2.74	2.74	3.08
Dry mass of tails [Mt]	41.7	10.5	0.83	121.2	121.2	1.4
Storage area [Ha]	35.2	20.6	2.1	42.6	42.6	36.2
Overburden pressure [kPa]	400	0	0	0	0	0
Height of solids	43.9	16.1	14.1	103.8	103.8	1.29
A [1/kPa] (Eqn 2)	1.482	1.5	0.718	1.26	0.94	0.64
b (Eqn 2)	-0.1171	-0.12	-0.35	-0.076	-0.0813	-0.76
$e_{\min}$ (Eqn 2)	0	0	0.58	0	0.25	0.4
Final height [m]	73.9	29.8	24.6	183.9	187.4	2.1
Predicted final dry density [t/m^3]	1.60	1.71	1.61	1.55	1.52	1.89
Measured final dry density [t/m^3]	1.62	1.70	1.62	1.51	1.51	1.71[7]

[1] Uranium mine – source of information not publicly available

[2] McArthur River Mine – parameters extracted from GHD (2016) and GHD (2017)

[3] Gold mine in Australia – source of information not publicly available

[4] Gold mine in Canada – parameters extracted from Tripathi *et al.* (2020)

[5] Gold mine in Canada – parameters extracted from Tripathi *et al.* (2020) with refitting

[6] Yangibana Rare Earth Project – parameters extracted from column testing data provided in GHD (2020)

[7] Estimate based on linear shrinkage testing given in GHD (2020)

4.5 APPLICATION 5 – COMBINING DIFFERENT COMPRESSIBILITY TESTING

The analytical equations presented here can be used to facilitate a combined fit of compressibility parameters under both low stress (such as column tests) and elevated stress (such as oedometer and Rowe cell testing). This allows derivation of a single set of parameters over an extended stress range thus better capturing overall tailings consolidation behaviour.

The combined fit can be undertaken simply by using the fitting techniques described in Section 4.2 in unison with fitting of the same parameters to Rowe cell, oedometer and/or any other relevant data (e.g. slurryometers). Figure 11 shows an example whereby compressibility relationships have been fitted to data from column settling tests and Rowe Cell testing, both separately and combined.

Note that it is not possible to plot column settling data on as compressibility as column settling data measures average and not provide single point values of void ratio and effective stress. Column test data in Figure 11 has been plotted as the average effective stress in the column against the average void ratio attained after consolidation for illustrative purposes only. Although the plot does not accurately reflect column test data, the compressibility parameters upon which the fitted curve is based are accurate as they are fitted to the analytical solution through the use of equations 12 or 13.

Figure 11 includes three fitted relationships:

- column settling test data fitted to Equation 15,
- Rowe Cell test data fitted to Equation 2,
- both datasets fitted to equations 15 and 2 in unison.

Figure 11 demonstrates the following:

- no single test method (for those methods used for this particularly set of tests) covered the full range of stress expected in a column of tailings; and
- the combination of data sets provided the ability to develop a single relationship that works reasonably well across a large stress range.

The void ratios derived from column testing are significantly less than for the Rowe cells tests. This difference is due to the low stress conditions simulated in column tests, which rely entirely on self-weight. This is opposed to Rowe cells where the initial void ratio is dictated by the lowest applied stress. Columns tests that are around 0.5 m tall, will have average effective stress due to self-weight of around 0.5 to 2 kPa after settling, whereas the minimum applied stress for Rowe cells is around 25 kPa. This difference in stress results in differences in the void ratio attained during the test.

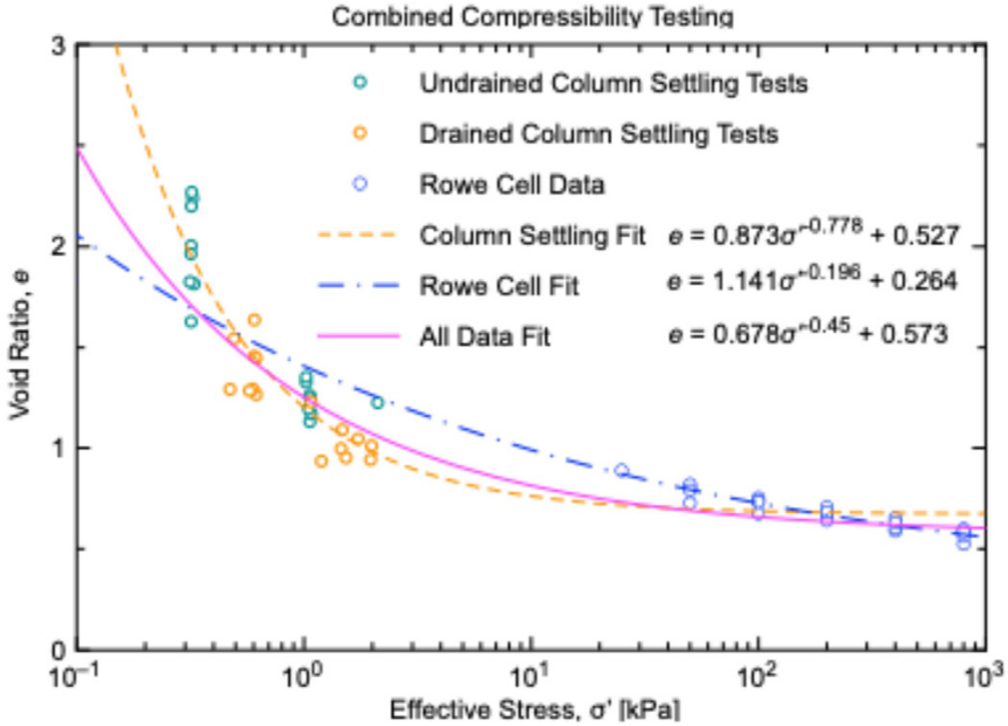

Figure 11: Combined compressibility data from settling columns and Rowe Cell testing

Although the data sets are based on a combination of average and point measurements, the combined data sets demonstrate that they are governed by a common compressibility relationship, notwithstanding the variation observed in column testing results and the discussion on the potential effects of high initial solids content as discussed in Section 1.2. Column test variation is usually attributed to adhesion of the tailings to the internal walls of the column test apparatus, causing 'hang up' and overestimation of the void ratio in some cases. This effect worsens as the diameter and height of test samples decrease.

5 CONCLUSIONS

The equations provided here (i.e. Equation 12 and 13 in particular) provide analytical solutions to the height of tailings (or other slurry) material at the end of primary consolidation. Key assumptions in the derivation of these solutions are:

- tailings can be represented as homogenous with reasonable accuracy for the problem being assessed or analysed,
- compressibility of the tailings can be represented as a single, smooth continuous relationship, with reasonable accuracy for the problem being assessed or analysed

It is noted that the second assumption must be valid to attain a solution to large strain theory generally.

The relationships derived here, by virtual of their analytical nature, provide several capabilities in the area of tailings consolidation, including:

- determination of the consolidated height under self-weight and surcharge loading based on two well-known compressibility relationships,
- determination of void ratio and effective stress profiles after consolidation from the compressibility relationship,
- back-calculation of compressibility parameters from column settling tests or from soil profiles; and
- estimation of consolidated height from the height of solids and compressibility parameters.

The author has used these relationships over many years to estimate compressibility parameters and undertake predictions under a number of different conditions. It is hoped that these relationships prove useful to others.

CRediT authorship contribution statement

Gareth Swarbrick: Methodology, Writing - original draft.

6 REFERENCES

Ahmed M., Beier N.A. and Kaminsky H. (2023), *A Comprehensive Review of Large Strain Consolidation Testing for Application in Oil Sands Mine Tailings*, Mining, ISSN:2673-6489, 3(1):121-150, https://www.mdpi.com/2673-6489/3/1/8.

Carrier W.D. and Beckman J.F. (1984), *Correlations between index tests and the properties of remoulded clays*, Géotechnique, 34(2):211-228.

Carrier W.D., Bromwell L.G. and Somogyi F. (1983), *Design capacity of slurried mineral waste ponds*, Journal of Geotechnical Engineering, ASCE, 109(5):699-716.

Fourie A., Verdugo R., Bjelkevik A., Torres-Cruz L.A. and Znidarcic D. (2022), *20ICSMGE State of the Art - Tailings*.

GHD (2016), *Tailings Deposition into Open Pit Consolidation Modelling*, McArthur River Mine Overburden Management Project - Draft Environmental Impact Statement, 32/17476/05 65221, 3 November, pp 22, https://ntepa.nt.gov.au/__data/assets/pdf_file/0010/407566/mrm_overburden_draft_eis_appendixAC_tailings_deposition_open_pit_consolidation_modelling.pdf.

GHD (2017), *Appendix P – Revised Tailings Consolidation Report*, McArthur River Mine Overburden Management Project - Supplement to Environmental Impact Statement, G:\32\1747605\WP\65221, 5 December, pp 18, https://ntepa.nt.gov.au/__data/assets/pdf_file/0020/554024/mrm_overburden_supplement_AppendixP_Updated_Tailings_Consolidation_Report.PDF.

GHD (2020), *TSF Design Development – Pre-Construction Report*, Yangibana Rare Earths Project, YGB-31-100-ENG-CIV-REP-0001, 2 September, pp 218, https://www.epa.wa.gov.au/sites/default/files/Referral_Documentation/Hastings%20APPENDIX%204-7.pdf.

Gibson R.E., England G.L. and Hussey M.J.L. (1967), *The theory of one-dimensional consolidation of saturated clays, I. Finite non-linear consolidation of thin homogeneous layers.*, Géotechnique, 17:261-273.

Gibson R.E., Schiffman R.L. and Cargill K.W. (1981), *The theory of one-dimensional consolidation of saturated clays, II. Finite nonlinear consolidation of thick homogeneous layers*, Canadian Geotechnical Journal, 18(2):280-293.

GWP (2021), *FSConsol: Program*, Edmonton, Canada, GWP Geo Software

Holtz R.D. and Kovacs W.D. (1981), *An Introduction to Geotechnical Engineering*, Newmark N.M. and Hall W.J. eds, New Jersey, Prentice-Hall.

Imai G. (1981), *Experimental Studies on Sedimentation Mechanism and Sediment Formation of Clay Materials*, Soils and Foundations, ISSN:0038-0806, 21(1 March:7-20 https://www.sciencedirect.com/science/article/pii/S0038080620324847.

Islam S., Williams D.J. and Bhuyan M.H. (2021), *Consolidation testing of tailings in a slurry consolidometer using constant rate and accelerated loading*, Proceedings of the Institution of Civil Engineers-Geotechnical Engineering, ISSN:1751-8563, 176(3):295-305.

Ito M. and Azam S. (2013), *Large-strain consolidation modeling of mine waste tailings*, Environmental Systems Research, ISSN:2193-2697, 2(1) 2013/07/24:7, https://doi.org/10.1186/2193-2697-2-7.

Krizek R.J., Casteleiro M. and Edil T.B. (1977), *Desiccation and consolidation of dredged materials*, Journal of Geotechnical Engineering, ASCE, 103:1399-1418.

Li A.L., Been K., Ritchie D. and Welch D. (2009), *Stability of Large Thickened, Non-Segregated Tailings Slopes*, Paste 2009: Proceedings of the Twelfth International Seminar on Paste and Thickened Tailings, R. Jewell A.B.F.S.B. and Wiertz J. eds., Viña del Mar, 21-24 April, Australian Centre for Geomechanics, https://papers.acg.uwa.edu.au/p/963_34_Li/.

Li Y. and van Zyl D. (2024), *Analysing the segregation of coarse tailings particles with a zone-formation differential settling model*, Paste 2024: Proceedings of the 26th International Conference on Paste, Thickened and Filtered Tailings, Fourie A.B. and Reid D. eds., Perth, 16-18 April 2024, Australian Centre for Geomechanics, https://papers.acg.uwa.edu.au/p/2455_22_Li/.

McBride R. and Baumgartner N. (1992), *A simple slurry consolidometer designed for the estimation of the consistency limits of soils*, Journal of terramechanics, ISSN:0022-4898, 29(2):223-238.

Murphy S.D. (1997), *Consolidation modelling of deep tailings deposition - A Case Study*, Geoenvironment 97 - 1st Australia-New Zealand Conference on Environmental Geotechnics, Bouzza A., Kodikara J. and Parker R. eds., Melbourne, Australia, 26-28 November, A.A. Balkema, 401-406.

Pane V. and Schiffman R.L. (1981), *FSCON4-I, Version 1, level A, one-dimensional finite strain consolidation of a thick, normally consolidated homogeneous layer*, Department of Civil Engineering, University of Colorado.

Pane V. and Schiffman R.L. (1997), *The permeability of clay suspensions*, Géotechnique, ISSN:0016-8505, 47(2):273-288.

Priestley D. (2011), *Modeling multidimensional large strain consolidation of tailings*, The Faculty of Graduate Studies (Mining Engineering), December, The University of British Columbia, https://open.library.ubc.ca/collections/24/items/1.0053478.

Rowe P.W. and Barden L. (1966), *A new consolidation cell*, Géotechnique, 4(2):162-170.

Schiffman R.L. and Carrier W.D. (1990), *Large strain consolidation used in the design of tailings empoundments*, International Symposium on Safety and Rehabilitation of Tailings Dams, Sydney, Australia, Australian National Committee on Large Dams, Vol 1, 156-174.

Schiffman R.L., Vick S.G. and Gibson R.E. (1988), *Behaviour and properties of hydraulic fills*, in Hydraulic Fill Structures, van Zyl D.J.A. and Vick S.G. eds, Colorado State University, American Society of Civil Engineers, pp. 166-202.

Seddon K.D. (2021), *The geomechanics of thickened and paste tailings*, Paste 2021: Proceedings of the 24th International Conference on Paste, Thickened and Filtered Tailings, Fourie A.B. and Reid D. eds., Online and Perth, Australia, 21-23 September, Australian Centre for Geomechanics, https://papers.acg.uwa.edu.au/p/2115_18_Seddon/.

Somogyi F. (1980), *Large strain consolidation of fine-grained slurries*, Canadian Society for Civil Engineering Annual Conference, Winnipeg, Manitoba, Canada, Canadian Society for Civil Engineering.

Suthaker N.N. (1995), *Geotechnics of Oil Sand Fine Tailings*, Department of Civil Engineering, Faculty of Graduate Studies and Research, Fall, University of Alberta, https://era.library.ualberta.ca/items/e3747c37-c2d9-4da8-bf46-508004b16b6d.

Swarbrick G.E. (1992), *Transient unsaturated consolidation in desiccating mine tailings*, School of Civil Engineering, Sydney, April, University of NSW.

Swarbrick G.E. (1995), *Large strain consolidation in unsaturated porous media*, First International Conference on Unsaturated Soils, Alonso E.E. and Delage P. eds., Paris, 6-8 September, ENDPC, ISBN:2-85978-241-9, Vol 2, 583-590.

Swarbrick G.E. and Fell R. (1992), *Modelling the desiccating behaviour of mine tailings*, Journal of Geotechnical Engineering, ASCE, ISSN:0733-9410, 118(4):540-557.

Terzaghi K. (1943), *Theoretical soil mechanics*, John Wiley and Sons.

Tito L. (2015), *Numerical evaluation of one-dimensional large-strain consolidation of mine tailings*, Department of Civil & Environmental Engineering, Department of Civil and Environmental Engineering, Colorado State University, 110.

Tripathi S., Scheremeta J., Coffin J. and Reiva J. (2020), *Consolidation Modeling for Design of Complex In-pit Tailings Storage Facility*, proceedings of the 24th international conference on tailings and mine waste, Virtual Event, November 15–18, UBC Studios, https://tailingsandminewaste.com/2020/TMW2020_V3.pdf.

Yang J.W., Yin X.T., Liao Y.F. and Ou X.D. (2011), *Experimental Study on Mechanism for Self-Weight Consolidation of the Red Mud Tailings Placed in the Karsts*, Applied Mechanics and Materials, ISSN:1660-9336, 90-93:3102-3107.

Zhou H., Amodio A. and Boylan N. (2019), *Informed mine closure by multi-dimensional modelling of tailings deposition and consolidation*, Mine Closure 2019: Proceedings of the 13th International Conference on Mine Closure, Fourie A.B. and Tibbett M. eds., Perth, 3-5 September 2019, Australian Centre for Geomechanics, https://papers.acg.uwa.edu.au/p/1915_23_Zhou/.

Znidarcic R., Schiffman R.L., Pane V., Croce P., Ko H.-Y. and Olsen H.W. (1986), *The theory of one-dimensional consolidation of saturated clays, V. Constant rate of deformation testing and analysis*, Géotechnique, 36(2):227-237.