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Large Deformation Analysis of Strip Footing on Layered Purely Cohesive Soils

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Summary The bearing capacity of footings on layered soils has already received significant attention of researchers, however, most of the reported studies are limited to footings resting on the surface of soil, and are based on the assumption that the displacement of the footing is very small. In this paper, an approach developed by Hu and Randolph (15), which combines the Arbitrary Lagrangian Eulerian (ALE) approach and the finite element (FE) method, is adopted to allow the large penetration analysis of footings on layered soil. The FE configurations and ALE approach were verified by comparing the FE small deformation analysis results with available analytical and semi-empirical predictions, and the results of ALE large deformation analyses with available solutions for homogeneous soil.

Large deformation analyses of strip footings on strong-over-weak layered cohesive soils were carried out. The load - displacement curves and the bearing capacity factors predicted by the large deformation analysis are given in the paper. It is shown that during footing penetration, both the increased bearing resistance from the soil above the footing base and the decreased bearing resistance from the weaker bottom soil affect the overall bearing capacity. Bearing capacity factors are given for various cases of different layer thickness and different cohesion ratios for the two soil layers. Unlike the small deformation analysis for cohesive soil, it is found that the soil self-weight can have a significant effect on the bearing capacity in large penetration problems.

1 INTRODUCTION

The bearing capacity of footings comprises quite an extensive literature today. The methods for calculating the bearing capacity of multi-layer soils range from averaging the strength parameters (1), using limit equilibrium considerations (2, 3, 4), to a more rigorous limit analysis approach (5, 6, 7). Semi-empirical approaches have been adopted based on some experimental studies (8, 9). The Finite Element Method (FEM) was also applied to this problem more recently (10, 11, 12, 13).

However, most of these studies are limited to footings resting on the surface of the soil, and are based on the assumption that the displacement of the footing is very small. These solutions may be applied to a footing which partially penetrates the soil, by ignoring the strength contribution of the adjacent soil above the plane of the footing base and the change of depth of the top layer soil during footing penetration. In many cases, the footings or other structures will experience a significant settlement, and sometimes even penetrate through the top layer to the deeper layer. In these cases, the small displacement assumption is no longer appropriate, and a large displacement theory should be introduced into the analysis.

For large deformation or large strain analysis, it is well known that the two main approaches are the Eulerian and Lagrangian formulations. The latter can be further classified as Total Lagrangian and Updated Lagrangian methods. As pointed out by Cheng and Tsui (14) and Hu and Randolph (15), each of these three approaches has its own limitations.

In an attempt to overcome the limitation of the pure Lagrangian and Eulerian approaches, a more flexible approach called the Arbitrary Lagrangian-Eulerian (ALE) method has been developed by Ghosh and his co-workers (16). Hu and Randolph (15, 17) adopted the ALE approach along with a fully automatic finite element mesh generation method to analyse large deformation problems of footings on a single-layered soil and spudcan penetration. In this paper, Hu and Randolph's method is used, and the analysis was extended to two-layered purely cohesive soil. The top layer is assumed stronger than the bottom layer. In the cases investigated, the ratio of the bottom layer strength to the top layer strength varies from 0.1 to 1, and the thickness of the top layer is either one half or the same as the footing width.

First, small deformation analysis was done to verify the FEM adopted for this problem. Then large deformation analysis of the ALE approach was carried out to give the full load-displacement curve and the mobilized bearing capacity of the layered soil. A discussion about the bearing resistance of layered cohesive soils under large penetration is made, based on the comparison of the small and large deformation analysis results. The effects of soil weight on the large deformation analysis were also investigated.

2 NUMERICAL MODELS

In this paper, six-noded triangular elements with a three-point Gauss integration rule were adopted. The small displacement analysis was carried out using the AFENA finite element package developed by Carter et. al. (18), and the large displacement analysis was based on using Hu and Randolph's program with some modification to suit two-layered soils. The solution algorithm adopted was based on a tangent stiffness approach.

Plane strain meshes were designed such that the elements were concentrated in highly stressed zones and the boundaries were sufficiently distant from the footing to ensure that the mesh contained the entire plastic zone. To ensure that the equilibrium errors and the discretization errors were small, several calculations of different meshes and increment sizes

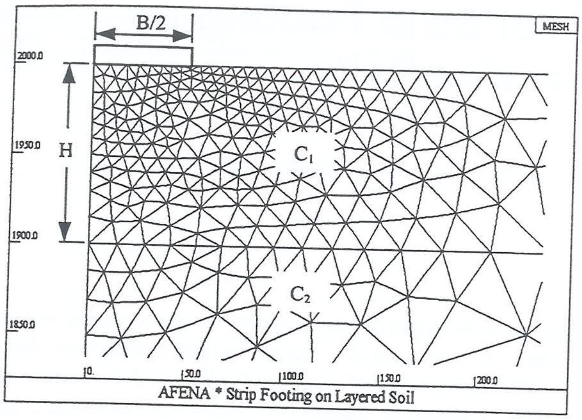


Figure 1 Typical mesh used in the analysis

were repeated. The typical mesh configuration finally adopted in this paper is shown in Fig. 1. The loading of a rigid footing was simulated by specifying incremental displacements of appropriate surface nodes. The increment size was taken as 0.02% of the footing width. The footing base and sides were assumed to be perfectly smooth.

The soil was assumed to be an elastic-perfectly plastic Tresca material, with modulus ratio $G/c_1 = 100$ or 200 (where G is the shear Modulus of both layer, c_1 is the cohesion of the top layer). Undrained analysis was carried out by adopting Poisson's ratio $\nu = 0.49$.

As to the large deformation analysis, full details of the algorithm are given by Hu and Randolph (15). While for the small deformation analysis, the mesh configuration remains the same as the original mesh; in the ALE large deformation analysis, the mesh was updated after certain steps of penetration. To obtain the results, in this paper, the mesh was updated every 20 steps. As the displacement increment size is 0.02% of the footing width, this means the updating interval is 0.4% of the footing width.

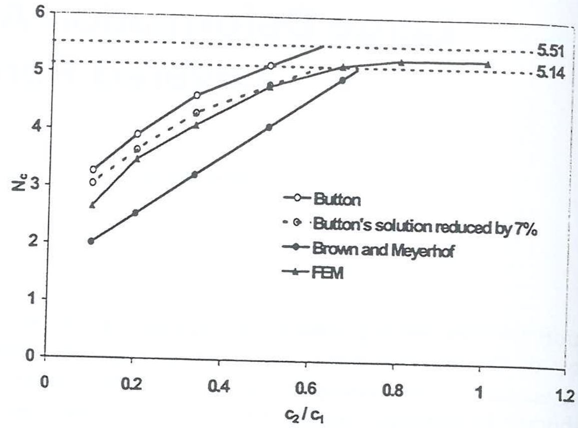
3 BEARING CAPACITY FACTOR BY SMALL DEFORMATION ANALYSIS

The ultimate bearing capacity q_m of a strip footing without a surcharge on two-layered cohesive soil can usually be expressed as

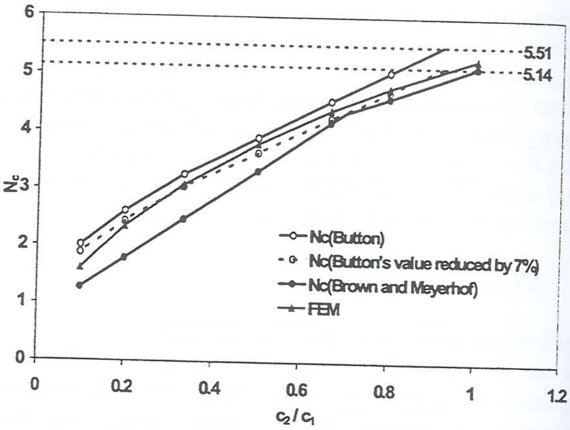
$$q_m = c_1 N_c \qquad (1)$$

where N_c is the modified bearing capacity factor that depends on the ratio of the cohesion of the two layers c_1/c_2 and H/B (the thickness of the top layer divided by the footing width).

The available values of N_c for two-layered cohesive soil have been given by Button (2) and Reddy et al. (3) using the assumption of a cylindrical yield surface, and Brown and Meyerhof (8) based on some experimental studies. Button and Reddy et. al. gave very similar figures for the values of N_c indexed by c_2/c_1 and H/B. Their approach gave the upper bound solution to this problem, and returned the bearing capacity factor for homogeneous soil (a special case for two-layered soil) to be 5.51, which is 7% above Prantl's exact solution of $(2+\pi)$. The factors given by Brown and Meyerhof



(a) H/B = 1



(b) H/B = 0.5

Figure 2 Comparison of bearing capacity factors for layered soil under surface footing

are lower, and in better agreement with the value of $(2+\pi)$ for homogeneous soil, and are given by the following equation:

$$N_m = 1.5\frac{H}{B} + 5.14\frac{c_2}{c_1} \leq 5.14 \qquad (2)$$

The bearing capacity factors given by the FE small deformation analysis were compared with the values given by Brown and Meyerhof (8) and Button (2). Various cases were investigated, and the comparison is shown in Figs. 2(a) and (b).

It can be seen from Fig. 2, that the bearing capacity factors given by the FEM are lower than Button's solution, but higher than the empirical values given by Brown and Meyerhof. As Button's solution is an upper bound, and for the special case $c_1/c_2 = 1$ it is 7% higher than the accurate value $(2+\pi)$, it can reasonably be assumed that for other cases, his solution is also approximately offset by about the same amount. The dashed curve in Fig. 2 shows Button's solution reduced by 7%. The FE results agree well with the reduced Button's solution.

The FE prediction is about 10-15% higher than the empirical values given by Brown and Meyerhof's formula. Georgiadis and Michalopoulos (19) proposed a "slip surface"method for the same problem, and their prediction was also 10% higher than the value given by Brown and Meyerhof, but agrees well with their FE analysis results and Giroud (20)'s prediction. Thus the adopted FE mesh configuration and solution algorithm can be regarded as suitable for solving this bearing capacity problem.

4 LIMITATIONS OF SMALL DEFORMATION ANALYSIS

The above-mentioned analytical solution and the FEM small displacement analysis for the bearing capacity of layered soil are only applicable to footings resting on the surface of the soil deposit. For the large penetration problem, they are not valid approaches. First, when the footing penetrates into the top layer, the actual value of H/B under the footing base decreases. Second, in the case that the footing penetrates through the top layer and force it into the bottom layer, it will trap a certain amount of top layer soil into the bottom layer. Also, the penetration will cause the soil heave close to the footing edges, which will affect the bearing capacity. All of these factors make it difficult and unreasonable to use H/B to refer to the existing figures and formula in the literature to determine the bearing capacity factors for footings undergoing deep penetration.

The geometry change makes assumptions for some analytical solutions (eg. cylindrical yield surface) not suitable to the penetration problem. The FEM with ALE, however, makes no a priori assumption about the failure mechanism, and thus it can reflect the natural development of the failure zone and give good predictions of the bearing behaviour.

5 LARGE DEFORMATION ANALYSIS AND RESULTS

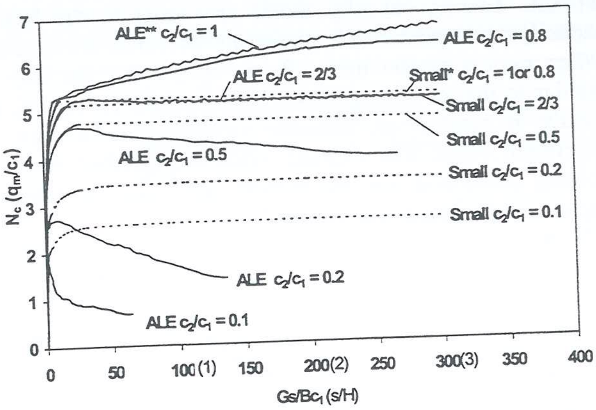
5.1 The Bearing Response

Hu and Randolph (17) used the ALE approach to compute the bearing response of shallow foundations penetrating into a single layered non-homogeneous soil. In this paper, the bearing response of the strong-over-soft two-layered soil under a strip footing is examined by comparing the results given by the small and large deformation analysis. Various cases of H/B = 0.5 and 1, and $c_2/c_1 = 0.1, 0.2, 1/3, 0.5, 2/3$ and 1 (homogeneous soil) were investigated. Typical normalized load-displacement curves are shown in Fig. 3(a).

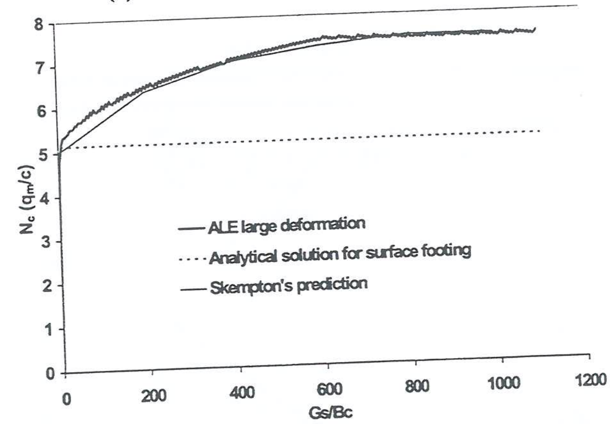
The curve given by the small deformation analysis reaches an ultimate value after a relatively small displacement (penetration), this value is the bearing capacity factor shown in Fig. 2(a). The curves given by the large deformation analyses are quite different from those given by the small displacement analysis.

For the homogeneous soil ($c_2/c_1 = 1$), the curve continues to rise until an ultimate value is reached (see Fig. 3(b) for the whole curve). The ultimate bearing capacity for a deep footing in homogeneous soil was investigated by Meyerhof (21) and Skempton (22), among others. Skempton's prediction was based on both theoretical and experimental results and can be simply estimated from the equation:

$$N_c = 5(1 + 0.2\frac{s}{B}), \qquad (3)$$



(a) H/B = 1, $G/c_1 = 100$, $c_2/c_1 = 0.1 \sim 1$



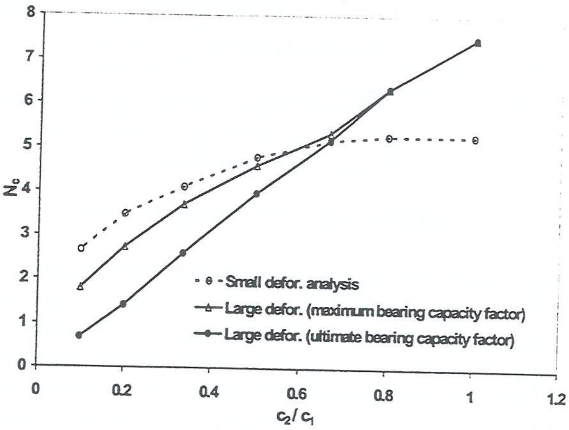
(b) G/c = 100, $c_2/c_1 = 1$ (complete curve)

Figure 3 Typical normalized load-displacement curves

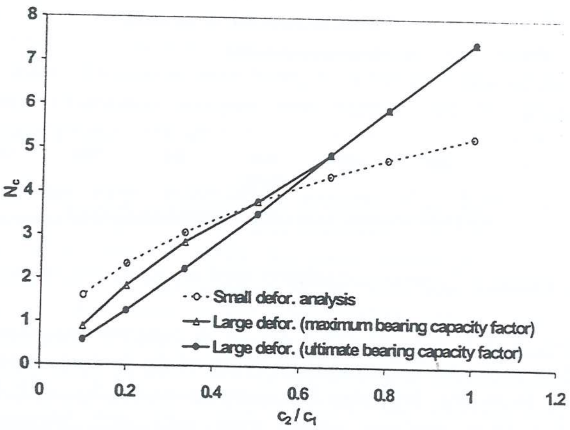
when $s/B < 2.5$. At $s/B \geq 2.5$, N_c *is* constant and equal to 7.5, where s is the penetration depth of the footing. It can be seen from Fig. 3(b) that the curve given by the ALE large deformation analysis agrees very well with Skempton's prediction. The FE analysis scheme and the ALE method were verified again by this agreement.

For the layered soil, it is difficult to give an analytical solution for the bearing capacity factor of a large penetration problem, thus it is the very problem that needs the FEM to conduct the analysis. For the cases of a top layer over a much weaker bottom layer (eg. $c_2/c_1 = 0.1$, 0.2, and 0.5), the curve given by the ALE method rises to a peak value (see Fig.3(a)), which is lower than the bearing capacity predicted by the small deformation analysis, then it declines with further penetration, finally it reaches a stable value after the footing settles into the bottom layer. The peak values of these curves are the maximum bearing capacity factors given by the large deformation analysis and the values reached after large penetration are referred to as the ultimate bearing capacity factors in this paper. For the small deformation analysis, the ultimate bearing capacity factor is also the maximum bearing capacity factor.

It is interesting to note that at $c_2/c_1 = 2/3$, the curves given by the large deformation analysis and small deformation analysis are almost coincidental. Thus both the maximum and the ultimate bearing capacity factor are equal to the bearing capacity factor predicted by the small deformation analysis. When c_2/c_1 is greater than 2/3, eg. $c_2/c_1 = 0.8$, the curve is similar to that of homogeneous soil, it continues to rise, and finally it reaches an ultimate value, which is also the maximum value. The ultimate value was reached when the footing penetrated into the bottom layer and the top layer broke into two parts (see section 5.2).



(a) H/B = 1



(b) H/B = 0.5

Figure 4. Bearing capacity factors given by large deformation analysis

The analyses of the cases of H/B = 0.5 gave similar curves after normalization but with different maximum and ultimate bearing capacity factors. In Fig. 4(a) and (b), the values of the bearing capacity factors for the case H/B = 1 and H/B = 0.5 are shown respectively with those predicted by the small deformation analysis.

It can be seen that for the case H/B = 1, when $c_2/c_1 = 0.5$ to 2/3 (or $c_2/c_1 = 0.4$ to 0.6 for H/B = 0.5), the maximum bearing capacities given by the small deformation analysis and large deformation analysis are very close to each other. When the bottom layer is weaker than this, i.e. the ratio of c_2/c_1 is less than the above-mentioned values, the ALE predicted maximum values of N_c are lower than those given by the small deformation analysis. For a stronger bottom layer, i.e. the ratio of c_2/c_1 is greater than the above-mentioned values, but still less than 1, the ALE predicted maximum values of N_c are higher than those given by the small deformation analysis. The reason for this can be explained as follows.

When a rigid footing penetrates into a two-layer soil system, the bearing capacity is contributed by two parts of the soil. One is the soil beneath the footing. The other is the soil above the level of the footing base and besides the footing edge. This part of the soil usually heaves at the surface during penetration. If a small deformation analysis is done, only the first part is considered, as the mesh configuration remains unchanged during the analysis and thus the soil is always beneath the footing base. For the large deformation analysis, the mesh is updated regularly and thus the ratio of the contribution to the bearing capacity from the two parts varies during penetration, i.e. the surcharge and burial effect varies during penetration.

At early stages of penetration, most of the soil is beneath the footing, and thus the load-displacement curves are almost identical to those given by the small displacement analysis. As the deformation gets larger, the footing penetrates into the soil, the soil of the top layer flows from beneath the footing, leading to less top layer soil under the footing. As the bottom layer is weaker than the top layer, the contribution of the bearing capacity from the soil beneath the footing becomes less. Meantime, more and more soil is above the level of the footing base, and additional bearing resistance comes from this part of the soil. Whether the predicted bearing capacity by the ALE analysis is higher or lower than that given by the small deformation analysis depends on how these two factors interact. From the above analysis, it can be seen that when $c_2/c_1 < 0.5 - 2/3$ for the case of H/B = 1 (or $c_2/c_1 < 0.4 - 0.6$ for H/B = 0.5), the loss of the bearing capacity because of the top layer's "flow away" beneath the footing is greater than the gain from the soil above the footing base. Thus the predicted bearing capacity factor is lower than that given by the small deformation analysis. The smaller the value of c_2/c_1, the lower is the bearing capacity.

When $c_2/c_1 = 0.5 - 2/3$ for H/B = 1(or $c_2/c_1 < 0.4 - 0.6$ for H/B = 0.5), the loss and the gain are approximately balanced as the footing settles to a certain distance into the soil, and at this point, the soil provides the maximum bearing capacity. Then the loss starts to surpass the gain and the load-settlement curve begins to decline. But there is a critical value for c_2/c_1, at this value, the loss and the gain can always be compensated, so the curve given by the large deformation analysis is the same as the one given by the small deformation analysis. For the case H/B = 1 this value is approximately $c_2/c_1 = 2/3$; while for H/B = 0.5 it is about $c_2/c_1 = 0.6$. From Fig. 4, it can be seen that at this point, the three curves give almost the same bearing capacity factors.

In the cases of $c_2/c_1 > 0.5 - 2/3$ for H/B = 1 (or $c_2/c_1 > 0.4 - 0.6$ for H/B = 0.5), the gain is always greater than the loss, which leads to the load - settlement curve continuing to rise. Thus the maximum bearing capacity factors are the ultimate bearing capacity factors and they are greater than those given by the small deformation analysis.

5.2 The Development of Plastic Zones

As the footing settles, the top soil layer gradually flows out from under the footing base. However, not all of the top layer soil will flow out of the footing base, some soil will be trapped under the footing. Thus the top layer soil tends to break into two parts, one part remains under the footing during further settlement, the other flows to the side of the footing. In the FE program, if the minimum width of the long strip of top layer soil at the side of the footing became less than 3% of the footing width, the top layer soil was broken into two parts. This is shown in Fig. 5. Then the footing and the trapped soil settle together into the bottom layer soil, at this stage, the value for the bearing capacity factor usually does not change much. This is because the contribution of the bearing capacity from the above-defined two parts of soil vary very little after this, and thus the final stable value for the ultimate bearing capacity is obtained.

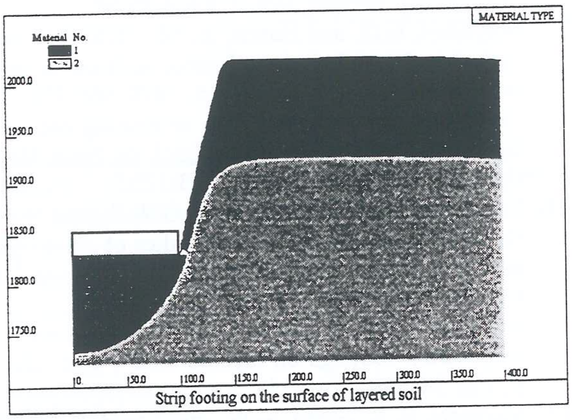


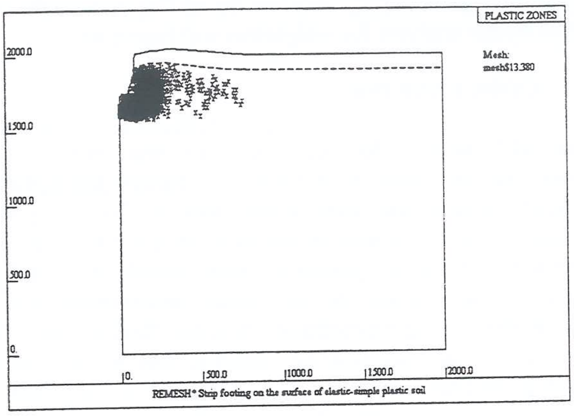
Figure 5 Footing settles into the bottom layer with some top layer soil trapped underneath (H/B = 0.5)

As mentioned before, to ensure the mesh contains the entire plastic zone for various cases, an extensive mesh was used in this paper. In Fig. 6(a), the plastic zone upon failure for the case $c_2/c_1 = 0.5$ of $H/B = 1$ is shown. This figure shows that the plastic zone was restricted to a very small area. A much smaller mesh domain could have been used for analyzing this case. However, for a weaker bottom layer soil, eg. $c_2/c_1 = 0.1$ of $H/B = 1$, the plastic zone is much more extensive and developed to the boundary of the FE mesh (Fig. 6(b)).

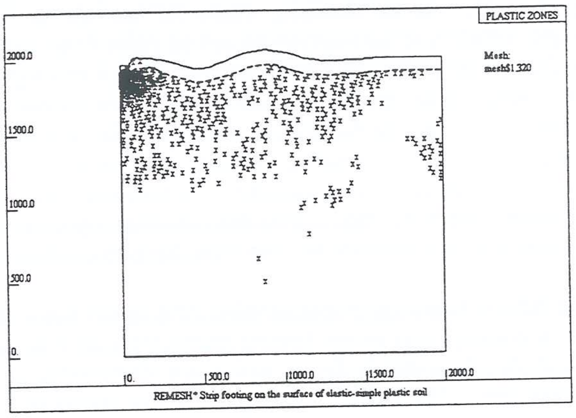
It is found that for case $c_2/c_1 = 0.5$ of $H/B = 1$, the failure of the soil occurred first in the top layer soil, and immediately after that, the bottom layer just beneath the top layer began to fail. Then the failure developed in both layers, but faster in the bottom layer. For a weaker bottom layer soil than this, e.g. $c_2/c_1 = 0.1$ of $H/B = 1$, failure began from the bottom layer, and only after it developed to a considerably large area within the bottom layer, did failure start to occur in the top layer. After that the plastic zone developed very fast in the bottom layer. The weaker the bottom layer compared to the top layer, the more extensive is the plastic zone developed in the bottom layer. The values of the ultimate bearing capacity factors, which were obtained when the footing settled into the bottom layer, were mainly affected by the bottom layer's strength and plastic zone development.

5.3 The Effect of Soil Weight

It is well known that for a surface footing, the ultimate bearing capacity given by the undrained analysis is independent of the soil self-weight for purely cohesive soil. However, in large deformation analysis, the footing is not a surface footing once it begins to settle and penetrate, and the self-weight of the soil affects the results. The previous analyses were carried out without considering soil self-weight



(a) $c_2/c_1 = 0.5$ of $H/B = 1$



(b) $c_2/c_1 = 0.1$ of $H/B = 1$

Figure 6 Plastic zone upon failure

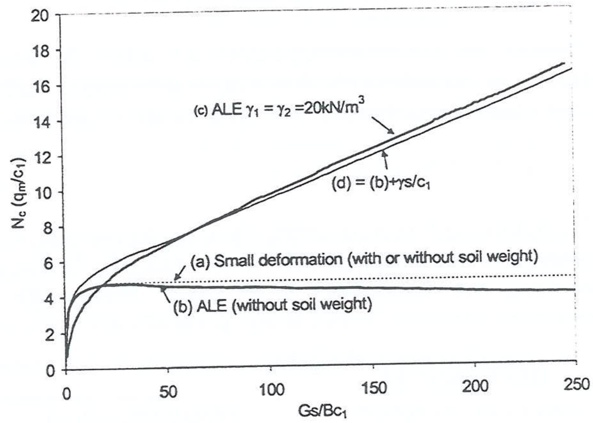


Figure 7 Normalized load-deflection curves given by analysis with and without soil self-weight

to investigate the effects of the large deformation analysis on the bearing capacity factor N_c. If the self-weight is included, the large deformation analysis results, for $c_2/c_1 = 0.5$ and $H/B = 1$, are as given in Fig.7.

Fig. 7 shows the small and large deformation analysis results of weightless soil and soil of unit weight 20kN/m^3. It can be seen that if soil weight is included, the results given by large deformation analysis are quite different from those without considering soil weight. However, if both layers are of the same unit weight, the large deformation analysis result (curve c) can be approximately obtained by adding $\gamma s/c_1$ to the result given by the analysis for weightless soil (curve d).

6 CONCLUSIONS

The ALE large deformation analysis was carried out to investigate the bearing response and failure mechanism of layered cohesive soil under a strip footing. For the specified strong-over-soft problem investigated in this paper, the load-displacement curves generally were significantly different from the ones given by the small deformation analysis. During the footing's penetration process, there are two major factors affecting the bearing capacity, one is the increased bearing resistance from the soil above the footing base and the other is the decreased bearing resistance from the weaker underlying soil. For a specified value of H/B, a corresponding value of c_2/c_1 can be obtained, where the two effects can be exactly offset, and the load-displacement curve is the same as the one given by the small deformation analysis. For a greater c_2/c_1 value than this, the load-displacement curve rises continuously until the footing penetrates into the bottom layer and a stable ultimate bearing capacity is achieved, which is also the maximum bearing capacity. For a smaller c_2/c_1 value, the curve rises to the maximum bearing capacity, then declines until the ultimate bearing capacity is reached.

The failure zone may either develop first in the upper layer (for a greater c_2/c_1) or the bottom layer (for a smaller c_2/c_1), but it develops much faster and more extensively in the bottom layer. The smaller is c_2/c_1, the more extensive is the plastic zone developed in the bottom layer. Thus a very extensive area should be modelled in the FE large deformation analysis for a small value of c_2/c_1, eg. 20B×20B or bigger for $c_2/c_1 = 0.1$ for the case of H/B = 1.

Although the soil self-weight does not affect the bearing response of the cohesive soil in a small deformation analysis, it has a significant effect in the large deformation analysis.

7 ACKNOWLEDGMENT

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