A New Model for the Behaviour of Granular Filters

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Summary Filters are used in Geotechnical Engineering to control seepage and to prevent erosion of soil due to the drag forces of seeping water. Filters act as barriers to retain the base soil while allowing seepage flows to exit without causing high hydraulic gradients or pore pressures which may damage the structure. This paper describes a new analytical model of filtration. The model is based on a three dimensional lattice model of the filter pores, and the equations of conservation of mass and momentum which govern the rate of particle transport. The model has application in the design of granular filters for non-cohesive base soils in embankment dams, retaining walls, drainage wells or road pavements.

1. INTRODUCTION

Filters are used, in geotechnical engineering, where it is necessary to protect soils from erosion due to seepage and groundwater. As water flows through a soil, fine soil particles can be washed out, leading to internal erosion (or piping) and eventual failure. A correctly designed filter will retain the loose soil particles while allowing seepage water to flow; thus preventing piping and avoiding a build up of high internal pore pressures. Filters are used in embankment dams, road pavements, behind retaining walls, coastal protection, in landfills and wastewater treatment, sand beds in oil wells and chemical engineering filtration. This study deals predominantly with the problem of filters for embankment dams.

Filters are used where water seeping out of fine grained soils may cause erosion of the soil, by removing particles under hydraulic forces. To function correctly, filters must be:

1. Fine enough that the pore spaces between the filter particles are able to capture some of the larger particles of the protected materials in place (see Figure 1).
2. Coarse enough to allow seepage flow to pass through the filter, preventing build up of high pore pressures and hydraulic gradients.
3. Non cohesive, so that no cavities or cracks can form within the filter.

Figure 1 shows a stable base - filter interface. Seepage forces have washed some base soil particles into the filter. Initially, some fine base particles may be washed completely through the filter, but in a stable filter the larger base particles will be trapped by the void constrictions (connections between voids) of the filter material. These trapped particles will then form smaller voids, retaining smaller base particles and the entire interface becomes stable. This process is called "self filtration". Water flow will be reduced during this process but generally reaches a steady state as self filtration occurs. If a filter is too coarse, the base soil particles will be able to move through the pores of the filter material and self filtration will not occur. If a filter too fine, it may not have sufficient permeability to allow the seepage flows to leave the base soil and high pore pressures can develop. Also, manufacturing a fine filter is often considerably more expensive than a coarse filter; hence the economic benefits of correct filter design are significant.

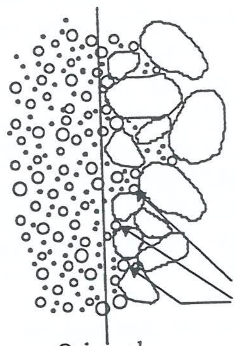

Figure 1 Stable Base-Filter Interface During Seepage.

There is an increasing push to replace granular filters with geotextiles which perform the same function. The advantages of geotextiles are numerous, often they are cheaper to install than granular filters and they are manufactured and placed under strict specifications, so the uncertainties involved with using natural materials are removed. However, there is still a concern that the long term performance of geotextiles (remembering a dam usually has a design life in excess of 50 years) may be unsatisfactory. A particular concern is that a geotextile may tear due to differential settlement within the structure, or earthquake motion. Because of these concerns, granular filters are more commonly used in important structures such as embankment dams. This study will focus solely on the performance of granular materials as filters.

2. EXISTING DESIGN METHODS

Terzaghi (1) was the first to develop filter design requirements. He envisaged two requirements which must be fulfilled:

- The filter should be many times more pervious than the base soil, to allow the free seepage of water, without causing excessive head loss (the permeability requirement). To ensure this he recommended:

$$D_{15F}/d_{15B} \geq 4 \qquad (1)$$

- The filter should be fine enough to prevent the washing through of the base soils and arrest piping (the stability requirement). To ensure this he recommended:

$$D_{15F}/d_{85B} \leq 4\text{-}5 \qquad (2)$$

Where D_{15F} is the diameter of filter particles where 15% by pass of particles are smaller, and d_{85B} is the diameter of base particles with 85% of the particles, by mass, smaller. These requirements describe, basically, the conflicting requirements on grain size, of a suitable filter. Some engineers still use these criteria for designing granular filters.

Subsequent research into filter behaviour has been predominantly empirical; a series of experiments on sets of base soil - filter combinations, has lead the researcher to recommend an empirical relationship for a stable combination. Research has lead to empirical design criteria which provide simple to apply relations for stable base soil - filter combinations. The most widely accepted empirical criteria are those of Sherard and Dunnigan (2). A review of the application of empirical methods can be found in Indraratna and Locke (3). These empirical criteria are extensively used, in preference to other methods, for filter design. However they are only applicable to the range of soils tested, and have certain laboratory bias due to different testing methods, definitions of failure etc. They also do not provide an understanding of the mechanisms involved with base soil - filter interaction. Hence they do not give the designer a clear picture of what may occur within the dam and the level of safety involved with design decisions.

Many researchers are now concentrating on numerical analysis of filtration, particularly modelling particle movement through filters. These approaches recognize that soil masses are made up of a random distribution of many sized particles. The most important part of base soil movement through a filter is the geometric requirement, that a base soil particle must be smaller than the pore void (and void constriction joining pores) through which it is passing. Additionally, some researchers have considered the hydraulic conditions (seepage forces) necessary to carry soil particles through the filter. The basis of the numerical analysis is:

1. to represent the filter by some form of a pore model, usually based on the particle size distribution of the filter material;
2. to simulate the movement of base soil particles by an analysis of the movement of individual base soil particles through the pores of the filter, caused by seepage forces, up to a point where the particle passage is blocked by a pore constriction, or the seepage forces are insufficient to move the particle further.

There are two general approaches to modelling particle movement, either: geometric - probabilistic, where the expected depth of infiltration of a particle, into the filter, is determined by probabilistic analysis of particle and void sizes; or mass transport equations using flow laws and conservation of mass and momentum to examine the rate of particle movement.

Analytic methods provide detailed models of what may be occurring at a base soil - filter interface. They give an idea of the thickness of filter required and also can estimate a probability of failure. The assumptions used in developing the model are very important. Often the assumption of spherical particles or a certain particle size distribution curve shape etc. cannot be applied to a real soil. The models are often difficult to apply to real design situations because of their reliance on a number of empirical parameters or impractical mathematical models.

3. NEW ANALYTICAL FILTER MODEL

Existing numerical models of filtration have some limitations. They generally adopt simplified void models, and very few consider the time rate of formation of a stable filter interface. Indraratna and Vafai (4) have developed a particle migration model, considering conservation of mass and momentum to model particle movement. This model is capable of showing the variation with time of particle size distribution, permeability and porosity of elements of the base soil and filter. Some criticism has been directed at the simplified void model adopted in this analysis. Hence there is some room for improvement and adaptation of Indraratna and Vafai's method. A new model for filtration is described below, based on the model of Indraratna and Vafai (4), and a modified three dimensional pore void model, based on the work of Schuler (1996). The entire model includes:

Filter void model - based on a cubic lattice network described by Schuler (5). Void sizes are determined by an adaptation of the method described by Silveira (6).
Particle infiltration depth - Schuler (5) developed an equation, based on Monte Carlo simulation, for infiltration depth, dependent on particle size.
Particle transport equations - the equations of conservation of mass and momentum, developed by Indraratna and Vafai (4), are used to determine the rate of particle transport.

3.1 Filter Void Model

There are many models of filter voids which have been adopted in modelling filtration. The commonly used models include; parallel channels of varying diameter (Indraratna and Vafai (4)), layers of filter voids perpendicular to the direction of flow (Silveira (6)), or pore networks (Schuler (5)). A granular soil is a three dimensional collection of particles which form voids of different size and shape, with constrictions between the voids, also of different size, shape and orientation. It is not feasible to model this completely random arrangement of voids and constrictions. However, the pore network models appear to provide a reasonable model of the soil structure. Schuler (5) suggests that after examination of the pore voids of a real soil, there are on average 5.7 constrictions from every pore. Based on this, Schuler (5) developed a regular cubic lattice model of pores and constrictions, shown in Figure 2.

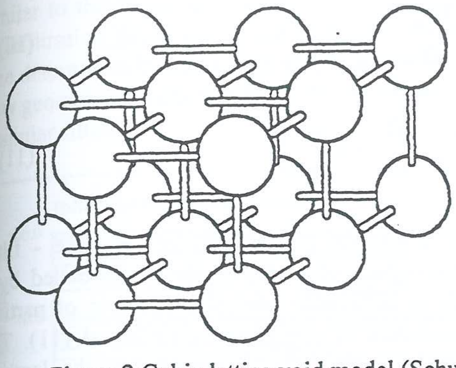

Figure 2 Cubic lattice void model (Schuler (5))

It remains then, to determine the size of the voids and void constrictions in the void model. The void constrictions, represented as bonds between the voids in Figure 2, form the smallest link between voids, capturing moving particles. Hence the important factor for modelling filtration is the void constriction size distribution, hereafter called the CSD. Schuler (5) has examined the CSD of a soil at varying relative density and found that the CSD curves all have the same shape. Hence, if we find the CSD for most dense and least dense states, then the actual CSD will have the same shape and lie somewhere in between. A reasonable assumption is that there is a linear change in CSD size from the most dense to least dense states.

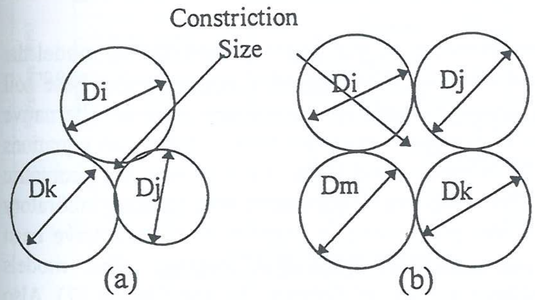

Figure 3 Void Constriction Size for a) Most Dense and b) Least Dense states.

The two geometric conditions to be considered, are shown in Figure 3. Humes (7) presents a method to calculate the CSD for the most dense case (Figure 3a), based a method described by Silveira (6) using the filter PSD. The PSD by mass (as determined by sieve analysis) tends to over-estimate the influence of larger particles, which form a large proportion of the mass of the soil, but are few in number and unlikely to meet to form large voids. Humes (7) recommends using the PSD by surface area of grains, and has shown this to better represent the pore constriction sizes for well graded materials. Equation (3) can be used to convert the PSD by mass to a PSD by surface area, assuming all particles have the same specific gravity.

$$P_{i,SA} = \frac{P_{i,mass} / D_i}{\sum_{j=0}^{n} P_{j,mass} / D_j} \qquad (3)$$

The PSD is divided into a number of discrete particle diameter intervals (D_0, D_{10}, D_{20}, etc.), so that it then represents the cumulative frequencies (P_0, P_{10}, P_{20} etc.) of the medians of these intervals. Humes (7) presents an equation, (4), to find the void size, D_v, for the most dense particle packing, when the void is formed by three tangent spheres of diameter D_i, D_j and D_k, as shown in Figure 3a.

$$\left(\frac{2}{D_i}\right)^2 + \left(\frac{2}{D_j}\right)^2 + \left(\frac{2}{D_k}\right)^2 + \left(\frac{2}{D_v}\right)^2 = 0.5\left[\left(\frac{2}{D_i}\right) + \left(\frac{2}{D_j}\right) + \left(\frac{2}{D_k}\right) + \left(\frac{2}{D_v}\right)\right]^2 \qquad (4)$$

The probability of occurrence, P_v, of void size D_v, is a function of the probability of occurrence of the three particles, taken from the discretised PSD. P_v is calculated using (5), where r_i, r_j and r_k represent the number of times particle diameters D_i, D_j, and D_k appear in the combination of three particles being considered. Hence r_i, r_j and r_k = 1,2 or 3 and $r_i + r_j + r_k = 3$.

$$P_v = \frac{3!}{r_i!\,r_j!\,r_k!}(P_i)^{ri}.(P_j)^{rj}.(P_k)^{rk} \qquad (5)$$

Silveira et al. (8) present equations for the least dense packing of a granular material; where the void constrictions are formed by four tangent grains as shown in Figure 3b. Silveira (8) note that the geometry of the problem is very difficult to solve directly and hence assumes the void is equivalent to a circle with the same area as that formed by four tangent particles as shown in Figure 4b and c. Silveira's equations for the diameter of the equivalent circle are not presented here.

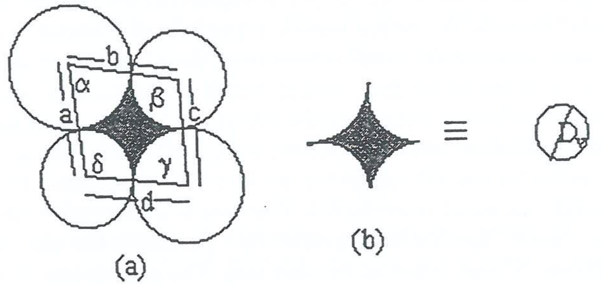

Figure 4a) Pore constriction formed by 4 particles
b) Constriction Area formed by tangent particles
c) Circle of equivalent area

Pore constrictions usually do not form on a plane through the centres of the three or four particles making up the constriction. Hence it is suggested that the mean of all possible chords through the circular particle be used to represent the size of particles, rather than the diameter. Thus the particle diameter to be considered for determining constriction size, D_{model}, is given by (6).

$$D_{model} = 0.82\, D_{actual} \qquad (6)$$

The CSD of a granular material, at different relative densities, will have the same shape. Also the assumption was made, that the difference between two constriction size distributions will be directly proportional to the difference in relative density. Hence, using relative density, defined in (7), then the actual CSD can be calculated from (8). We then have a pore void model, consisting of a 3D cubic lattice of pores with six constrictions connecting each pore to its neighbours, as shown in Figure 2. The size of each constriction is randomly generated from the CSD.

$$RD = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \tag{7}$$

Actual Void Diameter = Most Dense Diameter +
RD x (Most Dense Diameter - Least Dense Diameter) (8)

3.2 Particle Infiltration Depth

Previous laboratory and analytical research has shown that a filter has a controlling constriction size, d_c* (Kenney et al., 9). Where base particles finer than d_c* can pass through a filter of large thickness. As base particles larger than d_c* are considered, their depth of infiltration into the filter decreases rapidly as the particle diameter increases. Schuler (5) executed a detailed Monte Carlo simulation of the cubic lattice pore model, shown in Figure 2. It was shown that, as predicted by percolation theory, a percolation threshold, p_c, exists for this model. The percolation threshold represents a probability of meeting a constriction larger than the particle, so the particle can move through that constriction. Above the percolation threshold, a particle can move an infinite distance through the lattice. Below the threshold the infiltration depth reduces very quickly. The percolation threshold was found to be, approximately, the size of the 63% largest pore constriction. Based on the results of the analysis, Schuler (5) determined an equation, (9), for the depth of infiltration, up to the percolation threshold. In (9), E represents the number of levels, through the lattice model, a particle will travel. The physical distance of travel depends on the distance between pores or levels in the filter model, this is called the unit step. Since constrictions form near the centre of a particle and the next constriction will form near the centre of the next particle, the constrictions are separated by two half diameters. This suggests the mean filter particle diameter is a reasonable unit step. Finally, the distance travelled by a particle is the number of levels, E, multiplied by the unit step. The probability, p, is the probability density of constrictions larger than the particle.

$$E(p) = 123(36.3 - p)^{-0.88}; p < 36.3\% \tag{9}$$

3.3 Particle Migration Model

Indraratna and Vafai (4) have developed a particle transport approach to model particle movement. They also consider the hydraulic forces required to mobilise the particles. If seepage forces exceed the critical hydraulic gradient and the particle is smaller than the pore constriction, it will move. Moving particles are controlled by governing differential equations of conservation of mass (10) and momentum (11).

$$\frac{d(\rho_m u)}{dz} = \frac{d\rho_m}{dt} \tag{10}$$

$$\sum F = \rho_m V_m \left(\frac{du}{dt} + u\frac{du}{dz}\right) \tag{11}$$

By considering a number of elements at the base - filter interface, the movement of particles can be modelled by a forward step, finite difference analysis. The rate of particle erosion and movement is governed by (10) and (11). The geometric constraint to movement is modelled by the depth of infiltration into the cubic lattice (9). If the predicted infiltration of a particle size is equal the length of an element then particles smaller than that diameter can pass through; larger particles will be captured. The base and filter particle size distributions can be recalculated at each time step and the procedure repeated. This analysis predicts the gradual change in particle size distribution of the base and filter elements and hence describes what is occurring at the base - filter interface with time for the entire particle size range. Indraratna and Vafai (4) describe a method to determine permeability of a non-cohesive soil based on the particle size distribution, thus the time dependent changes in filter permeability can also be modelled.

3.4 Application of the Model

The model presented in this paper is intended to model the time rate of particle migration of a non-cohesive base soil through a granular filter. No laboratory data or alternative models are available to verify the particle migration equations proposed by Indraratna and Vafai (4). However, the geometric model of filter voids can be compared with existing laboratory data and model predictions. A number of models have been proposed for particle infiltration depths. The models considered here are that of Schuler (5), and Humes (7). Also the equations for controlling constriction size developed by Kenney et al. (9) and Witt (10), from experimental and theoretical modelling, describe the particle range at which a large infiltration depth will occur. The pore channel model adopted by Indraratna and Vafai (4) gives a minimum pore diameter, where smaller particles will pass through the filter. These five models are compared with the newly developed model for two cases. These are a uniform sand, with C_u=2, as shown in Figure 5, and a well graded sand, with C_u=6, as shown in Figure 6.

The models of Schuler (5) and Humes (7) both predict particle infiltration as a function of particle diameter, and these models are plotted in Figure 5 and Figure 6. Kenney et al (9), Witt (10) and Indraratna and Vafai (4) predict a single value for the approximate boundary between the retained and free particle diameters. These models are shown as a single vertical line in Figure 5 and Figure 6. As can be seen, in both examples the model of Schuler (5) predicts a percolation diameter very close to that of Kenney et al. (9) and Witt (10). The model of Humes (7) predicts a lower infiltration depth. The pore channel model of Indraratna and Vafai (4) is reasonable for uniform filters but overestimates pore channel diameters for broadly graded materials. The new model, similar to the model of Schuler (5), predicts a rapid increase in infiltration depth very close to the controlling constriction size determined by Kenney et al. (9) and Witt (10). Hence the new geometric model is suitable for modelling infiltration into granular filters.

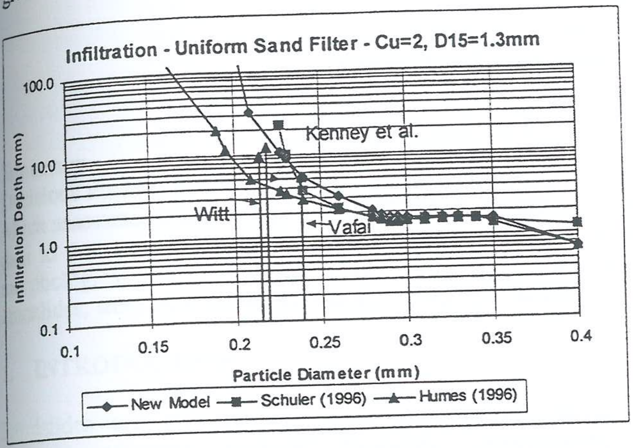

Figure 5 - Comparison of Predicted Infiltration Depth of Base Particles into a Granular Filter - Uniform Sand

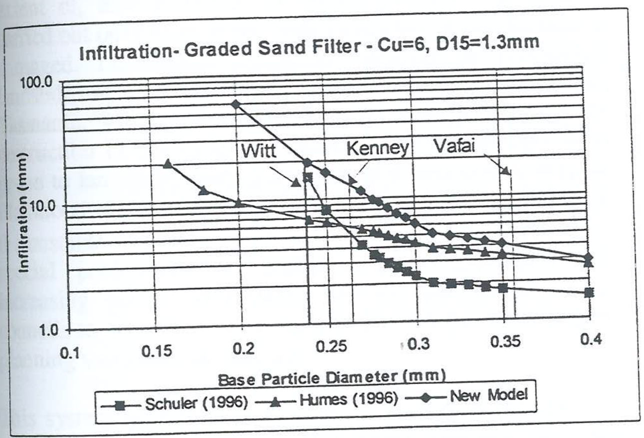

Figure 6 Comparison of Predicted Infiltration Depth of Base Particles into a Granular Filter - Well Graded Sand

4. CONCLUSIONS

Granular filters are an essential element of earthfill dams, providing protection from erosion and piping to the dam core. The role of a filter is to retain any eroded particles, while allowing seepage water to drain from the base material. This requires careful selection of filter particle size; the pores of the filter must be small enough that the larger base soil particles are captured within the pore constrictions. A new analytic model has been presented which describes the time rate of infiltration of base soil into a filter. The filter pores are modelled by a three dimensional lattice of pores, connected by constrictions. The size of these constrictions is randomly generated from the constriction size distribution; which is determined from the filter particle size distribution and relative density. The rate of movement of particles is modelled by a finite difference approximation of the differential equations of conservation of mass and momentum. The model has been shown to predict infiltration depths of base soil particles; similar to those determined, by other researchers, from both experimental and theoretical work. The analytical model presented here has several advantages over others in the literature including:

- A three dimensional pore model, more representative of real soil conditions than simple pore channel or layered void models.
- Modelling of the rate of particle movement. Hence predicting the time to reach steady state or large scale piping of the base soil.
- Modelling of time dependent changes in the base soil and filter. As particles are eroded or captured they alter the particle size distribution of the soil or filter. By re-calculating the constriction size distribution and permeability of each element in the model at each time step, time dependent changes are considered.

5. REFERENCES

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