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DETERMINATION OF FIELD STRESS RATIO AND YOUNG'S MODULUS USING THE UNDER EXCAVATION TECHNIQUE

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ABSTRACT

The under-excavation technique (UET) is a simple method of assessing the Field Stress Ratio (k) and Young's Modulus (E) of a rock mass from its response to an advancing excavation. The key to this method is that a hole in an elastic medium under stress will have a unique convergence signature which can be measured. The advantages of this method are its simplicity, low cost, utility, applicability to excavations of irregular dimensions, is non-invasive and views the rock mass at a suitably large scale. Further, it can be applied to any excavation in relatively homogeneous material , being particularly suited to tunnels and shafts.

This paper (1) outlines the theoretical background for the UET, (2) explains a practical and simple way to apply it, and (3) offers the case study of the Eastern Distributor Tunnel where it has been successfully applied.

1. INTRODUCTION

For thousands of years humans have been developing underground space, largely in the pursuit of minerals. With growing pressure on today's crowed cities, there is an increased push towards the development of underground space for civil engineering applications. This push has resulted in the development of new and the refinement of old design and monitoring tools. The Underground Excavation Technique (UET) is one such tool.

Techniques for analysing a rock mass's response to an advancing excavation are not new (Pells et al, 1981), however until the development of numerical analysis software they were limited to tunnels of regular profile. With increasingly available and simple to use numerical modelling software, it is now relatively simple to estimate the geometric influence an excavation has on sidewall convergence.

From a practical view point, the UET allows the continual assessment of insitu stresses in a rock mass to be during the excavation phase of a project, providing a useful tool to assist in the design of excavation support and an aid to recognise zones of abnormal ground conditions. It also allows a check to be made on insitu stress measurements that may have been made during the investigation phase of a project.

2. THEORETICAL BACKGROUND

In 1898 Kirsch published one of the first closed form solutions describing the relationship between stresses and strains around an excavation in a homogeneous elastic medium. The Kirsch Solution has become the basis for numerous methods for assessing the behaviour of a hole in a media, including the UET.

The mathematical derivation of the Kirsch Solution is beyond the scope of this paper, and no attempt is made to reproduce it here. Rather, this paper limits itself to the principles of the UET. Readers seeking a more complete coverage of the Kirsch Solution are referred to standard texts on the subject - Love (1927) and Jaeger & Cook (1971).

The Kirsch Solution for radial displacements at the surface a circular excavation in an infinite homogeneous isotropic linear elastic medium (plain strain) can be represented as:

$$u = \frac{\left(1-\nu^2\right)}{E} dp\left[\tfrac{1}{2}\left(1+\tfrac{1}{k}\right)+\left(1-\tfrac{1}{k}\right)\cos\theta\right] \qquad \text{Eqn 1}$$

where:

u = radial displacement of a point on the perimeter of the hole;
p = field stress at the level of the hole;
d = diameter of circular hole;

E = Young's modulus (elastic parameter);
υ = Poisson's ratio (elastic parameter);
θ = angle from the x axis to the point on the perimeter of the hole being considered; and
k = stress ratio (typically horizontal stress divided by vertical stress).

Figure 1 shows the geometric relationship between the parameters of eqn 1.

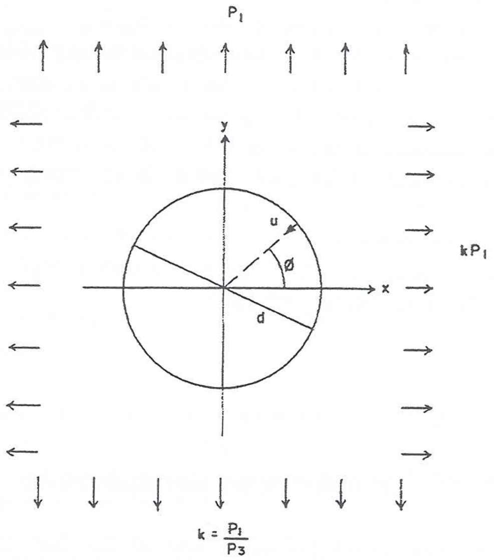

FIG 1: Circular Hole in an Infinite Elastic Medium.

The UET is based on the principle that for each independent displacement vector that can be measured one of the independent material property variables described by the Kirsch Solution can be calculated. The remainder of the variables must either be assessed using testing procedures or estimated. As two displacement vectors in most excavations can be effectively measured (sidewall convergence and crown sag for tunnels and sidewall convergence in two planes for basements and shafts) three parameters must be known. It should also be noted that in some cases displacement vectors can only be measured in one plane, the consequence being that an additional parameter must be determined through testing or assumed.

Of the five parameters presented in the Kirsch Solution, two can be simply accounted for: the geometry of the excavation; and the Poisson's ratio (ν), which can usually be assessed by laboratory testing or estimated from comparison with published values for similar materials. The results of the UET are also relatively insensitive to ν because of its limited range for most rocks (0.15 to 0.3).

For tunnels a third parameter, vertical stress, can be estimated based on the thickness and density of the overburden. This leaves the field stress ratio (k) and Young's modulus (E) to be determined.

If a material property has to be tested for or assumed it is usually E as there are laboratory techniques that can do it, albeit with some difficulty, and there are published ranges of E for most rock types.

The Kirsch Solution in its original form has limited application in civil and mining engineering because of its geometric constraints. However, a simplified form of it referred to as the "Lame Solution" in Pells et al (1981) does. The Lame solution combines the geometric term, presented in the square brackets in eqn 1, into a single "displacement influence factor"(I_δ), giving:

$$\delta = pd\frac{(1-\upsilon^2)}{E}I_\delta \qquad \text{Eqn 2}$$

where: $I_\delta = nk$

The displacement influence factor is unique for each location on the excavation perimeter. Often for tunnels of regular geometry this can be assessed using a closed form solution, however for tunnels of irregular geometry or foundation excavations this is not possible.

Instead, a numerical model is used to estimate I_δ. More specifically, it estimates the change in the convergence under a range of each subject variable. The results of this model can usually be simplified to a linear relationship for each of the monitored axes of convergence, viz:

$$\delta_1 = (a + bk)E \qquad \text{Eqn 3.}$$

$$\delta_2 = (c + dk)E \qquad \text{Eqn 4}$$

Equations 3 and 4 can be solved simultaneously or graphically, with the point of commonality being the unique convergence signature for a particular point on a given excavation. This is perhaps most easily understood by looking at the example in section 5 and its results presented by Figure 7.

In order to apply the above theory to practice a number of simplifying assumptions need to be made. These assumptions can be divided into two groups: general assumptions and model specific assumptions. These assumptions and the errors that they introduce are as follows.

General Assumptions

1. Faulty readings - poor monitoring results may be hard to distinguish from real results.

2. Discontinuity affected readings - where local in-elastic displacement has occurred due to discontinuities.

3. Pre-monitoring convergence - convergence occurs prior to the installation of monitoring apparatus. This is considered further in the following section.

4. Numerical modelling - does not produce exact solutions.

5. Isotropy - many rock masses do not behave isotropically.

6. Liner influence - the influence on convergence of liners and supports can be difficult to assess.

7. Temperature - assumed to be constant (minor).

Model Specific Assumptions

1. Homogeneous medium - only a single value for the Young's modulus is accepted.

2. Elastic medium - plastic behaviour is not modelled.

It should be noted that some numerical models can accommodate in-homogeneous and plastic behaviour, UDEC being one example.

3. PRE-MONITORING CORRECTION

A certain amount of displacement occurs prior to the installation of monitoring points. Indeed, some converge occurs in front of the excavation face. As such, a correction must be made to the monitored convergence to account for pre-monitoring convergence. This can be achieved using a Boundary Integral Element Model (BIEM).

The theoretical background for the BIEM is essentially the same as discussed in section 2. A 3-D model is produced of the excavation extending some way into the rock mass. From this model it is possible to estimate the total convergence experienced at any distance back from the face. It should be noted however that this relies on an estimate being made for E, ν and k. The first runs of the BIEM model are likely to be inaccurate because of the assumed parameters, however, these parameters can be improved by using the results of the UET in an iterative manner until such time as the two match. An example of a BIEM model is presented in Section 5.

As a first pass it is reasonable to estimate the pre-monitoring convergence from published solutions, such as those by Brady & Brown (1985). It should be noted that when using these solutions no account is taken of the specific geometry of the tunnel or the material properties of the rock mass. As such, for tunnels of geometry that considerably differs from circular or for highly heterogeneous stress fields, these solutions can be misleading. Figure 2, which was taken from Brady & Brown (1985), shows the radial displacement of a circular tunnel in a hydrostatic stress field.

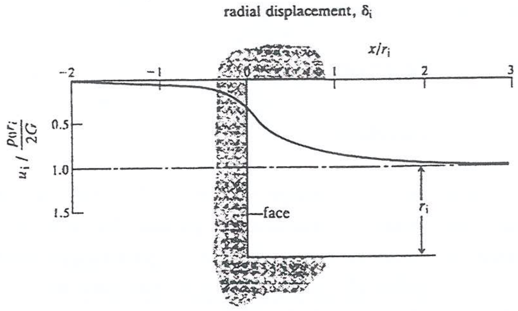

FIG 2: Proportion of Radial Displacement Experienced in Relation to Face Advance (Brady & Brown, 1985)

As shown by Figure 2 and concluded by Hocking (1976) "80% of the vertical elastic displacement of the crown of a tunnel will occur within 0.75 radii from the face". While the amount of convergence within the first radii of a tunnel will vary depending on its profile, the importance of having monitoring records from as close to the face as possible is clear.

4. PRACTICAL APPLICATION

4.1 Fields of Application

Strictly speaking the theory on which the UET is based is only valid for homogeneous isotropic linear elastic mediums, a situation not found in nature. However, in practice it can be applied to any rock environment where either the elastic component of deformation is significantly greater than the plastic component or the amount of plastic deformation is known. The former includes most hard rock environments and thus the UET can be expected to work reasonably well provided significant stress induced fracturing does not occur around the excavation.

The more displacement vectors measured the larger the redundancy in the Kirsch Solution and the greater the confidence in the answers from the UET. As such the UET is best suited to tunnels because convergence can be measured along two planes, and the magnitude of one of the stresses (vertical stress) can be calculated. This gives two independent convergence signatures which can be used to back-calculate E and k. The UET can still be used for vertical excavations (shafts and foundations), but the magnitude of one of the horizontal stresses or E has to be estimated.

4.2 Numerical Modelling

The role of numerical modelling is to ascertain a suitable I_δ and to develop the BIEM. This can be achieved with a variety of numerical techniques. As there are ample examples of easy to use software that can be used for such modelling in the market place today no further discussion is given to it here.

4.3 Instrumentation

4.3.1 Location

A significant advantage of the UET is that it can utilise the same suite of monitoring points that are included in most conventional excavations. Typically in tunnels this includes sidewall convergence monitoring and crown sag monitoring. However, any monitoring arrangement is acceptable, with the better result being achieved the closer the measured displacement vectors are to the principal stress planes.

4.3.2 Type of Monitoring Apparatus

The type of monitoring apparatus required depends on the precision required. The regularly employed convergence measuring equipment can be divided into three categories, viz:

- Borehole extensometers: measure the movements of both the perimeter of the excavation as well as the differential movements of the rock mass surrounding the excavation. Because they can be measured remotely and their accuracy they are the most attractive monitoring option, sadly they are also the most expensive (accuracy ±0.01mm).
- Direct measurements: the distance between two points on the excavation perimeter can be assessed directly using tape convergence measurements. The results of this method are typically accurate enough for the UET, however there can be some difficulty in recording measurements as they must be taken across the face of the excavation (accuracy to ±0.1mm).
- Surveying: total station surveying has limited application in collecting data for the UET because of its relatively poor accuracy (accuracy to ±1.0mm).

5. CASE STUDY - EASTERN DISTRIBUTOR TUNNEL

5.1 General

The Eastern Distributor Tunnel is a double-decker road tunnel in Sydney's eastern suburbs. It forms the backbone of a road link that connects the Sydney Harbour Tunnel in the north with the Kingsford Smith International Airport in the South. The tunnel is approximately 1.7km in length and carries 3 lanes of traffic in each direction and is excavated almost exclusively in slightly weathered Hawkesbury Sandstone (Bertuzzi & Justice, 1999). The tunnel is complicated by a series of on-load/off-load ramps that result in a continually changing profile, with a maximum unsupported span of 22m. The crown of the tunnel is not greater than 35m below the ground surface and the tunnel is typically 12m high.

The Eastern Distributor Tunnel was well suited to the UET for the following reasons:

- the low stress environment meant that the Hawkesbury Sandstone would behave in a pseudo-elastic manner, as has been shown by monitoring of other excavations in Sydney;
- the low insitu stress due to the shallow depth of the tunnel made conventional stress assessment methods (over coring, flat jacking and hydraulic fracturing) unreliable;
- access from both within the tunnel and from the surface for conventional stress measurements was limited;
- the changing ground conditions due to faulting and varying tunnel depths required a large suite of stress measurements to account for the changing ground conditions for which the cost of conventional methods was prohibitive; and
- an extensive array of monitoring stations was installed for tunnel observations as part of the excavation contracts.

An additional advantage for the application of the UET at the Eastern Distributor was that the Young's modulus for Class I/II Hawkesbury Sandstone is generally accepted to be between 1500 and 2500MPa.

5.2 Monitoring Array

The monitoring used for the UET consisted of crown sag and sidewall convergence at 50 to 100m internals.

Figure 3 shows the typical tunnel cross section and monitoring array.

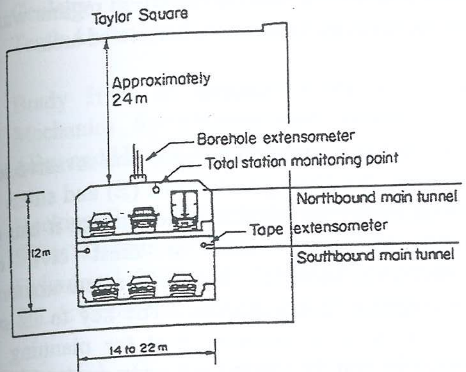

FIG 3: Typical Tunnel Cross Section with Monitoring Array

5.3 I_8 Numerical Model

The numerical model to assess the convergence signature of the tunnel was developed using Phase², a finite element package from the Rock Engineering Group, University of Toronto. The model incorporated three stages: (1) balancing of insitu stresses; (2) excavation of the north bound tunnel; and (3) excavation of the south bound tunnel. An eight node finite element mesh was used with node spacing reduced to not greater than 200mm at the perimeter of the tunnel. Figure 4 shows the output of the model after complete tunnel excavation for a 13m span section of the tunnel.

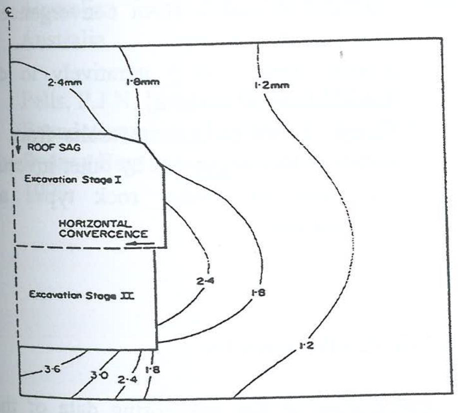

FIG 4: Total Displacement Contours from I_8 Model

The model was run for a series of E and k. The results from which were plotted to obtain a graphical solution for E and k for varying sidewall and crown convergence. An example of this is presented as Figure 7 with the monitoring results plotted onto it.

5.4 BIEM

A three dimensional BIEM was developed to estimate the pre-monitoring convergence, as discussed in Section 3. This was conducted using Examine 3D, a boundary element numerical package from the Rock Engineering Group, University of Toronto. A cross section of the displacement output of the model is presented as Figure 5 and the results are summarised by Figure 6.

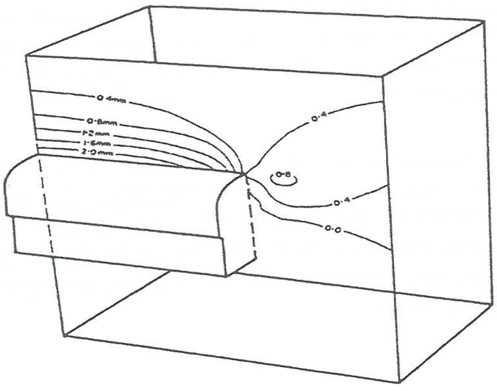

FIG 5: BIEM Model - Assessment of the Convergence Prior to Convergence Monitoring.

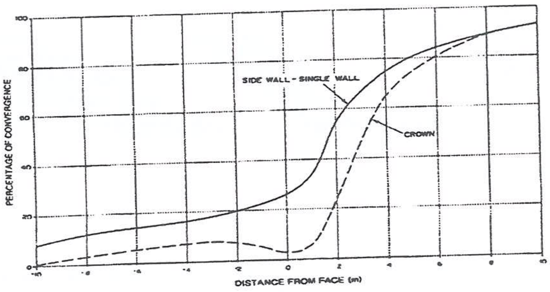

FIG 6: Summary of BIEM Results

5.5 Fault Zone

The 13m span of the tunnel modelled as part of this study passed through the Woolloomolloo Fault Zone (WFZ), as described in Bertuzzi & Justice (1998). The WFZ is a reverse fault with throw of up to about 6m. Significant strike slip movement is though to also be associated with the faulting action; however, there is little evidence to definitively support this postulation.

As part of the investigation phase of the project hydro-fracture testing was conducted in a single borehole that intersected the WFZ. The results of this testing indicated a k of 2 to 6, with the higher values recorded towards that base of the borehole which was approximately at the level of the tunnel crown.

5.6 Results

The sidewall convergence and crown sag results, corrected for pre-monitoring convergence, have been plotted on the graphical solution for E and k which was developed using the numerical model described in section 5.3. This is presented as Figure 7.

The sag results varied from the crown rising by 6mm to it lowering by 7mm. There was no apparent pattern to these results that was discernible by the author through the noise introduced by the inaccuracy of total station surveying. As such, the sag monitoring data had to be discarded for the purposes of assessing k.

To establish k it was necessary to assume a value for E. For Class I/II Hawkesbury Sandstone this is generally accepted to be between 1500 and 2500MPa and typically about 2000MPa (Pells, Mostyn & Walker, 1998). The sensitivity of the prediction of k to this assumption can be seen at Figure 7.

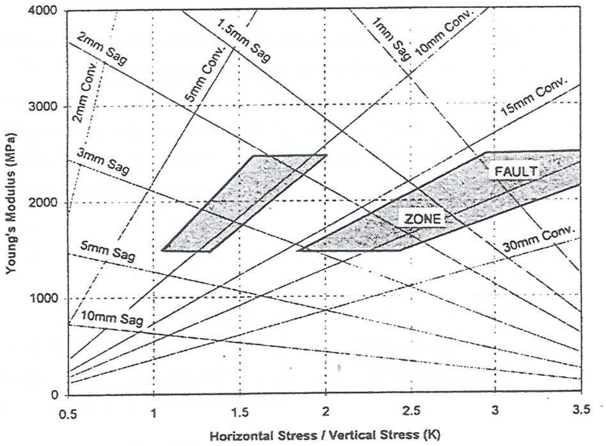

Fig 7 Stress Regime versus Sag and Horizontal Convergence with Monitoring Results

The results for the 12 sidewall convergence monitoring points recognised two discrete stress regimes; one for the general rock mass, and the other for the fault zone. The stress ratios for these two areas are summarised by Table 5.1.

Table 5.1
Summary of Assessed Stress Regime

AREA	STRESS RATIO (k)	
	RANGE	MEAN
General rock mass	1.1 to 2.0	1.6
Fault zones	1.8 to 4.0	2.8

Given the structural complexity associated with faulting, these results compare favourably with those obtained from the hydro-fracture testing which predicted a range of k from 2 to 6. No tests of the stress regime was conducted for the areas outside the fault affected area.

6. CONCLUSIONS

The UET is a simple, low cost, non invasive technique of assessing the Young's Modulus (E) and stress ratio (k) of a rock mass. Its advantage lies in that it can employ the monitoring array used in most civil engineering excavations, including borehole extensometers and tape convergence measurements. The key to its successfully application is to consider it at the planning stage of a project so that the monitoring array can be developed so as to be applicable to it.

The UET has it limitations, as do all insitu stress measurement techniques, however its advantages well outweigh them. There is no doubt that the UET will become an increasingly popular tool for both the design and analysis of civil excavations.

Summary of the UET process:

1. Convergence signature model,
2. BIEM Model to assess pre-monitoring convergence,
3. Install and monitor convergence vectors,
4. Correct convergence results for pre-monitoring convergence using BIEM,
5. Estimate E and k from convergence signature model,
6. Repeat steps 1 to 5 iteratively to correct for model errors in E and k,
7. Check E and k against results from stress test and/or values suggested by other investigations in the same or similar rock types and stress environments.

ACKNOWLEDGMENTS

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