Third A-NZ Young Geotechnical Professionals Conference, Cameron, Collingwood & Slatter (eds), Feb. 1998, Melbourne, Australia.

The Bearing Capacity of Strip Footings on Two-Layered Clays

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ABSTRACT

This paper applies numerical limit analyses to evaluate the undrained bearing capacity of a rigid surface footing resting on a two-layer clay deposit. Rigorous bounds on the ultimate bearing capacity are obtained by employing finite elements in conjunction with the upper and lower bound limit theorems of classical plasticity. Results from the limit theorems typically bracket the true collapse load by approximately twelve per cent, and have been presented in the form of bearing capacity factors based on various layer properties and geometries. A comparison is made between existing limit analysis, empirical and semi-empirical solutions. This indicates that the latter can overestimate or underestimate the bearing capacity by as much as twenty per cent for certain problem geometries.

1. INTRODUCTION

The ultimate bearing capacity of surface strip footings resting on a single layer of homogeneous undrained clay has been studied by numerous investigators with practitioners generally using Terzaghi's (1943) expression to compute ultimate footing loads. In reality, however, soil strength profiles beneath footings are not homogeneous but may increase or decrease with depth or consist of distinct layers having significantly different properties. Whilst the effect of increasing strength with depth on bearing capacity has been addressed by several researchers, notably Davis and Booker (1973), rigorous solutions to the problem of footings resting on layered clays do not appear to exist.

To calculate the ultimate bearing capacity for surface strip footings resting on a horizontally layered clay profile, practitioners commonly use the approximate solutions of Button (1953), Reddy and Srinivasan (1967), Chen (1975), Brown and Meyerhof (1969) and Meyerhof and Hanna (1978). Button (1953) and Chen (1975) calculated upper bound solutions assuming a simple circular failure surface (Figure 2) while Reddy and Srinivasan (1967), assuming the same circular mechanism, obtained results using the method of limiting equilibrium. The solutions of Brown and Meyerhof (1969) and Meyerhof and Hanna (1978) were based upon a series of model footing tests from which empirical and semi-empirical solutions for the bearing capacity factor were derived.

In view of its simplicity, past research into the bearing capacity on layered clays using the limit theorems appears to be restricted exclusively to the upper bound method of analysis. A more desirable solution for engineering practice is a lower bound estimate as it results in a safe design and, if used in conjunction with an upper bound solution, serves to bracket the actual collapse load from above and below.

The purpose of this paper is to take advantage of the ability of the limit theorems to enclose the actual collapse load by computing both types of solution for the bearing capacity of strip footings on a two layered clay profile. These solutions are obtained using the numerical techniques developed by Sloan (1988) and Sloan and Kleeman (1995), which are based upon the limit theorems of classical plasticity and finite elements. The methods assume a perfectly plastic soil model with a Tresca yield criterion and lead to large linear programming problems. The solution to the lower bound optimisation problem defines a statically admissible stress field and gives a rigorous lower bound to the ultimate bearing capacity. The solution to the upper bound optimisation problem defines a kinematically admissible velocity field and hence results in a rigorous upper bound to the ultimate bearing capacity. A statically admissible stress field is one which satisfies equilibrium, the stress boundary conditions and the yield criterion while a kinematically admissible velocity field is one which satisfies compatibility, the flow rule and the velocity boundary conditions.

2. PROBLEM DEFINITION

The plane strain bearing capacity problem to be considered is illustrated in Figure 2. A strip footing of width B rests upon an upper layer of clay with undrained shear strength c_{u1} and thickness H. This is underlain by a clay layer of undrained shear strength c_{u2} and infinite depth.

The bearing capacity solution to this problem will be a function of the two ratios *H/B* and c_{u1}/c_{u2}. Past research by Brown and Meyerhof (1969) and Meyerhof and Hanna(1978) indicates that a reduction in bearing capacity for a strong-over-soft clay system may occur up to a depth ratio of $H/B \approx 2.5$. In this paper solutions have been computed for problems where *H/B* ranges from 0.125 to 2 and c_{u1}/c_{u2} varies from 0.2 to 5. This should cover most problems of practical interest. Note that $c_{u1}/c_{u2} > 1$ corresponds to the common case of a strong clay layer over a soft clay layer, whilst $c_{u1}/c_{u2} < 1$ corresponds to the reverse.

The bearing capacity of a shallow strip footing on a clay layer can be written in the form

$$q_u = c_u N_c + q \tag{1}$$

where N_c is a bearing capacity factor and q is a surcharge. For a surface strip footing this equation reduces to

$$q_u = c_u N_c \tag{2}$$

For the case of a layered soil profile it is convenient to rewrite (2) in the form

$$N_c^* = \frac{q_u}{c_{u1}} \tag{3}$$

where c_{u1} is the undrained shear strength of the top layer and N_c^* is a modified bearing capacity factor which is a function of both *H/B* and c_{u1}/c_{u2}. The value of N_c^* will be computed using the results from both upper and lower bound analyses for each ratio of *H/B* and c_{u1}/c_{u2}. For a homogeneous profile with $c_{u1}=c_{u2}$, N_c^*equals the well known Prantl solution of $(2 + \pi)$. For the range of problem geometries considered, the bound solutions are typically able to bracket the exact bearing capacity factor to within twelve percent or better.

3. FINITE ELEMENT FORMULATION OF LIMIT THEOREMS

The following is only a brief summary of the numerical formulation of the limit theorems and only those aspects specifically related to the current study of bearing capacity are mentioned. Full details of the numerical procedures can be found in Sloan (1988) and Sloan and Kleeman (1995), and will not be repeated here.

Lower bound formulation

The lower bound solution is obtained by modelling a statically admissible stress field using finite elements where stress discontinuities can occur at the interface between adjacent elements. Application of the stress boundary conditions, equilibrium equations and yield criterion leads to an expression of the collapse load which is maximised subject to a set of linear constraints on the stresses.

Unlike the more familiar displacement finite element method, each node is unique to a particular element and therefore any number of nodes may share the same coordinates. This enables a wide range of stress fields to be modelled by permitting statically admissible stress discontinuities at all edges that are shared by adjacent elements, including those edges that are shared by adjacent extension elements.

To furnish a rigorous lower bound solution for the collapse load, it is necessary to ensure the stress field obeys equilibrium, the stress boundary conditions and the yield criterion. Each of these requirements imposes a separate set of constraints on the nodal stresses.

For many plane strain geotechnical problems, we seek a statically admissible stress field which maximises an integral of the normal stress σ_n over some part of the boundary. Denoting the out-of-plane thickness by h, these integrals are typically of the form

$$P = h \int_S \sigma_n \, ds \tag{4}$$

where P represents the collapse load.

By assembling the various constraints and objective function coefficients for the overall mesh, a statically admissible stress field which maximises the collapse load may be found. A lower bound solution for the footing problem is obtained by maximising the integral of the compressive stress along the soil footing interface.

Upper bound formulation

An upper bound on the exact collapse load can be obtained by modelling a kinematically admissible velocity field. To be kinematically admissible, such a velocity field must satisfy the set of constraints imposed by compatibility, velocity boundary conditions and the flow rule. By prescribing a set of velocities along a specified boundary segment we can equate the power dissipated internally, due to plastic yielding within the soil mass and sliding of the velocity discontinuities, with the power dissipated by the external loads to yield a strict upper bound on the true limit load.

The three noded triangle is again used for the upper bound formulation. Now, however, each node is associated with two unknown velocities and each element has p non-negative plastic multiplier rates (where p is the number of sides in the linearised yield criterion).

To define the objective function, the dissipated power (or some related load parameter) is expressed in terms of the unknown plastic multiplier rates and discontinuity parameters. As the soil deforms, power dissipation may occur in the velocity discontinuities as well as in the triangles.

Once the constraints and the objective function coefficients are assembled, a kinematically velocity field which minimises the internal power dissipation for a specified set of boundary conditions, may be found.

An upper bound solution is obtained by prescribing a unit downward velocity to the nodes directly below the footing along with the additional constraint that the footing cannot move horizontally. After the corresponding optimisation problem is solved for the imposed boundary conditions, the collapse load is found by equating the dissipated power to the power expended by the external forces. The results for the simple case of a surface footing resting on a homogeneous soil profile are shown in Figure 1, where the bearing capacity factor N_c^* was found to equal 5.32 (approximately 3 percent above the exact Prantl solution of $N_c^* = (2 + \pi)$

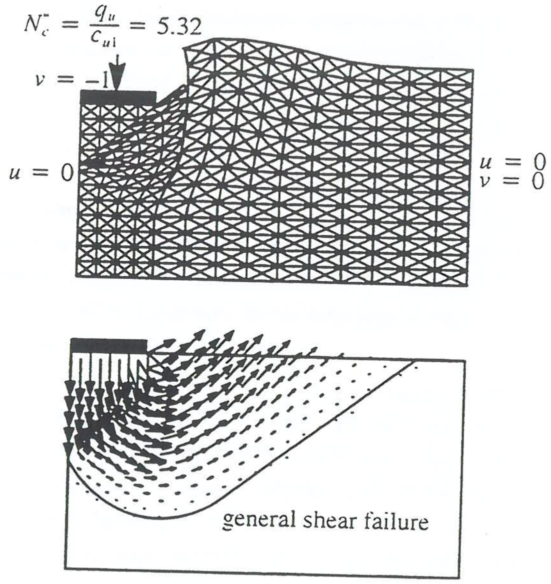

Figure 1 Deflected mesh and velocity diagram for a homogeneous soil profile

4. RESULTS AND DISCUSSION

The computed upper and lower bound estimates of the bearing capacity factor N_c^* for layered clay soils are shown graphically in Figure 3 and Figure 4. These results indicate that, for practical design purposes, sufficiently small error bounds were achieved with the true collapse load typically being bracketed to within 12% or better.

Figure 3 and Figure 4 also compare the numerical bounds and the available upper bound solutions of Chen (1975), the empirical solutions obtained by Brown and Meyerhof (1969), and the semi-empirical solutions of Meyerhof and Hanna (1978).

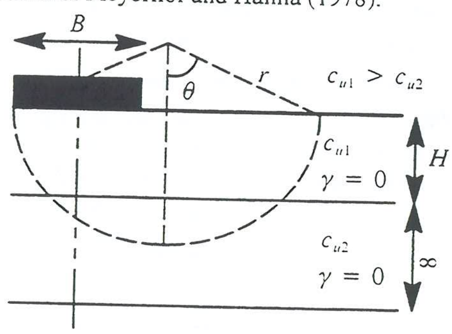

Figure 2 Circular mechanism – Chen

The bearing capacity factors obtained by Chen (1975) were obtained by assuming a circular failure mechanism as shown in Figure 2.

By equating the rate of internal and external work an upper bound expression for the bearing capacity factor can be derived. For a homogeneous soil profile this expression can be solved analytically to give a value of $N_c^* = 5.53$. This is approximately 8 percent above the exact Prantl solution of $N_c^* = (2 + \pi)$.

The ultimate bearing capacity of a footing resting on a strong-over-soft clay deposit, as determined by Meyerhof and Hanna (1978), is based on the assumption that failure occurs by punching shear through the top layer followed by general shear failure of the bottom layer. The ultimate capacity is given by

$$q_u = c_{u2}N_c + 2c_a H/B \qquad (5)$$

where $N_c = 5.14$.

In terms of physical behaviour, the second term in this equation is representative of some type of punching shear through the strong top layer, with the first term reflecting full general shear failure in the bottom layer. The term c_a is defined as the unit adhesion acting on the assumed punching shear plane through the strong top crust and was derived from experimental results. Equation (5) can be rearranged to give the bearing capacity factor N_c^* as

$$N_c^* = \frac{q_u}{c_{u1}} = N_c\left(\frac{c_{u2}}{c_{u1}}\right) + 2\left(\frac{c_a}{c_{u1}}\right)\left(\frac{H}{B}\right) \qquad (6)$$

Brown and Meyerhof (1969) provided charts of the bearing capacity factor N_c^* for both strong-over-soft and soft-over-strong clay profiles based on a series of model laboratory tests. Their results for the soft-over-strong case are reproduced in Figure 3 and Figure 4 for comparison purposes.

Footings on strong clay overlying soft clay

The upper and lower bound results clearly indicate that a complex relationship exists between general, local and punching shear failure and the ratios c_{u1}/c_{u2} and *H/B*. Failure generally occurs by either partial or full punching shear through the top layer followed by yielding of the bottom layer. The distinction between these two failure modes is illustrated in Figure 5. Full punching shear is characterised by a complete vertical separation of the top layer which then effectively acts as a rigid column of soil that punches through to the bottom layer. Conversely, only a small vertical separation of the top layer is evident for partial punching shear (Figure 5(a)), with both local vertical and lateral deformation of the soil column below the footing now apparent. Full punching shear typically occurs for ratios of $H/B \leq 0.5$, regardless of the ratio c_{u1}/c_{u2}, while for $H/B > 0.5$ the division between full and partial punching shear occurs at c_{u1}/c_{u2} approximately equal to 2.5. The extent and form of yielding within the bottom layer is dependent on both the depth and strength of the overlying top layer. This is best illustrated by the velocity diagrams shown in Figure 6.

For the case of moderately strong crusts ($c_{u1}/c_{u2} \leq 2.5$), failure is generally caused by partial punching shear. For thin top crusts with $H/B < 0.5$, the overall failure mechanism is similar to that depicted in Figure 1. As the depth of the top crust approaches the footing width *B*, upward deformations within the bottom layer become restricted causing an increase in the extent of plastic yielding(see Figure 6)

As the top crust becomes very strong compared to the bottom layer ($c_{u1}/c_{u2} \geq 2.5$) full punching shear through the top layer occurs. The very strong top layer then serves to greatly restrict both lateral and vertical movement of the soil contained in the soft layer below (see Figure 6). This results in the formation of a deep zone of plastic shearing within the bottom layer and, for thicker crusts ($H/B > 0.75$), a local elastic zone is formed within the top layer immediately adjacent to the footing.

The limit analysis results indicate that a reduction in bearing capacity for a strong-over-soft clay system occurs up to a depth ratio of $H/B \approx 1.5$ - 2.0. This lower limit is applicable for soil profiles where $c_{u1}/c_{u2} \leq 2.5$, but for profiles that have a very strong top crust with $c_{u1}/c_{u2} \geq 2.5$, punching failure through the top layer is likely to occur up to depth ratio of $H/B = 2$. For ratios of $H/B > 2$, failure is contained entirely within the top layer and is independent of the ratio c_{u1}/c_{u2}. These results are similar to those predicted by Chen (1975), but are lower than those estimated by Meyerhof and Hanna (1978), who suggest a reduction in bearing capacity may occur up to a depth ratio of $H/B = 2.5$.

The analytical upper bounds obtained by Chen (1975), who assumed a simple circular failure mechanism, compare favourably with those obtained from the finite element upper bounds for smaller values of H/B but become rather unconservative when $H \geq B$.

For ratios of $H/B \leq 0.5$, the solutions of Chen are less than 5% above the upper bound limit analysis results for all values of c_{u1}/c_{u2}. For larger values of H/B, the solutions of Chen become increasingly inaccurate as c_{u1}/c_{u2} increases, with a maximum error of approximately 15% for H/B=1.5 and c_{u1}/c_{u2}=5. The reason for this is that for larger values of H/B and c_{u1}/c_{u2}, the assumed mechanism of Chen (1975) is no longer a good representation of the true collapse mechanism. The optimal mechanism using a circular failure surface is found to lie almost entirely within the top layer. This is in contrast to the finite element limit analysis results which clearly indicate that the failure mechanism that yields the best upper bound penetrates deeply into the soft bottom layer.

With reference to Figure 3 and Figure 4 it can be seen that for a soil profile having a moderately strong top crust ($c_{u1}/c_{u2} \leq 2.5$) the solutions of Meyerhof and Hanna (1978) typically lie either within or just outside the upper and lower bound solutions. For very strong top crusts ($c_{u1}/c_{u2} > 2.5$), these solutions tend to become over conservative as H/B increases, and lie 12-16% below the lower bound solution. This margin is likely to increase to as much as 20% upon further refinement of the lower bound mesh.

As with the solutions of Chen (1975), the solutions of Meyerhof and Hanna (1978) are limited by their assumption that a single type of failure mechanism exists. Only for the restricted case of thin, moderately strong crusts, where $H/B \leq 0.5$ and $c_{u1}/c_{u2} \leq 2.5$, does the assumption of punching through the crust followed by general shear failure in the bottom layer appear, to some degree, to be valid. This assumption is clearly not correct for larger top crust thicknesses or if the crust is substantially stronger than the bottom layer. For these cases, failure tends to be either a combination of general shear failure through both layers or a deep rotational mechanism, depending on the ratio of the layer strengths c_{u1}/c_{u2}.

Footings on soft clay overlying strong clay

The upper and lower bound results indicate that for ratios of $H/B \leq 0.5$, the bearing capacity increases as the relative strength of the bottom layer rises. For all of these cases the proportion of yielding within the bottom layer decreases as its strength increases. At a limiting ratio of c_{u1}/c_{u2}, no further increase in bearing capacity is achieved as the failure surface becomes fully contained within the top layer. This is represented by the sudden change in the curvature of the plots shown in Figure 3 and Figure 4. As an example, for H/B=0.125 the bearing capacity increases as c_{u1}/c_{u2} decreases until a limiting value of c_{u1}/c_{u2}=0.5 is reached. After this point failure is fully contained within the top layer.

For all values of $H/B > 0.5$, the bound solutions indicate that failure occurs entirely within the top layer and the bearing capacity is independent of the strength of the bottom layer.

The upper bound solutions of Chen (1975) overestimate the bearing capacity factor for all cases where $c_{u1}/c_{u2} < 1$ and are 5-22% above the upper bound finite element solutions. The overestimate is greatest for small top layer thicknesses where $H/B \leq 0.375$. The reason for this is that the optimal slip circle determined by Chen (1975) penetrates deeper into the underlying strong layer than the mechanism predicted by the finite element solution. When the failure mechanism is contained within the thin top layer, failure is by lateral squeezing and local failure at the footing edge and is therefore not accurately modelled by a circular slip mechanism.

As H/B increases above 0.375, the accuracy of the Chen (1975) solutions improves and typically lies 3-7% above the upper bound finite element solutions. This is because the majority of yielding occurs within the top layer and the actual failure mode can now be adequately modelled by a rotational failure mechanism. For $H/B \geq 0.75$ failure occurs entirely within the top layer and the exact solution for these cases will be $N_c^* = 5.14$, the Prandtl solution. This implies that the error in the upper bound finite element solutions is $\approx$3%, while the error in the Chen (1975) solutions is $\approx$6-7%.

The empirical results given by Brown and Meyerhof (1969) are limited to c_{u1}/c_{u2} ratios between 1 and 0.5. For relatively thin top layers with $H/B \leq 0.5$, the solutions of Brown and Meyerhof typically lie near the lower bound finite element solutions. As the top layer thickness increases above $H/B > 0.5$, failure becomes contained within it and the Brown and Meyerhof

(1969) solution lies central to both upper and lower bound finite element solutions.

5. CONCLUSIONS

The undrained bearing capacity of a surface strip footing resting on a layered clay profile has been investigated. Using recent numerical formulations of the upper and lower bound limit theorems, rigorous bounds on the bearing capacity for a wide range of problem geometries have been obtained, with the exact collapse load typically being bracketed to within 12%. The results obtained have been presented in terms of a modified bearing capacity factor N_c^* in both graphical and tabular form to facilitate their use in solving practical design problems.

The following conclusions can be made based on the limit analysis results :

- For a strong-over-soft clay profile a number of different failure mechanisms exist that are functions of both the crust thickness and its strength relative to the underlying weaker layer. For this reason, existing upper bound and semi-empirical solutions that are based on a single assumed failure surface are unable to model the likely failure mode over a large range of problem geometries.
- Existing upper bound,empirical and semi-empirical solutions can differ from the bound solutions by up to $\pm 20\%$. The existing solutions are in greatest error when the top layer is very strong compared to the bottom layer ($c_{u1}/c_{u2} > 2.5$) and/or its depth is greater than half the footing width ($H/B > 0.5$).
- A reduction in bearing capacity for a strong-over-soft clay system occurs up to a depth ratio of $H/B \approx 1.5$ - 2.0, where the lower limit is applicable for soil profiles with $c_{u1}/c_{u2} \leq 2.5$. For depth ratios of $H/B > 2$, failure is likely to be fully contained within the top layer and the bearing capacity is given by the Prandtl solution $N_c^* = 2 + \pi$.
- For a soft-over-strong clay system where $H/B \leq 0.5$, the bearing capacity is likely to increase as the relative strength of the bottom layer rises. For thicker top layers where $H/B > 0.75$, failure occurs entirely within the top layer and the bearing capacity is given by the Prandtl solution $N_c^* = 2 + \pi$.

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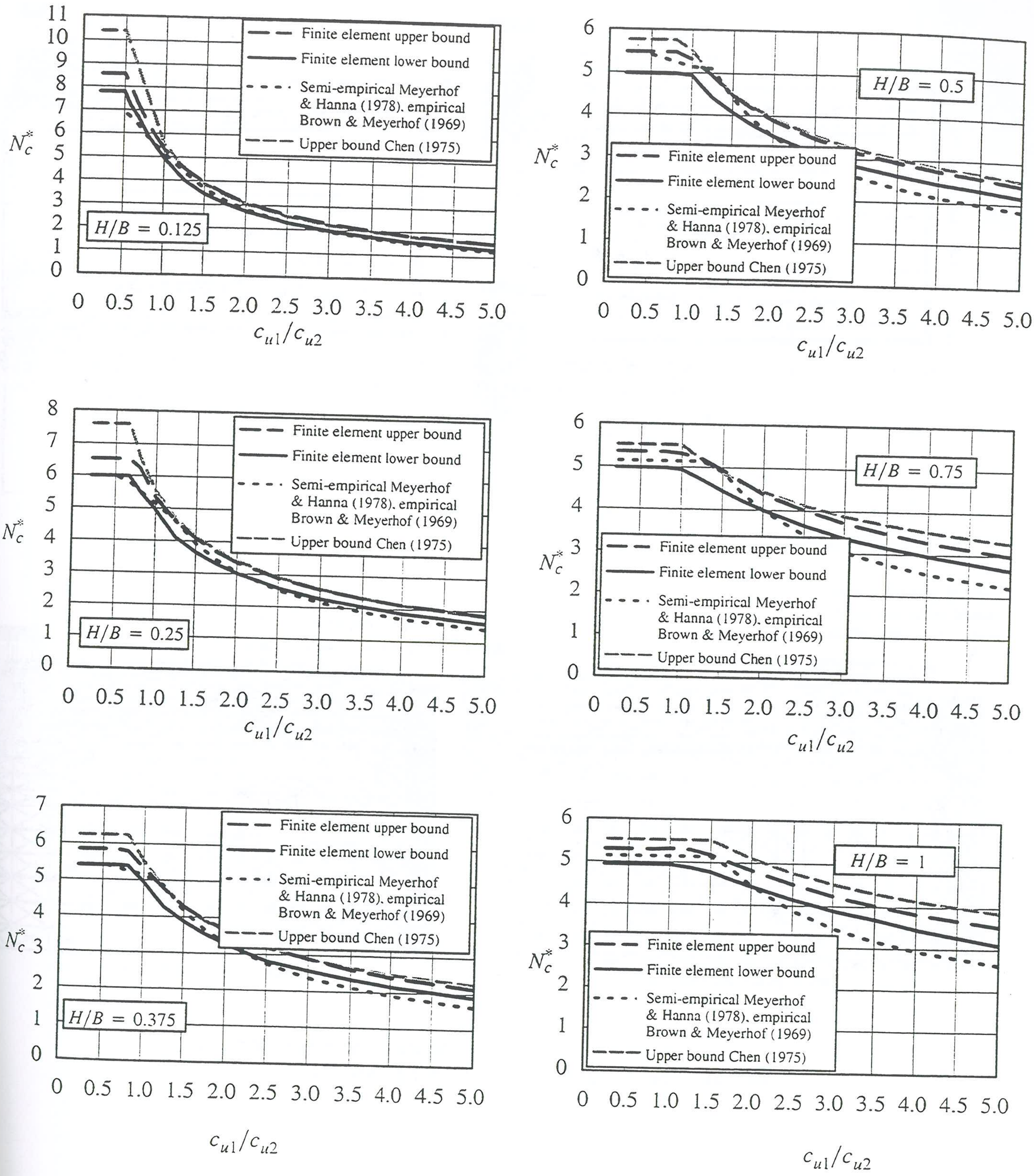

Figure 3 Variation of bearing capacity factor N_c^*
(H/B=0.125 to H/B=1.0)

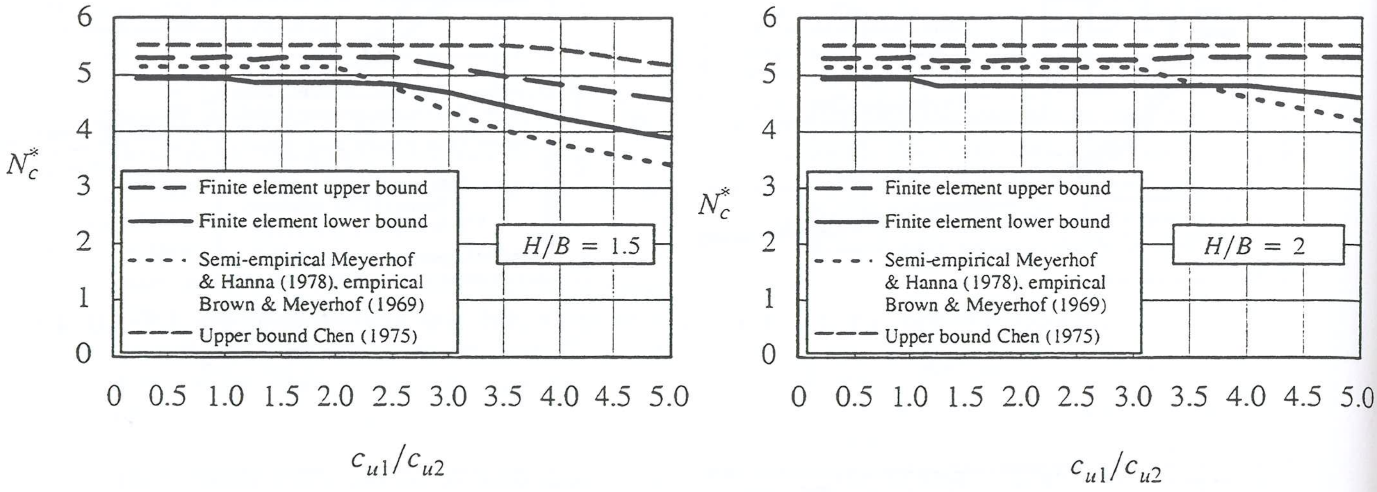

Figure 4 Variation of bearing capacity factor N_c^* (H/B=1.5 and H/B=2)

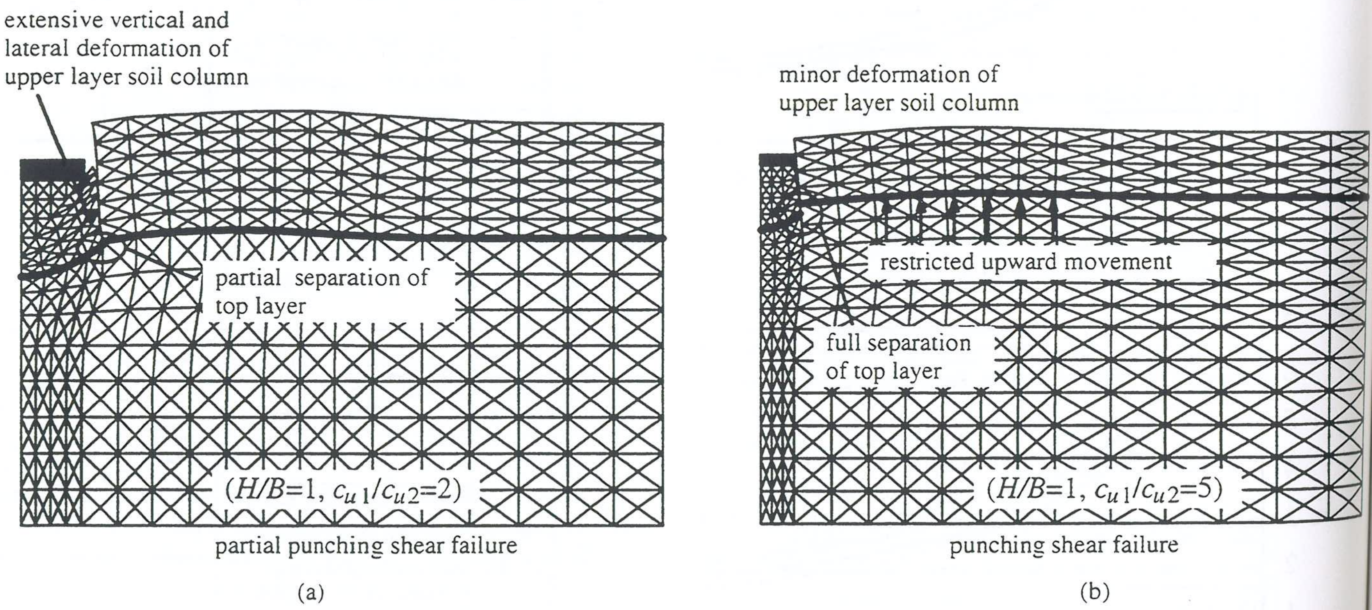

Figure 5 Deflected mesh and zone of plastic yielding for partial punching shear failure and full punching shear failure.

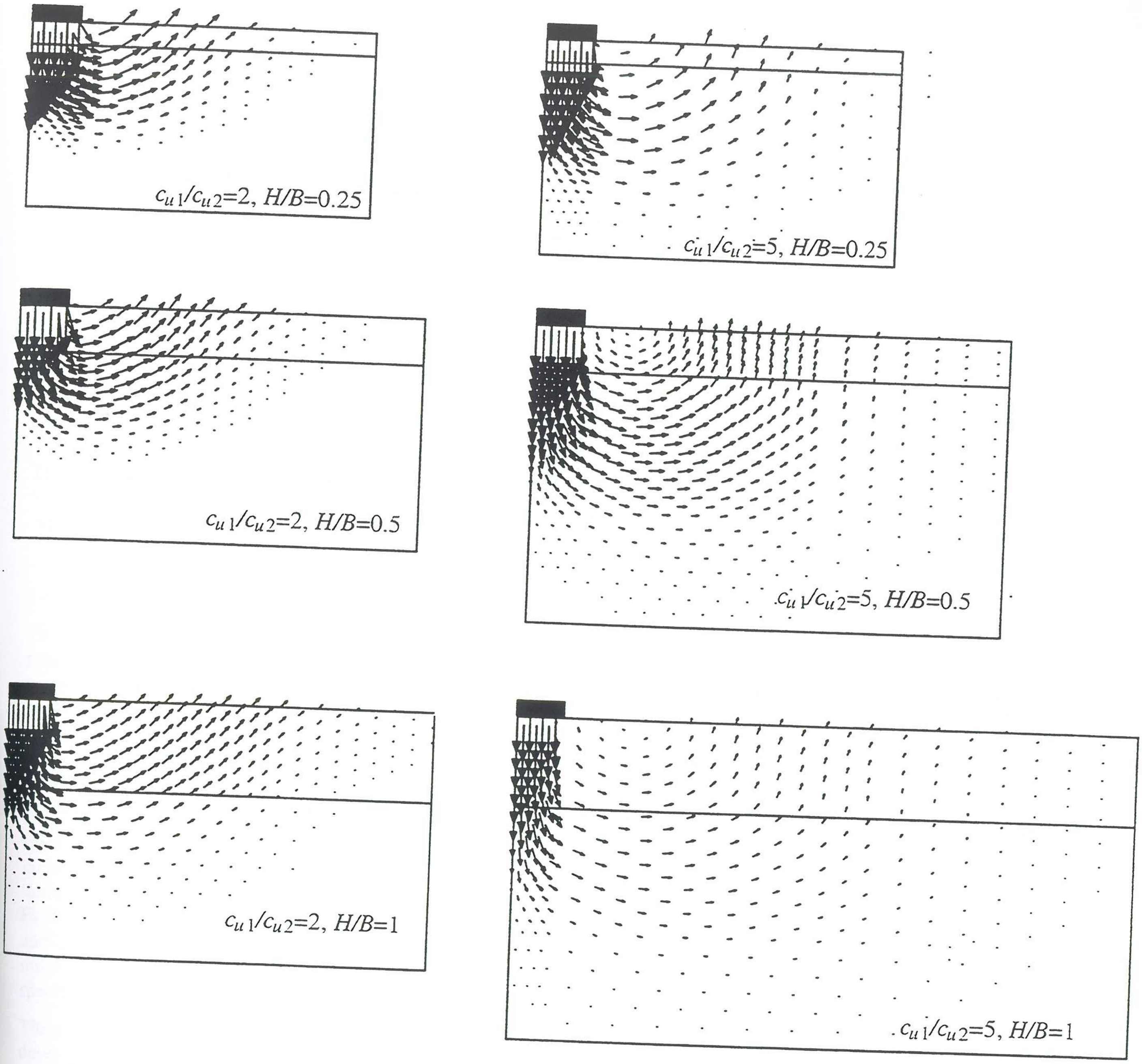

Figure 6 Velocity diagrams for strong-over-soft layers
(c_{u1}/c_{u2}=2,5 and H/B=0.25,0.5)