

Use of Simple Model for Dynamic Foundation Design

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SUMMARY: The dynamic response of a structure can be significantly influenced by the motion of the foundation and the underlying soil, increasing both the flexibility and damping of the system. These additional degrees of freedom can be included into a simple analysis of a structure using widely available stiffness and damping coefficients. This investigation compared the prediction of the response of a structure resting on an elastic soil layer using two models; a simple three degree of freedom model and a more complex finite difference program, using transfer functions as a means of comparison. It was found that the simple model closely predicted the first mode frequency and damping found in the complex model, but did not predict the deviation in response observed in the finite difference program. These differences were more significant for softer soils where the influence of the foundation was greater on the response of the structure. The deviation between the models was attributed to frequency dependence of the foundation coefficients and to the effect of the shallow soil layer causing shear waves to reflect back to the surface from the base rock.

1. INTRODUCTION

This paper presents some of the results of a research project into the influence of soil structure interaction (SSI) on the response of a structure subject to seismic loading. The response of a structure can be significantly altered by the inclusion of additional modes of vibration due to the translation and rotation of the foundation on the underlying soil. This becomes more influential for soft soils and for relatively rigid structures, where the first mode of the system tends to be strongly influenced by foundation modes of vibration.

Modeling of the structure and soil were made using two levels of sophistication. The first used a simple three degree of freedom system to model the structure and foundation. The second model was analysed using the finite difference program FLAC (Fast Lagrangian Analysis of Continua). The soil was modelled as a linear elastic layer with Rayleigh damping resting on a rock base, and the structure as a series of beams.

The aim of the investigation was to compare the simple model of a structure and foundation system with the FLAC model, with the soil remaining linear. The shear modulus of the soil profile was held constant over the depth of the soil layer for simplicity and a range of shear moduli were analysed. This had the effect of altering the relative dominance of the foundation on the response of the structure and altering the site response characteristics of the soil profile. Two aspects of this interaction were investigated; firstly the influence of the structure on the free field site response of the soil layer and secondly the effect of the soil layer on the response of the structure.

2. SIMPLE MODEL

2.1 Introduction

The simple model of the structure and foundation system used for this investigation was one with only three degrees of freedom (3DOF); one for the structure and two for the foundation to model translation and rotation of the foundation on the soil, refer to figure 1. The stiffness and damping coefficients for the foundation were found from solutions for a rigid foundation on an elastic halfspace, and were dependent on the soil shear modulus and Poisson's ratio (Wolff, 1988, Gazetas, 1990). Using these parameters the equations of motion were worked out for the system and the model solved in the time domain for a base acceleration time history. As the system was linear modal superposition could be used to uncouple the equations of motion. Of these uncoupled equations, it was necessary to discard one as its frequency was too high to allow a numerically stable solution to be found. But typically for the situations investigated this mode could be shown to have a negligible contribution to the response due to its low participation factor.

The foundation stiffness and damping parameters used in the analysis were chosen at the static value of the parameter. The foundation stiffness and damping in rotation and translation have been found to vary with the applied frequency of the loading, making it difficult to select the appropriate value of the parameter for use in a time domain analysis. This frequency dependence is further complicated by the presence of a relatively rigid layer at shallow depth by causing waves to reflect back to the structure. Including this frequency dependence is not possible for a solution in the time domain.

2.2 Equations of Motion

The equations of motion were determined for the structure as represented by figure 1. The degrees of freedom for the

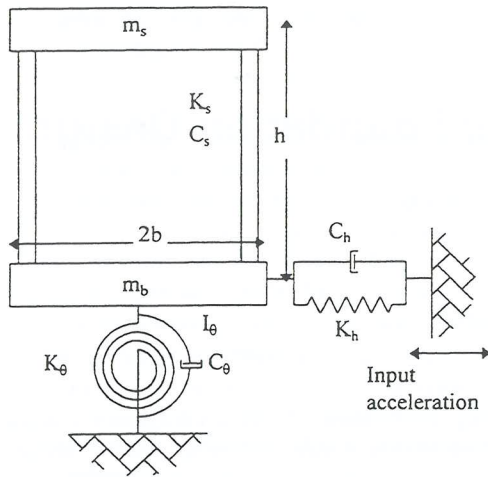


Figure 1. 3DOF model of structure and foundation.

system were the top of the structure relative to the base, and the foundation translation and rotation relative to an at rest position.

$$M_{ssi} \begin{bmatrix} \ddot{u}_s \\ \ddot{u}_b \\ \ddot{\theta} \end{bmatrix} + C_{ssi} \begin{bmatrix} \dot{u}_s \\ \dot{u}_b \\ \dot{\theta} \end{bmatrix} + K_{ssi} \begin{bmatrix} u_s \\ u_b \\ \theta \end{bmatrix} = -M_{load} \ddot{u}_g$$

For convenience the rotational degree of freedom was changed to the equivalent lateral motion of the structure due to rotation at the base. This was a relatively simple modification, but required the alteration of the mass, stiffness and damping matrices to maintain consistency within the model.

$$u_\theta = h\theta$$

The mass matrix for the system, including this change of coordinates, was

$$M_{ssi} = \begin{bmatrix} m_s & m_s & m_s \\ m_s & m_s + m_b & m_s \\ m_s & m_s & \frac{I_\theta}{h^2} \end{bmatrix}$$

where m_s was the mass of the structure, m_b was the mass of the base and I_θ was the rotational mass of the whole structure about the centre of the base. The stiffness, damping and loading matrices were as follows

$$K_{ssi} = \begin{bmatrix} K_s & 0 & 0 \\ 0 & K_h & 0 \\ 0 & 0 & \frac{K_\theta}{h^2} \end{bmatrix}$$

$$C_{ssi} = \begin{bmatrix} C_s & 0 & 0 \\ 0 & C_h & 0 \\ 0 & 0 & \frac{C_\theta}{h^2} \end{bmatrix}$$

$$M_{load} = \begin{bmatrix} m_s \\ m_s + m_b \\ m_s \end{bmatrix}$$

The stiffness and damping values used for the translation and rotation of the strip foundation were found using the following formulae, taken from Wolff (1988)

$$K_h = G(1 + 5v^2)$$

$$K_\theta = Gb^2(1.8 + 5.2v^2)$$

$$C_h = (2 - 2.2v) \frac{b}{V_s} K_h$$

$$C_\theta = (0.14 - 0.24v^2) \frac{b}{V_s} K_\theta$$

where G was the soil shear modulus, V_s was the soil shear wave velocity, v was the soil Poisson's ratio, and b was the semi width of the foundation.

2.3 Modal Response of System

The equations of motion were uncoupled into a set of independent equations of motion using the principal of modal superposition. A requirement of modal superposition was that the coefficients of the mass, stiffness and damping matrices remained constant. This restricted the use of any modification the stiffness and damping values for the foundation to account for frequency dependence. The foundation coefficients used were the static values of stiffness and damping.

The modal frequencies and eigen vectors were determined according to the following equation

$$[K_{ssi} - \omega_j^2 M_{ssi}] \phi_j = \tilde{0}$$

where ω_j was the modal frequency and ϕ_j was the eigen vector of that mode. For this system there were three solutions to this equation, of which one had a very high frequency. This high frequency mode had a relatively minor contribution to the response of the system, and was left out of the subsequent analysis. This frequency became higher as I_θ became closer to $m_s h^2$. If I_θ was set to equal $m_s h^2$ this third mode disappeared altogether.

The relative influence of each mode was determined by the modal participation factor, Γ_j

$$\Gamma_j = \frac{\phi_j^T M_{load}}{\phi_j^T M_{ssi} \phi_j}$$

and the damping ratio, ξ_j , for each mode

$$\xi_j = \frac{1}{2\omega_j} \frac{\phi_j^T C_{ssi} \phi_j}{\phi_j^T M_{ssi} \phi_j}$$

The response of each mode of the system could then be solved for in the time domain and, using the modal participation factors and the eigen vectors, the response of the system determined.

This system was strongly influenced by the first mode. This mode was one in which all three contributing degrees of freedom were in phase with one another, denoted by the eigen vector components being either all positive or all negative. This dominance of the first mode meant that the influence of the foundation on the response of the system could be illustrated by comparison of the frequency and damping ratio of the structure and of the first mode of the structure and foundation system.

3. FLAC MODEL

The model of the soil and structure in FLAC was composed of a layer of soil overlaying a rigid rock base. For the linear analyses the soil was modelled as linear elastic with Rayleigh damping, centred around the natural frequency of the soil profile. The structure was modelled using beam elements to form a foundation and a structure which would behave as a single degree of freedom (SDOF) oscillator. The structure was comprised of two columns with a connecting beam at the top. The beams and columns were of equal length, and the Young's modulus of the columns was set at one tenth that of the top beam and the foundation, as was the density. This ensured that almost all of the flexure in the structure was in the columns with little rotation at the ends. Making that assumption the equivalent stiffness of this 'column sway' structure could be easily worked out and used in the simple model. Damping was included in the structure again using Rayleigh damping centred at the first mode of the structure.

The spacing of zones (the FLAC equivalent of elements) was set to meet two criteria. The first was to accurately pick up the high frequency components of the earthquake time history. This set the maximum spacing possible for the grid. The other was to model the soil accurately in the region of the foundation, where the gradients of stresses would be expected to be highest and to allow comparison between FLAC results with the formulae used in the 3DOF model for the foundation stiffness and damping values. This region of close spaced zones was set to a depth equal to the foundation width and extending half the foundation width either side. The remaining zones were then progressively scaled, minimising any rapid change in zone size. Care was also taken to keep the aspect ratio of zones within recommended range (less than ten). The model is shown in figure 2.

The analysis was composed of two section; static and dynamic. For the first part the soil zones and structure were allowed to find static equilibrium due to gravity. This was sped up considerably by applying initial internal stresses and boundary stresses to the zones based on the soils self weight, requiring that only the effect of the weight of the structure needed to be solved for.

In the second part of the analysis an earthquake time history was applied at the base. Acceleration time histories were recorded at the base and top of the soil profile and at the base and top of the structure. The acceleration time history at the base of the structure was used as the input to the 3DOF model. From these acceleration time histories the behaviour of the structure was investigated in both the time and frequency domain. By performing a fast Fourier transform (FFT) on the time history the frequency composition of the record could be examined, and transfer functions calculated for the structure. A FFT decomposes the time history into an equivalent series of sinusoidal waves with a magnitude and a phase angle (often as a complex number) over a range of frequencies. A transfer function between two points A and B, (the top and bottom of the structure for example) is the magnitude FFT at B divided by the FFT at A. This produces a curve which is independent of the original time history, and a means of interpretation and comparison of the results. Results in this form were used to compare between the 3DOF and FLAC models.

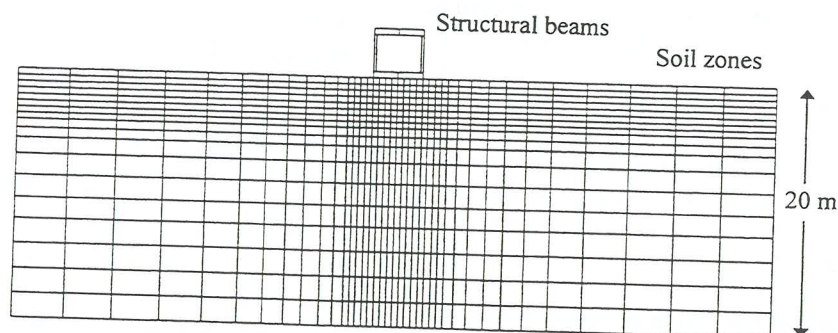


Figure 2. FLAC model of soil and structure

4. COMPARISON OF MODELS

4.1 Introduction

The FLAC model of soil and structure had the following properties:

Soil properties:

- Density $\rho = 2000 \text{ kg m}^{-3}$
- Shear modulus $G_{\text{max}} = 10, 20, 40, 60 \text{ MPa}$
- Poisson's ratio $\nu = 0.40$

Structure properties:

- Frequency $f_s = 3.883 \text{ Hz}$
 $\omega_s = 24.398 \text{ rads}^{-1}$
- Width $2b = 4 \text{ m}$
- Mass $m_s = 12600 \text{ kg}$
- Mass of base $m_b = 12600 \text{ kg}$
- Rotational mass $I_s = 203.6e3 \text{ kg m}^2$
- Foundation contact pressure $q = 61.8 \text{ kPa}$

Acceleration time histories were recorded at soil nodes at the bottom of the soil layer and at the top of the soil layer beneath the foundation. The response of the structure was recorded at the centre of the foundation beams and at the centre of the top beam. From these time histories the response of each position was put into the frequency domain using a FFT, and the transfer functions for the site and the structure calculated.

For each linear analysis the soil shear modulus was different. This had two key effects; firstly it changed the response characteristics of the site, changing the magnitude and frequency composition of the excitation into the base of the structure. Secondly the shear modulus changed the dynamic properties of the foundation, which in turn changed the response of the structure. The shear moduli chosen ranged from having a considerable effect upon the behaviour of the structure to only a small influence upon its behaviour. The transfer function for the structure and the site are presented in figures 5, a to d.

For each FLAC analysis a simple 3DOF analysis was performed using the input motion into the structure at the base and the appropriate shear modulus for the determination of the foundation parameters. From this analysis the acceleration time history at the top of the structure was solved for and, using the method outlined earlier, the transfer function for the structure found. This was then compared to that for the FLAC model, also shown in figure 5, a to d.

4.1 Effect of SSI on response of structure

The relative influence of SSI on the response of the structure can be summarised by Table 1. These results were determined from the 3DOF, using modal superposition to solve for the first modes of vibration and the damping ratios of the structure and foundation system. This table shows the decrease to the first mode of the system as the soil softens and the foundation motion becomes more dominant in the

vibration of the structure. There was also the expected increase in damping for this first mode.

Table 1. Frequencies and damping ratios of first mode of structure and foundation system for 3DOF model

Shear modulus	First mode frequency, f_1	First mode damping, ξ_1
10 MPa	2.390 Hz	0.0649
20 MPa	2.888 Hz	0.0437
40 MPa	3.282 Hz	0.0298
60 MPa	3.454 Hz	0.0253
Structure	3.883 Hz	0.0211

4.2 Influence of Structure on Site Response

A comparison of the transfer functions of a site with the structure present and of a site with free field conditions is shown in figures 3 and 4 for two shear moduli, $G = 10 \text{ MPa}$ and 20 MPa . The general trend from these comparisons was of a general lessening in the amplification of the site with the structure present, shown by the reduction in the peaks of the transfer functions where the structure is present. The other main point of interest was the distortion of the transfer curve in the frequency range corresponding to the first mode of the structure and foundation system. This distortion was particularly evident for the $G = 10 \text{ MPa}$ transfer function. This is discussed further in the following section.

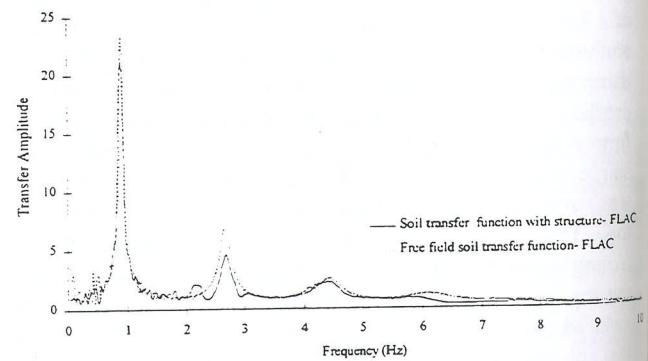


Figure 3. Site response transfer function for free field and with structure, $G = 10 \text{ MPa}$.

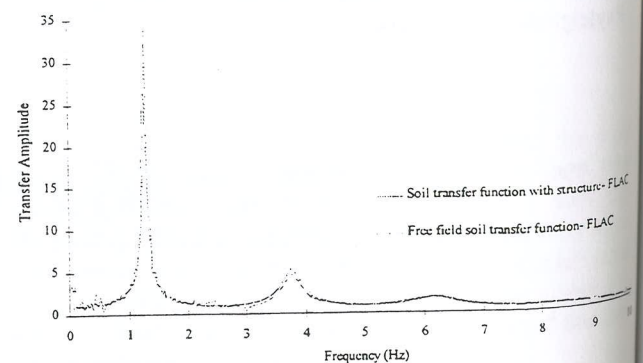


Figure 4. Site response transfer function for free field and with structure, $G = 20 \text{ MPa}$.

Both of these effects had an influence on the acceleration time history at the top of the soil layer. This is presented in Table 2. The effect of the structure was most significant for the softest site, with a 24 % drop in the peak ground acceleration at the surface. For the $G = 20$ MPa site the drop in peak ground acceleration was 7 %. For the stiffer sites the change was less significant. This reduction in response had significance for the use of the 3DOF model. As an alternative to the recorded acceleration time history at the base of the structure, the free field motion could be used, and would usually be conservative.

Table 2. Influence of structure on peak ground acceleration of site.

Peak ground acceleration	$G = 10$ MPa	$G = 20$ MPa
Base acceleration	2.174 ms^{-1}	2.174 ms^{-1}
Top acceleration: Free field	4.449 ms^{-1}	3.075 ms^{-1}
Top acceleration: Structure	3.400 ms^{-1}	2.854 ms^{-1}

4.3 Influence of Site on Structural Response

From a comparison of the transfer functions for the structure and the FLAC model a general fit between the two in terms of the frequency and amplitude of the peak response of the structure. The shape of the transfer function for the FLAC structure was not a regular as that predicted by the 3DOF model however. The FLAC transfer function tended to dip more rapidly than the 3DOF model and was generally over predicted by the 3DOF model over a range of frequencies around the peak. This was most evident for the softest site ($G_{\text{max}} = 10$ MPa). As the shear modulus of the site increased the difference between the FLAC and the 3DOF models was also very evident in the comparison of the acceleration time histories of the top of the structure. The 3DOF model predicted significantly higher accelerations than were recorded in the FLAC model.

The cause of the difference between the FLAC and the 3DOF models can be strongly linked to frequency dependent factors not incorporated into the 3DOF model. When the site transfer function at each site was looked at for each analysis there was significant distortion of the transfer function in the same region as the FLAC structure transfer function deviating from what was predicted by the 3DOF model. This again was most evident for the softest site. The first mode of the structure and foundation system (2.39 Hz) corresponds roughly to the second mode of the site (2.65 Hz), and the expected peak in the site transfer function was considerable reduced by the influence of the structure. For the stronger sites the influence reduces to only a small deviation in the site transfer function for the stiffest site investigated. Also for the stiffer sites the frequencies of major structural amplification did not coincide with those of the site.

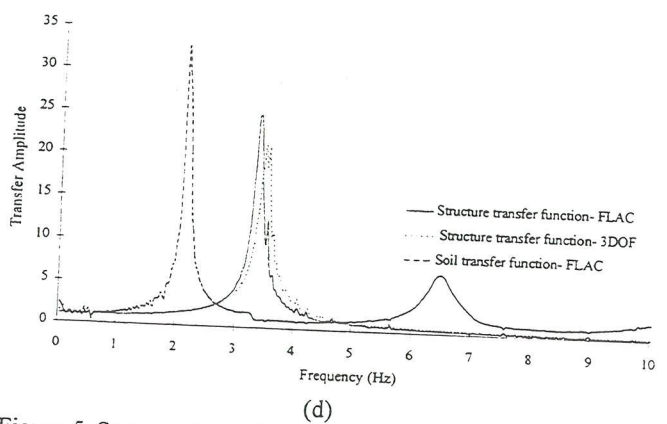
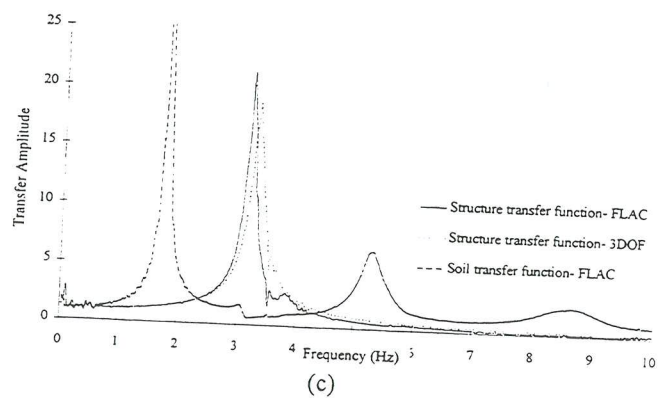
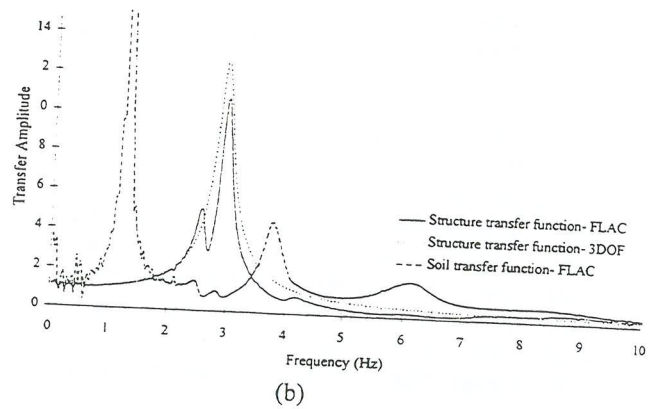
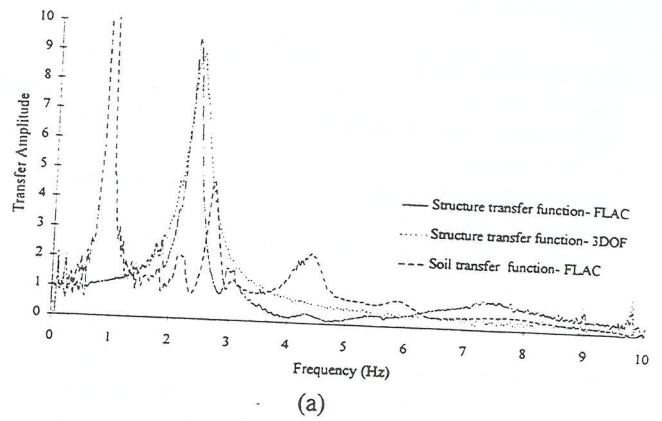


Figure 5. Structural transfer functions for FLAC and 3DOF models; (a) 10 MPa, (b) 20 MPa, (c) 40 MPa and (d) 60 MPa

5. CONCLUSIONS

Soil structure interaction had a significant effect on the response characteristics of the structure, generally increasing the flexibility and the damping of the first mode of the system by the inclusion of additional modes of vibration. The relative influence of SSI on the response increases for stiffer structures, and for structures with little damping, where the foundation may provide the dominant stiffness and damping to the system.

The structure has an influence on the response of the site, and hence the input acceleration it experiences. For the sites investigated the structure reduced the acceleration at the top of the soil profile. This was evident by a reduction in the peak ground acceleration and a general reduction in the transfer function for the site when the structure was present, relative to the free field response.

The response of the structure was reasonably well predicted by the 3DOF model, particularly in the peak amplification of the transfer function at the first mode and the damping at this peak. However, there were significant variations between the two models in the ranges of frequency around the first mode. These were due to the frequency dependence of the foundation stiffness and damping parameters, and due to seismic waves rebounding to the surface from the rock base. Both of these effects were evident by the distortion of the

transfer functions for the site and structure for the FLAC model. These frequency dependant effects were most significant for the softest site ($G = 10$ MPa), and decreased for stiffer sites.

6. ACKNOWLEDGMENTS

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