

Joint Inversion of Phase Velocity and Apparent Resistivity using Genetic Algorithms: A Synthetic Model

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Abstract: Independent inversions of apparent resistivity or phase velocity have assisted in obtaining global subsurface physical parameters such as resistivity and S-wave velocity. Restrictions on the attainability of actual observations, makes the inversion of observation data to a true model difficult. A joint inversion of Rayleigh wave phase velocities and apparent resistivity is proposed using genetic algorithms, in order to obtain appropriate model parameters from insufficient data in shallow geophysical exploration. The inversion of synthetic data supported the effectiveness of the method.

INTRODUCTION

Apparent resistivity and Rayleigh wave phase velocity tests, used in the analysis of soil properties, have many applications in the evaluation of natural resources, environmental and civil engineering problems.

For instance, Resistivity tests can detect water bodies and ground fractures through the current's identification of air as a resistor and water as a conductor. Known as a Vertical Electrical Sounding (VES) test, current is passed between two stationed poles and the dispersion recorded at various distances from the central current circuit. In doing so a direct relation between depth, spacing and apparent resistivity at the respective distances can be observed.

Rayleigh wave phase velocities are highly effective in the assessment of soil stiffness. As a

phase velocity wave continues through its medium, with consistent shape of peak, trough, or zero-point these test readings give valuable information regarding the layer stiffness and thus the dispersion of the Rayleigh waves (Haskell, 1953) at varying subsurface depths. Wave amplitude also diminishes exponentially with depth, measurable by the period and amplitude of the observed surface wave.

Forward modeling is the standard mathematical process of interpreting a model into graphical data. That is to say, taking a three-layer soil model with given layer thickness, resistivity, and phase velocity values, and interpreting the phase velocity over period, or resistivity over tested spacings and therefore relative to depth. Where it is difficult to know first these model parameters of thickness, resistivity, and phase

velocity, data from the abovementioned tests can be taken to then generate the model. This reversed mathematical process is known as an inversion.

The aim of inverting apparent resistivity and phase velocity data separately, (an independent inversion) obtained from field observations which generates subsurface parameters, is to minimise misfit between the observed data and theoretical values calculated in forward modeling. However, as various obstacles are often encountered in the field, the data observed on site may at times be incomplete, parameters obtained may consequently be far from those of the actual soil profile, this profile of which the inversion is seeking to attain to is herein referred to as the 'true' synthetic model.

In belief that the joint inversion of the two elements would compliment one another in the case of incomplete data, the assessment of joint inversion effectiveness was undertaken, focusing specifically on the applicability of a joint inversion to insufficient data. In this study, having proposed the joint inversion procedure, data of a synthesised model will be used to then investigate its applicability in shallow geophysical explorations.

METHOD

Rayleigh wave propagation is assumed in a layered subsurface model, with the phase velocity represented as a function of the S-wave (secondary, rotational wave) and P-wave (primary, dilatational wave) velocity, thickness, and density of each layer (Ohta and Goto, 1976). Once frequency-dependent phase velocities are obtained, through the optimisation of influential S-wave velocity and thickness, upper and lower search parameters for the two are determined in order to

minimise misfit, Φ_p , as defined by

$$\mathbf{f}_p = \frac{1}{N_p} \sum_{i=1}^{N_p} |c^c(f_i) - c^o(f_i)| \quad (1)$$

where N_p is the number of observed phase velocity data, $c^c(f_i)$ and $c^o(f_i)$ are the calculated and observed phase velocities at frequency f_i .

Apparent resistivity, observed on site at various electrode spacings, can be assimilated to the one-dimensional model assumed in the resistivity inversion. Resistivity and thickness of each layer are determined inversely, likewise from within assigned search parameters, for which the misfit used, Φ_r , is

$$\mathbf{f}_r = \frac{1}{N_r} \sum_{i=1}^{N_r} |r_a^c(a_i) - r_a^o(a_i)| \quad (2)$$

where N_r is the number of observed resistivity data, $r_a^c(f_i)$ and $r_a^o(f_i)$ are the calculated and observed resistivities at electrode spacing a_i .

As resistivity units, measured in ohms, are directly unrelated to that of phase velocity units of ms^{-1} , the use of a dimensionless misfit is necessary. In this study, the following dimensionless misfit in joint inversion, Φ_j , was used.

$$\mathbf{f}_j = p\mathbf{f}_r + (1-p)\mathbf{f}_p \quad (3)$$

Throughout the study, the weighting factor p was taken to be 0.5, highly applicable to the fundamental three-layered theoretical model. Particularly useful in the case of insufficient site data, the p factor weighs the contribution of each test's results to the other in the joint inversion's profile modeling process, where stronger test results would call for a higher value of p .

In conducting a joint inversion, Genetic Algorithms (GA) were used. These proved to be a successful tool in independent inversions

(Yamanaka, H and Ishida, H; 1995) and were seen as applicable to an apparent resistivity and phase velocity joint inversion.

Genetic Algorithm variables included the probability of cross-over and mutation of individuals in each generation which were set at 0.7 and 0.3 respectively, as is generally characteristic of genetics. Mutation discourages premature convergence towards the optimal models within the misfit function, whilst the probability of crossover can be assimilated to genetic inheritance. In this sense the cross-over function contributes coded aspects of the parenting individuals to individuals of the successive generation. The additional element of elite selection ensures the generation's strongest individual is successfully carried into the following generation, assisting in attaining the potential, optimal model resultant.

For each inversion, independent and joint alike, models with a 1%, 3%, 5%, 10% misfit variation between the models identified as closely attaining to the true synthetic model, referred to as 'good-models', were evaluated. This was in search for the optimum misfit percentage with maximum number of good-models along with a minimum standard deviation. As the results found minimal variation with respect to the misfit percentage, 5% was selected for all inversion bringing a common percentile to the misfit analysis.

NUMERICAL EXPERIMENT

A simple three-layer subsurface structure was adopted for the initial model (model-bb1). The simple model was chosen to keep the parameters to a minimum in order to allow the results to show the appropriate fundamental conclusions. The

eight given parameters for model-bb1 include a phase velocity, and resistivity parameter for each of the three layers along with thickness parameters for layers N1 and N2, where the half layer N3 is of infinite thickness. Depicted below in Figure 1, the given values are shown below in Table 1.

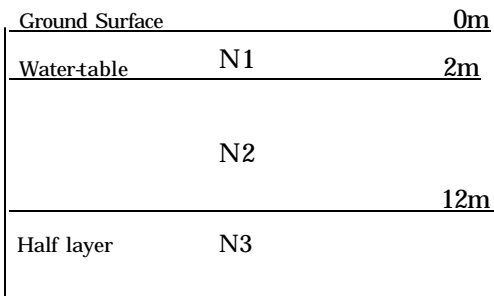


Figure 1

Three-layer, subsurface synthetic model.

Table 1 Given parameters of true model

Layer No.	Phase Velocity Vs (m/s)	Resistivity r (ohms)	Thickness h (m)
N1	200	100	2
N2	200	1	10
N3	600	10	*

*Half layer of infinite depth, shared interface with N2.

A Forward modeled computation of resistivity and phase velocity using the given parameters for model-bb1, produced a set of synthetic observed data which was then applied to all inversions.

Figures 2 and 3 show the forward modeled true functions of resistivity and phase velocity respectively, along with the complete and limited observed data used in the inversions. Inversions involving the complete set of twenty observations implied a full inversion, where as, of those twenty observations six were selected representing insufficient data and therefore a limited inversion, as typically obtained on site.

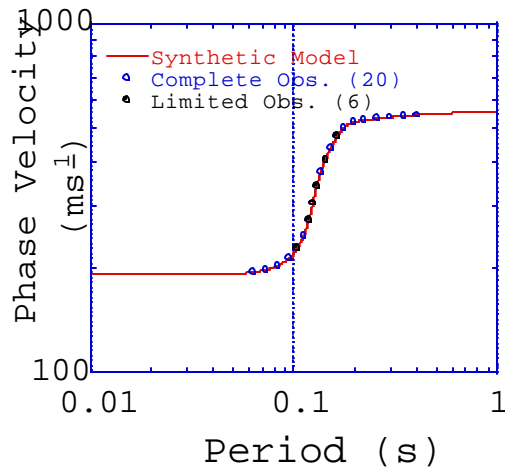


Figure 2

Points of observations taken from the synthetic phase velocity data used in complete and limited inversions.

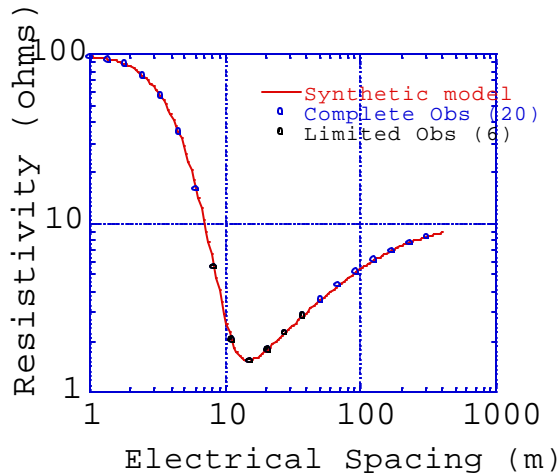


Figure 3

Points of observations taken from the synthetic resistivity data used in complete and limited inversions.

In a twenty observation full inversion of both apparent resistivity and phase velocity, a greater accuracy was achieved by the independent inversions, when compared with the jointly inverted models. Although the variance proved limited, it is recommended that independent inversions be used for profiling with complete

observations. Results obtained herein, are in reference to inversions carried out with the limited six observations.

As with an increasing subsurface depth, data for apparent resistivity becomes more difficult to obtain. Therefore, the limited observations chosen reflect those obtainable in a real situation. Similarly for the phase velocity inversion, the six observations chosen are within the limited period domain of 0.11s and 0.17s, assumed in the numerical experiment in making synthetic data.

Figure 4 shows the comparison between the synthetic, observed data and calculated apparent resistivity for models from independent and joint inversions.

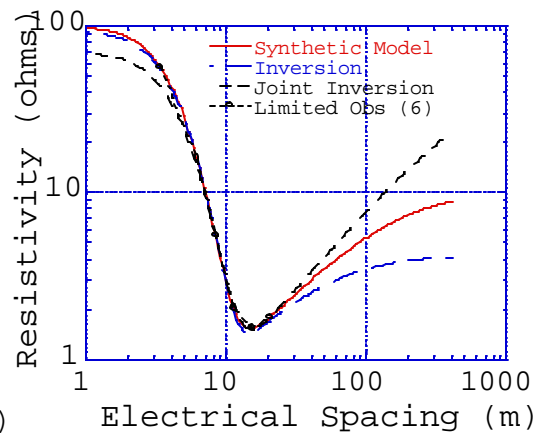


Figure 4

Comparison of synthetic resistivity with theoretical values for inverted models.

Table 2 Parameter Comparison: Resistivity

Layer	True Model		Independent		Joint	
	ρ	h	ρ	h	ρ	h
1	100	2	93.5	2.06	69.709	2.16
2	1	10	0.455	3.51	0.937	10.54
3	10	**	4.154	**	56.969	**

Table 2 shows that the parameters obtained in

the joint inversion for both layer depths and resistivity of N2 has achieved higher accuracy with respect to the synthetic model, in comparison to those obtained by independent inversion, particularly in the model's mid-depth layers.

Figure 5 below shows the comparison between the synthetic observed and calculated phase velocities of independently and jointly inverted models. Limited to six observations within the period range of 0.11s to 0.17s, the upper and lower bounds are exposed to significantly larger discrepancy outside this period range.

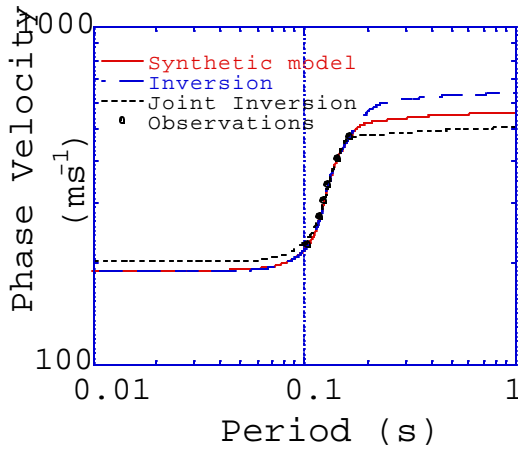


Figure 5

Comparison of synthetic phase velocity with theoretical values for inverted models.

Table 3 Parameter Comparison: Phase

Velocity	True		Independent		Joint	
	N	Model	Vs	h	Vs	h
1	200	2	198.24	9.60	211.86	2.16
2	200	10	231.09	3.26	212.14	10.54
3	600	**	685.44	**	542.23	**

Table 3 shows S-wave velocity and thickness from inversion of the phase velocities for both individual and joint inversions.

The effectiveness of the joint inversion in this instance of limited data can be seen as the joint inversion achieves a greater accuracy in the phase velocity profiling, with respect to the true model, especially for phase velocities of higher periods.

A significant contributing factor to the success of the joint inversions is relative to the reduction of unknown parameters. Specifically, the layer thickness parameters h1 and h2 are represented twice with each independent inversion, but only once in a joint inversion:

Independent Inversion Unknown Parameters:

$$(\rho_1, \rho_2, \rho_3, h_1, h_2) + (Vs_1, Vs_2, Vs_3, h_1, h_2) = 10$$

Joint Inversion Unknown Parameters:

$$(\rho_1, \rho_2, \rho_3, Vs_1, Vs_2, Vs_3, h_1, h_2) = 8$$

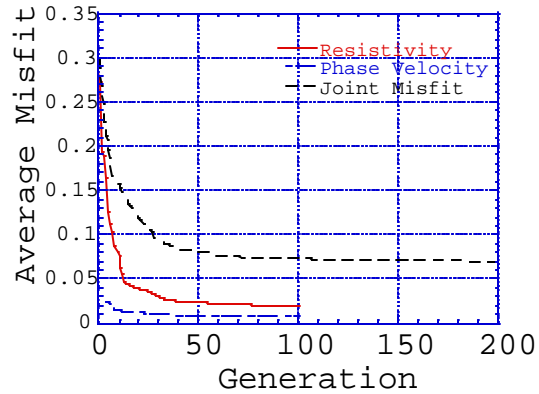


Figure 6

Comparison of misfit variations against generations in joint and independent inversions, for a misfit of 5%.

Shown in Figure 6 the final average misfit of the independent apparent resistivity and phase velocity inversions are lower than that of jointly inverted models. However, it is considered that the joint inversion results are within acceptable margins as the misfit stabilises, indicating the attainment of an optimum model of minimum

misfit for the 95th percentile.

CONCLUSIONS

Parameters obtained from the independent inversions aligned more effectively than those obtained through the joint inversion of apparent resistivity and phase velocity, for a profile modeled on a full set of observations.

Conversely, as anticipated the parameters generated by the joint inversion of a limited resistivity and phase velocity data set, attain more accurately to the true model.

Parameters of the joint inversion align more effectively overall, in comparison to those of the independent inversion, particularly in obtaining layer thickness parameters, resistivities of mid-depths and phase velocities of higher periods. Showing a greater effectiveness in attaining these parameters, the results confirm the credibility and applicability of joint resistivity and phase velocity inversions as a tool in shallow geophysical exploration.

As was to be expected in the case of decreased number of unknown parameters in the joint inversion, the final misfits of independent apparent resistivity and phase velocity inversion misfit functions are lower. However, as the joint inversion's 5% misfit function shows a steady misfit average around the 50th generation, it suggests the 95th percentile optimum model of minimum misfit is achieved.

Restricted to one jointly inverted model the depth of outcomes are limited, however, having chosen limited observations within small domains for the joint inversion, the average misfit and inversion results show an optimal model is being derived through the joint inversion of resistivity and phase velocity.

FURTHER RESEARCH

Investigation into a joint inversion of resistivity and phase velocity using observations obtained on site would be highly complimentary to the analysis carried out on this synthetic model. Allowing a comparative analysis of a theoretical model versus the practical site application, the success of a joint inversion on limited data would be challenged further, uncovering any obstacles imposed by the true limits of on-site observations.

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