

Effect of Design Models and Test Numbers on the Design of Pad Foundations

J S Goldsworthy and M B Jaksa – The University of Adelaide, Australia

Abstract: It is widely known that there are significant uncertainties in the design of foundation systems. Specifically these uncertainties are categorised into model errors, uncertainties due to the spatial variation of soils and test and procedural effects. Geotechnical engineering professionals have aimed to minimise the magnitude of these uncertainties by adopting refined testing techniques, numerical design procedures and increased site investigation schemes. However the magnitude of the uncertainty still remains unknown. This paper introduces a method to quantify the effectiveness of undertaking site investigations of increasing scope as well as the use of varying testing techniques and foundation design models. The process involves simulating soil profiles using random field theory with nominated statistical parameters. This allows the soil properties to be known in detail at all locations – a situation that is not possible with real sites. Several design methods are employed using either complete knowledge of the soil profile or data derived from simulated site investigations. Designs are compared on a failure and over-design basis, where conclusions regarding the sensitivity of the design model and scope of site investigation are drawn. A Monte Carlo analysis is undertaken where numerous soil profiles are generated to develop probabilities of failure and over-design.

INTRODUCTION

A typical foundation design involves undertaking a site investigation of nominated scope and using the results from such an investigation to define mechanical soil properties. These properties are used in any one of a number of different design models, developed using differing theories, to estimate the settlement or bearing capacity of the soil and foundation. However, inherent uncertainties exist in this design procedure. Firstly, soils are inherently variable and consequently, site investigations do not always accurately characterise the actual soil conditions. This can severely affect the results of the estimated soil property and the resulting foundation design. There are also inherent uncertainties in the models used to estimate the response of a foundation system. This is a result of assumptions and simplifications in the theory used to develop the model and the model itself.

The framework of using random field theory [1] and a three dimensional finite element analysis to investigate the reliability of varying site investigation scopes has been introduced by Jaksa et al. [2]. The method involves simulating soil profiles based on random field theory, thus enabling all soil properties to be known at all locations in detail. A “traditional” design is undertaken using the results of a simulated site investigation. This is compared to an “optimal” design determined using the complete knowledge of the soil profile and a finite element analysis. Preliminary results have been published by Goldsworthy et al. [3] for a single pad footing. The research presented in this paper extends these results to investigate the reliability of various traditional design models using the results obtained from various site investigation schemes. This allows conclusions to be drawn about the effectiveness of the current available settlement models with respect to the magnitude of information gained from a site investigation scheme.

UNCERTAINTIES IN A FOUNDATION DESIGN

The typical stages involved in a foundation design procedure as identified by Bowles [4] & Holtz [5] are:

1. Establish the scope of the problem including the location of the site and loads;
2. Investigate the conditions at the site;
3. Design and undertake a subsurface exploration programme including in-situ and/or laboratory tests;
4. Determine relevant design soil parameters from the exploration programme in Step 3; and
5. Undertake the design of the foundation.

There are complex uncertainties in all of the above stages of a foundation design. Phoon & Kulhawy [6] identify the uncertainties inherent in the process of obtaining a soil design parameter as being due to:

1. Inherent soil variability;
2. Measurement errors; and
3. Transformation errors.

The uncertainties indicated by Phoon & Kulhawy [6], as shown in Figure 1, in addition to the uncertainty of the design model itself (Step 5), contribute to the overall uncertainty of the foundation design process.

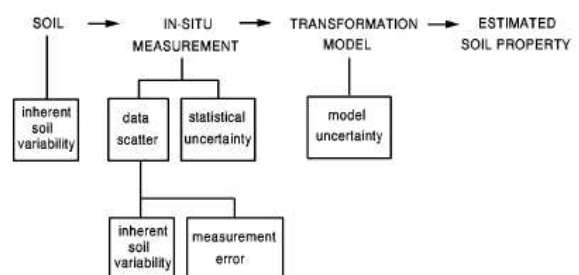


Figure 1. Uncertainty in soil property estimates [7].

Statistical methods have been used to quantify the uncertainty due to the inherent spatial variability of soils. The more common recent statistical methods include random field theory [1] and geostatistics [8], both of which involve modelling the spatial correlation of soil properties [9; 10]. In reality the spatial characteristics of a soil are rarely known for a foundation design and therefore, site exploration strategies are implemented to estimate them. The scope of these exploration strategies also affect the uncertainty of the process. Parsons & Frost [11], Azzouz & Bacconnet [12] and Subrahmanyam & Ng [13] have attempted to quantify the uncertainty due to limited site exploration strategies through the use of statistics and probability.

The contribution of measurement error to the overall uncertainty of a design process has been investigated by many authors [6; 14-18] and generally includes the effects due to sampling errors and testing errors. The transformation model uncertainties are sometimes difficult to distinguish from measurement errors, however Phoon & Kulhawy [19] and Ronold & Bjerager [20] have attempted to quantify these effects. The design model implemented also attributes to the overall uncertainty of the design process. This appears to be the least researched form of uncertainty and is one of the focuses of the research presented in this paper.

FOUNDATION DESIGN SCENARIO

A foundation system founded on six different soils based on the statistical parameters shown in Table 1 has been investigated. Analyses are restricted to serviceability criteria with a constant Poisson's ratio, requiring only the Young's Modulus, E , to be simulated by a random field. A foundation system of nine pad footings, evenly spaced and subject to column loads is modelled as shown in Figure 2.

Table 1. Statistical properties of soil profiles investigated

No	Young's Modulus (E)					ν
	Mean MPa	COV (%)	Scale of Fluctuation, θ			
			X	Y	Z	
20R	30	20	1	1	1	0.3
20M	30	20	4	4	2	0.3
20C	30	20	8	8	4	0.3
50R	30	50	1	1	1	0.3
50M	30	50	4	4	2	0.3
50C	30	50	8	8	4	0.3

The naming convention in Table 1 is based on the overall variability of the soil deposit (coefficient of variation of 20% or 50%) and the spatial variation of the field (random, medium or continuous). It is important to note the scale of fluctuation, which is a measure of the

distance within which soil properties are considered reasonably correlated [21].

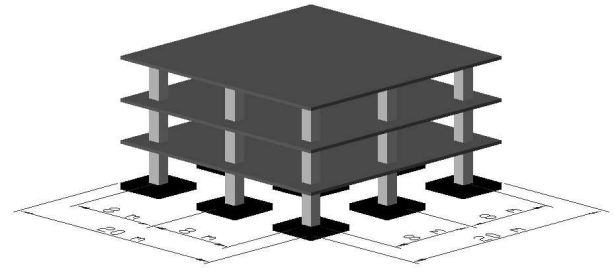


Figure 2. Layout of foundation system & design loads

The column loads applied to the nine rigid pad footings are indicative of a 20 metre \times 20 metre, 3-storey building with a 3 kPa live load and a 5 kPa dead load.

FOUNDATION DESIGN TECHNIQUES

There are numerous foundation design models that are used to estimate the settlement of a foundation for design purposes. Many of these models relate the results of in-situ tests directly to the settlement of a footing. However, these models are notoriously uncertain [4] and models that are based on soil mechanics are generally preferred. The model proposed by Timoshenko and Goodier [22], is based on the theory of elasticity (Eq. 2). This model uses an averaged Young's Modulus, E_s and a uniform Poisson's Ratio, ν , with three influence factors I_1 , I_2 and I_f .

$$\rho = \frac{qdI(1-\nu^2)}{E_s} \quad (2)$$

Schmertmann's [23] settlement relationship (Eq. 3) is recommended for use with granular soils, however Bowles [4] suggests it is suitable for most soils. The relationship uses two correction factors, C_1 and C_2 , which account for embedment and time effects, as well as an Influence factor, I_z , which idealises the strain profile into a triangle. Unlike the Timoshenko & Goodier method, Schmertmann's method integrates with depth, allowing effects from layering and varying Young's Modulus values.

$$\Delta H = C_1 C_2 \Delta p \sum_{i=1}^N \frac{\Delta z_i I_{z_i}}{E_{s_i}} \quad (3)$$

Numerous settlement models are based on linear elasticity (Eq. 4) theory. The models differ when determining the distribution of stresses throughout the soil mass. The "2:1 Method" was proposed by the US Army Corp of Engineers and involves scaling the magnitude of stresses in the soil at a ratio of 2:1. The resulting stress, q_v , at depth, z , is determined by Equation 5, where B is the width of the footing, L , is the length and Q is the applied load.

$$\Delta H = \sum_{H_i}^{H_{i+1}} \frac{\Delta q_i}{E_{si}} \quad (4)$$

$$q_v = \frac{Q}{(B+z)(L+z)} \quad (5)$$

The Bousinessq method develops a stress profile in the soil mass and Equation 5 is used to find the stress increment, q_v at depth z , for an applied load q_0 . The values, M , N , V and V_1 are all functions of the footing geometry, B & L and depth, z [4]. Alternatively, the Westergaard method [24] has the advantage of including Poisson's Ratio, accounting for varying lateral and vertical strains. The Westergaard equation is shown in Equation 6 and includes the a term given by Equation 7, incorporating Poisson's Ratio.

$$q_v = q_0 \frac{1}{4\pi} \left[\frac{2MN\sqrt{V}}{V+V_1} \frac{V+1}{V} + \tan^{-1} \left(\frac{2MN\sqrt{V}}{V-V_1} \right) \right] \quad (5)$$

$$q_v = \frac{q_0}{2\pi} \tan^{-1} \left(\frac{MN}{\sqrt{a}(M^2 + N^2 + a)^{1/2}} \right) \quad (6)$$

$$a = \frac{(1-2\nu)}{(2-2\nu)} \quad (7)$$

To obtain design values for use within the aforementioned design models, a simulated site investigation is undertaken. This site investigation consists of up to 8 test locations arranged as shown in Figure 3. It involves sampling from the random field representing the soil profile in a discrete manner. This is representative of a Standard Penetration Test (SPT) where soil properties are typically obtained at 1.5 metre depth intervals. It is important to note, at this stage, that no test uncertainty has been included in the results. Consequently, the results do not measure the effectiveness of the test method, rather the number and location of the test and the use of discrete sampling.

The dashed line in Figure 3 represents the boundary of the site investigation, while the solid line represents the site boundary. The naming convention used represents the scheme number, while the letters "RG" refer to a regular grid arrangement. The last number refers to the number of tests in the scheme. This convention is consistent throughout this paper.

It was concluded from non-compliance and over-design probabilities published by Goldsworthy et al. [3] that positioning a single test directly beneath the proposed footing location significantly influences the results. The test at the centre of the proposed footing location provides the best representation of the underlying soil conditions. However, few site investigations allow boreholes to be located at the centre of all of the proposed footing locations. Therefore, the test patterns

shown in Figure 3 are considered a more typical representation of most site investigation plans.

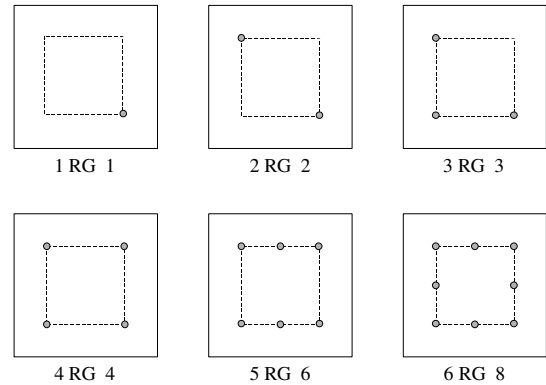


Figure 3. Site Investigation Schemes

Each design is compared with an optimal design achieved by utilising the completed knowledge of the simulated soil profile and a finite element analysis. The finite element analyses are three-dimensional and incorporate the stochastic variability of the soil properties.

EFFECTIVENESS OF SITE INVESTIGATION SCHEMES

Figures 4 and 5 show the probability of non-compliance and overdesign of the foundation system, designed with Schmertmann's settlement model for highly variable (COV=50%) soil profiles. Results from the soil profile with high COV are used exclusively for the remainder of the paper, as the uncertainty in the characterisation of the soil profile is proportional to the uncertainty in the properties of the soil profile. For the purposes of the research presented in this paper, non-compliance is defined as the design from the simulated site investigation being smaller than the design using complete knowledge of the soil profile or optimal design. Conversely, overdesign occurs when the design using site investigation data is greater than the optimal design. Future studies will investigate the use of the finite element analysis to analyse all designs to provide a better definition of non-compliance and overdesign.

Figure 4 shows an increasing trend of non-compliance probability for larger scope site investigations, however, the trend in Figure 5, suggests that the probability of overdesign reduces significantly with the increase in site investigation scope. This seemingly unusual trend in the probability of non-compliance is clarified in Figure 6, which presents the mean design area resulting from the Schmertmann model using the site investigation data and an optimal design using the complete knowledge of the soil profile. Figure 6 shows the mean design area reducing for increasing knowledge of the soil profile

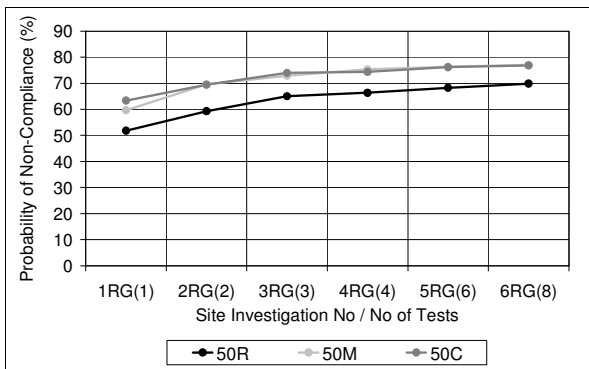


Figure 4. Probability of non-compliance for foundation system designed using *Schmertmann's* method

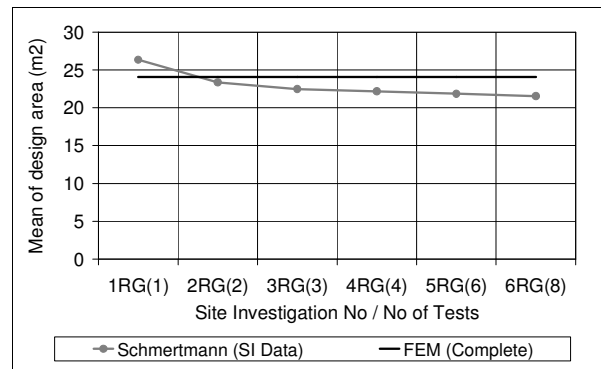


Figure 6. Mean of design area using *Schmertmann's* method and optimal design for soil profile 50C.

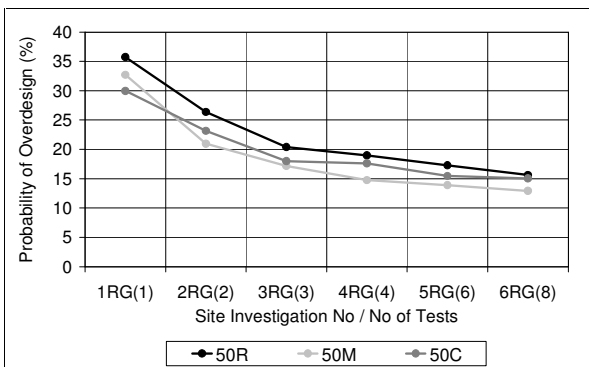


Figure 5. Probability of overdesign for foundation system designed using *Schmertmann's* method

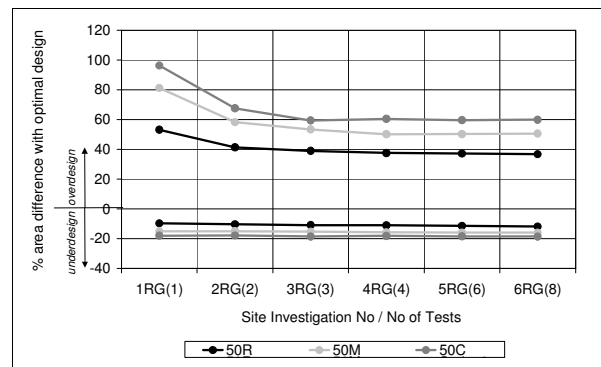


Figure 7. Design area comparison (underdesign & overdesign) with optimal for *Schmertmann's* method

(larger site investigation scope), however for site investigation 6RG(8), where there is the greatest knowledge of the soil profile, there is a 6 m² difference in the mean design area. This represents the error associated with the Schmertmann model for this load and soil profile scenario. Hence, the Schmertmann model provides a less conservative design method where the resulting foundation design is less than the optimal design, leading towards non-compliance, as defined in this paper.

Figure 7 compares the error in the design area when non-compliance (underdesign) and overdesign occurs for the Schmertmann model compared with the optimal design. Although Figure 6 indicates that the Schmertmann model is under-conservative and tends to underdesign foundations for this soil profile and design loading scenario, it appears from Figure 7, that when an underdesign occurs its magnitude is not large. Conversely, when overdesign occurs there appears to be a large difference in the design areas. Figure 7 also shows a strong trend indicating that an increased site investigation scope will approach the optimal design as the difference between the underdesign and overdesign lines narrow for increasing site investigation scope.

EFFECTIVENESS OF DESIGN MODELS

For each of the design methods, a reliability analysis was undertaken. This analysis involves investigating the differences in the resulting foundation design with the optimal design derived from complete knowledge of the soil profile. Figure 8 presents the mean design area of the foundation system for each design model and site investigation plan. The dashed line in Figure 8 indicates the mean design area resulting from the optimal design incorporating the complete knowledge of the soil profile. It is apparent from Figure 8, that all of the design models are less conservative showing smaller mean design areas than the optimal design for most of the site investigation schemes. This conforms with the conclusions drawn in the previous section. However, it is apparent from Figure 8, that the Schmertmann model and the 2:1 technique are more conservative than the other design models.

Figure 9 shows the uncertainty in footing design area for each model and site investigation plan. As knowledge about the soil profile increases (larger site investigation scope), the uncertainty in the design area is reduced. The trend for each of the design models appears to be the same, indicating this is a function of the soil profile and investigation strategy, rather than the model.

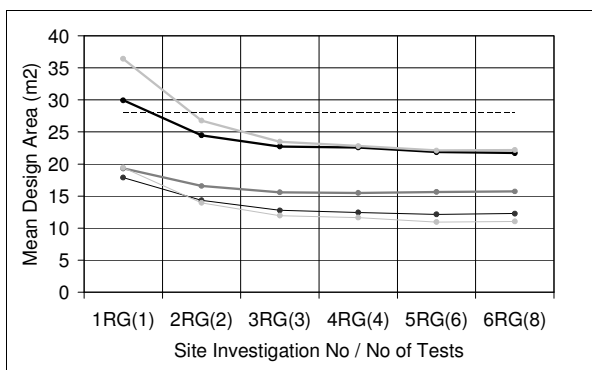


Figure 8. Mean of design areas for all foundation design models for soil profile 50C.

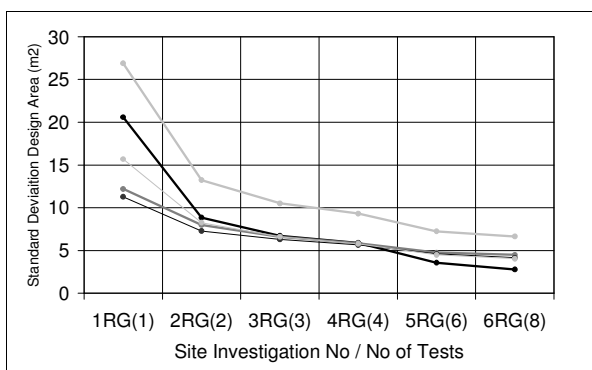


Figure 9. Standard deviation of design areas for all foundation design models for soil profile 50C.

Figure 10 provides the best comparison of design models, where the design area is compared with the optimal design area as a percentage error. The distance between the overdesign and underdesign line of each design method can be viewed as the uncertainty of the model, where a large distance represents large model uncertainty and a small distance indicates small model uncertainty. The model bias is measured by the distance the mean of the overdesign and undersign lines stray from zero. This is also a measure of the conservatism of each model.

Using these measures, it is apparent from Figure 10 that the Schmertmann and 2:1 models are very similar in both uncertainty and bias, while the other models (Bousinessq, Westergaard and Timoshenko) show similar behaviour. Figure 10 also further clarifies that the Schmertmann and 2:1 models are more conservative than the other models leading towards overdesigns of greater error than the other models. It is also concluded from Figure 10 that the Schmertmann model needs less information from a site investigation scheme to provide the best design. This is shown by the overdesign and underdesign lines reaching an asymptote faster than the other models.

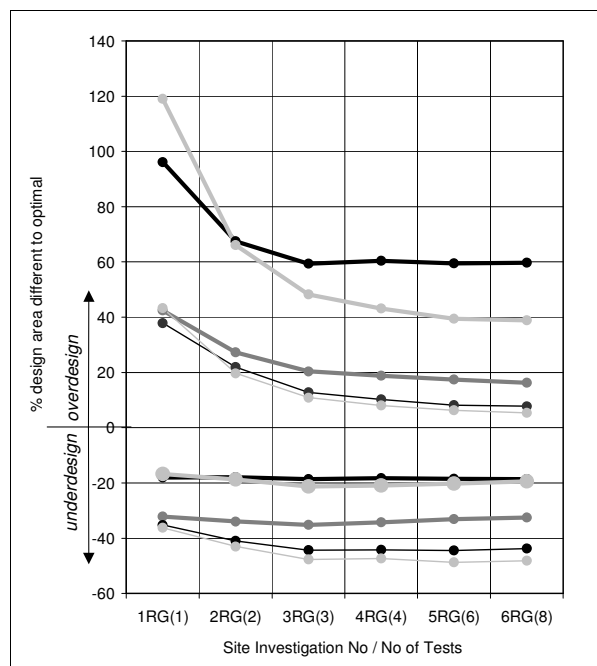
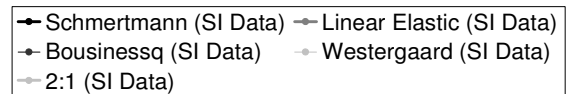


Figure 10. Design area comparisons (under-design and over-design) with optimal design for all design models for soil profile 50C.



CONCLUSIONS

The uncertainties in a foundation design due to a limited site investigation and the use of idealised and simplified design models have been quantified. It has been shown that an increased knowledge about the site from a larger scope site investigation will not only reduce the uncertainty associated with a foundation design, but will also reduce the possibility and magnitude of overdesign. The results also indicate that, although a greater knowledge of the soil profile reduces the probability of a foundation overdesign, it increases the probability of obtaining a design which does not meet all the design criteria (non-compliance). This is a result of the design approaching the optimal design and reducing the magnitude of overdesign. It has been shown that common design models have varying inherent conservatism, uncertainty and bias. It was also apparent that different design models approach their optimal design using different degrees of site knowledge. It is hoped that the results presented in this paper will assist geotechnical professionals to select site investigation plans and design models to suit the site conditions and design requirements.

ACKNOWLEDGMENTS

The contribution and support by Dr William Kaggwa, Professor Gordon Fenton, Professor Vaughan Griffiths and Professor Harry Poulos is greatly acknowledged. The research presented in this paper is supported by an Australian Research Council Discovery Project Grant, which is greatly appreciated.

REFERENCES

1. VANMARCKE, E.H., "Random Fields: Analysis and Synthesis", 1984, The MIT Press, Cambridge, Massachusetts, USA.
2. JAKSA, M.B., KAGGWA, W.S., FENTON, G.A., and POULOS, H.G., "A framework for quantifying the reliability of geotechnical investigations", 2003, 9th International Conference on the Application of Statistics and Probability in Civil Engineering, 1285-1291.
3. GOLDSWORTHY, J.S., JAKSA, M.B., FENTON, G.A., KAGGWA, W.S., GRIFFITHS, D.V., POULOS, H.G., and KUO, Y.L., "Influence of Site Investigations on the Design of Pad Footings", 2004, 9th Australia New Zealand Conference on Geomechanics, 282-288.
4. BOWLES, J.E., "Foundation Analysis and Design", 1997, McGraw-Hill, Singapore.
5. HOLTZ, R.D., "Stress Distribution and Settlement of Shallow Foundations", 1991, Foundation Engineering Handbook, Fang, H.Y., ed., Van Nostrand Reinhold, New York, USA, 923.
6. PHOON, K.K., and KULHAWY, F.H., "Characterization of geotechnical variability," 1999, Canadian Geotechnical Journal, 36(4), 612-624.
7. KULHAWY, F.H., and PHOON, K.K., "Observations on Geotechnical Reliability-Based Design Development in North America", 2002, International Workshop on Foundation Design Codes and Soil Investigations in view of International Harmonisation and Performance Based Designs, 31-48.
8. JOURNEL, A.G., and HUIJBREGTS, C.H., "Mining Geostatistics", 1978, Academic Press Inc, New York, USA.
9. FENTON, G.A., and VANMARCKE, E.H., "Simulation of Random Fields via Local Average Subdivision," 1990, Journal of Engineering Mechanics, 116(8), 1733-1749.
10. FENTON, G.A., "Simulation and Analysis of Random Fields", 1990, PhD, Princeton University, New Jersey, USA.
11. PARSONS, R.L., and FROST, J.D., "Evaluating Site Investigation Quality using GIS and Geostatistics," 2002, Journal of Geotechnical and Geoenvironmental Engineering, 128(6), 451-461.
12. AZZOUEZ, R., and BACCONNET, C., "Optimal approach to the planning of a network of in situ soil test", 1991, 6th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP6), 652-659.
13. SUBRAHMANYAM, M.S., and NG, I.T., "Probability-based design of soil exploration programme", 2000, 8th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP8), 1109-1114.
14. SPRY, M.J., KULHAWY, F.H., and GRIGORIU, M.D., "A Probability Based Geotechnical Site Characterisation Strategy for Transmission Line Structures", 1988, EL-5507, Cornell University, Ithaca, New York, USA.
15. ORCHANT, C.J., TRAUTMANN, C.H., and KULHAWY, F.H., "In-Situ Testing to Characterize Electric Transmission Line Routes", 1986, In Situ '86 - Use of In Situ Tests in Geotechnical Engineering (GSP 6), 869-886.
16. LEE, I.K., WHITE, W., and INGLES, O.G., "Geotechnical Engineering", 1983, Pittman Publishing Inc, Massachusetts, USA.
17. ORCHANT, C.J., KULHAWY, F.H., and TRAUTMANN, C.H., "Critical Evaluation of In-Situ Test Methods and their Variability", 1988, EL-5507, Vol. 2, Electric Power Research Institute, Palo Alto.
18. KAY, J.N., KULHAWY, F.H., and GRIGORIU, M.D., "Assessment of Uncertainties in Geotechnical Design", 1991, 6th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP6), 683-692.
19. PHOON, K.K., and KULHAWY, F.H., "Evaluation of geotechnical property variability," 1999, Canadian Geotechnical Journal, 36(4), 625-639.
20. RONOLD, K.O., and BJERAGER, P., "Model Uncertainty Representation in Geotechnical Reliability Analyses," 1992, Journal of Geotechnical Engineering, 118(2), 363-376.
21. VANMARCKE, E.H., "Probabilistic Modelling of Soil Profiles," 1977, Journal of the Geotechnical Engineering Division, 103(GT11), 1227-1246.
22. TIMOSHENKO, S., and GOODIER, J.N., "Theory of Elasticity", 1951, McGraw-Hill, New York, USA.
23. SCHMERTMANN, J.H., "Guidelines for Cone Penetration Test: Performance and Design", 1978, FHWA-TS-78-209, US Dept of Transportation, Federal Highway Administration, Washington, USA.
24. WESTERGAARD, H.M., "A Problem of Elasticity Suggested by a Problem in Soil Mechanics: Soft Material Reinforced by Numerous Strong Horizontal Sheets", 1938, Contributions to the Mechanics of Solids - Stephen Timoshenko 60th Anniversary Volume, Macmillan, New York, USA.