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RANDOM FIELD MODELLING FOR THE EFFECT OF CROSS–CORRELATION

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ABSTRACT: Probabilistic approaches are being used increasingly in the modelling of geotechnical problems, some of the assumptions made in most analyses to simplify the calculations in stochastic modelling have to be reviewed. One of those assumptions is that of zero correlation between soil strength parameters (cohesion and internal friction) in the analysis of soil slope stability. In the paper, it is shown that this assumption is far too conservative. A random field is used to model the covariance function of cohesion and internal friction. The first order second moment (FOSM) method and method of slices are then used to calculate the probability of failure of a slope by defining the performance function as the safety margin. Such a realistic soil model has not been adopted elsewhere and its use results in considerably increased reliabilities for slopes. The effect of this change is demonstrated by some examples.

1. INTRODUCTION

Probabilistic analysis of the stability of slopes is usually based on some assumptions which are chosen to simplify the calculations or to be conservative. Many researchers have undertaken probabilistic modelling assuming independence between the shear strength parameters cohesion, c, and angle of friction, ϕ (Li and Lumb, 1987; A–Grivas and Asaoka, 1982; Tobutt and Richards, 1979; Chowdhury, 1984; Felio et al., 1984; Oboni et al., 1984; Bergado and Anderson, 1983; Vanmarcke, 1980 and Alonso, 1976). This assumption is often made to simplify the analysis, many of these writers refer to the paper of Alonso (1976) in support of this assumption, others offer no support. The evidence offered by Alonso is not strong but does indicate a weak negative correlation for unsaturated residual soils. A–Grivas and Harrop–Williams (1979) state that cohesion and friction are negatively correlated (i.e. a high angle of friction is associated with a low cohesion) and provide one example. Some writers state that correlation can be ignored because the probability of failure for a negative correlation is lower than it is under the assumption of independence, thus this assumption is conservative (A–Grivas and Asaoka, 1982; Chowdhury, 1984; Bergado and Anderson, 1983; Li and White, 1987a; Li and Lumb, 1987 and Vanmarcke, 1980). This is supported by the cases analyzed by Cherubini et al. (1983) but as will be demonstrated the probability of failure obtained in this manner may be several orders of magnitude in error; this appears to be far more conservative than would be desired for such an elaborate method of analysis (i.e. probabilistic methods as compared with deterministic methods). This study explores the effect of cross–correlation in space of c and ϕ on the probability of failure of slopes adopting a random field model.

2. SPATIAL CROSS–CORRELATION AND AUTOCORRELATION

Since soil slopes are spatial structures, their measured properties or parameters are referred to as spatial data. Properties of soil slopes can be divided into two groups: geometric and mechanical. Geometric properties mainly include topography, boundaries between materials and orientation (dip and dip direction) of defects and failure surfaces. Mechanical properties are the strength (c and ϕ), density and pore pressure. These properties can all be treated as random variables due to their uncertainty in space and sometimes time. If data are not randomly or independently distributed in space, they are said to be correlated. Autocorrelation and spatial cross–correlation are the common terms describing respectively the correlation in space of a random variable with itself and that between a random variable and other random variables. The effect of each correlation is important and can be studied separately. Autocorrelation is what is commonly meant by spatial correlation and has been modelled as a random field (Li and White, 1987b). Its significant effect on the probability of failure of slopes is further demonstrated by Mostyn and Soo (1992) and the effect on rock joints is discussed by Yu and Mostyn (1993). Random fields can also be used to model spatial cross–correlation. It should be noted that most probabilistic slope analyses in the literature do not model the soil as a random field but model each parameter as a perfectly correlated single variable. Such analysis is built on fundamentally flawed models as is discussed at length in Li (1992) and Mostyn & Li (1993).

3. THE PERFORMANCE FUNCTION AND RELIABILITY INDEX

3.1 The performance function

To calculate the probability of failure of slopes, the common practice is to formulate a performance function, $G(\underline{X})$, in which $\underline{X}$ is a vector representing a collection of random variables (e.g. c and ϕ). This function is defined in such a way that failure is implied when $G(\underline{X}) < 0$ and safety when $G(\underline{X}) > 0$. The state when $G(\underline{X}) = 0$ is called the limit state. Often, $G(\underline{X})$ is formulated as the difference between the total resisting and disturbing forces or moments acting on the slope; this is termed the safety margin. The safety margin is more likely to be normally distributed than the factor of safety. The probability of failure of a slope can be calculated by direct integration of the entire failure domain for $G(\underline{X}) < 0$. However, direct integration is sometimes not practical or is very difficult. In this case, indirect methods can be used. The most common methods use the reliability index. Further details of the performance function are given in Li (1991).

3.2 The reliability index

The conventional definition of reliability index, denoted by β, is given as,

$$\beta = \frac{\mu_G}{\sigma_G} \quad (1)$$

where μ_G and σ_G are respectively the mean value and standard deviation of the performance function, $G(\underline{X})$. Then,

$$P_f = P(G(\underline{X}) < 0)$$

$$= P(\frac{G(\underline{X}) - \mu_G}{\sigma_G} < -\frac{\mu_G}{\sigma_G})$$

$$= P(Z < -\beta)$$

and if $G(\underline{X})$ is normally distributed then,

$$= \int_{-\infty}^{-\beta} \phi(Z)dZ$$

$$= \Phi(-\beta) \quad (2)$$

where P_f is the probability of failure of the slope, and is equal to the the value of $\Phi(\cdot)$ which is the cumulative distribution of a standardized normal variate and can be directly read from standard tables if β is known.

The value of μ_G can be estimated easily by using the sample mean of specified random variables, while the value of σ_G is generally estimated by using a Taylor series expansion of the performance function, $G(\underline{X})$, about the mean. By linearizing the performance function at the sample mean and then using the moment generating function, the variance of $G(\underline{X})$ can be expressed as,

$$Var(G(\underline{X})) = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial G}{\partial X_i}\Big|_{\overline{X}} \cdot \frac{\partial G}{\partial X_j}\Big|_{\overline{X}} \cdot cov(\overline{X}_i, \overline{X}_j) \quad (3)$$

where $\overline{X}$ is defined as a vector of sample means of random variables, n is the number of random variables and $\overline{X}_i$ and $\overline{X}_j$ are sample means of the individual random variables within the random field model. This technique is called the first order second moment (FOSM) method. The expressions for the partial derivatives of $G(\underline{X})$ can be obtained by determining the actual derivatives once the performance function is defined (Li & Lumb, 1987). To calculate the covariance between sample averages of random variables, cov($\cdot$), random field theory is applied.

In the present study, only two random variables are considered. They are the spatial local averages of cohesion, $\overline{c}$, and internal friction, $\overline{\phi}$. The spatial local average must be used instead of the sample average because, in most engineering measurements, the soil properties are measured by using samples of finite small volume and the variability of the values obtained in this manner is often significantly greater than that over a larger region. Thus, in this study, n in the Eq. 3 is 2. As already mentioned, $\overline{c}$ and $\overline{\phi}$ depend on their locations, $\underline{t} = (x, y, z)$ or $\underline{t}' = (x', y', z')$, in 3-dimensional Cartesian co-ordinates. Thus, Eq. 3 can be rewritten as,

$$Var(G(\underline{X})) = \frac{\partial G}{\partial c}\Big|_{\overline{c},\overline{\phi}} \frac{\partial G}{\partial \phi}\Big|_{\overline{c},\overline{\phi}} \cdot cov(\overline{c}_{\underline{t}}, \overline{c}_{\underline{t}'})$$
$$+ \frac{\partial G}{\partial c}\Big|_{\overline{c},\overline{\phi}} \frac{\partial G}{\partial \phi}\Big|_{\overline{c},\overline{\phi}} \cdot cov(\overline{c}_{\underline{t}}, \overline{\phi}_{\underline{t}'})$$
$$+ \frac{\partial G}{\partial \phi}\Big|_{\overline{c},\overline{\phi}} \frac{\partial G}{\partial c}\Big|_{\overline{c},\overline{\phi}} \cdot cov(\overline{\phi}_{\underline{t}}, \overline{c}_{\underline{t}'}) \quad (4)$$
$$+ \frac{\partial G}{\partial \phi}\Big|_{\overline{c},\overline{\phi}} \frac{\partial G}{\partial c}\Big|_{\overline{c},\overline{\phi}} \cdot cov(\overline{\phi}_{\underline{t}}, \overline{\phi}_{\underline{t}'})$$

More concisely, it can be written in matrix notation as,

$$Var(G(\underline{X})) = \begin{bmatrix} \frac{\partial G}{\partial c} \\ \frac{\partial G}{\partial \phi} \end{bmatrix}^T_{\overline{c},\overline{\phi}} \cdot \begin{bmatrix} cov(\overline{c}_{\underline{t}}, \overline{c}_{\underline{t}'}) & cov(\overline{c}_{\underline{t}}, \overline{\phi}_{\underline{t}'}) \\ cov(\overline{\phi}_{\underline{t}}, \overline{c}_{\underline{t}'}) & cov(\overline{\phi}_{\underline{t}}, \overline{\phi}_{\underline{t}'}) \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial G}{\partial c} \\ \frac{\partial G}{\partial \phi} \end{bmatrix}_{\overline{c},\overline{\phi}} \quad (5)$$

where sup T means the transpose of the column matrix, [$\cdot$].

4. RANDOM FIELD MODELLING AND CROSS-CORRELATION

Random field or random (stochastic) process models (Vanmarcke, 1977&1983) model random variables in n-dimensional space (time or space). Soil properties have different values at different locations even in a statistically homogeneous soil profile and a random field model is applicable.

It has been stated that autocorrelation of soil properties has been modelled successfully by Li and White (1987b) using the random field. In their study, the covariances of $\overline{c}$ and $\overline{\phi}$ in the principal diagonal of the 2 × 2 matrix (i.e. $cov(\overline{c}_{\underline{t}}, \overline{c}_{\underline{t}'})$ & $cov(\overline{\phi}_{\underline{t}}, \overline{\phi}_{\underline{t}'})$) in Eq. 5 have been determined and the covariances between $\overline{c}$ and $\overline{\phi}$ (i.e. $cov(\overline{c}_{\underline{t}}, \overline{\phi}_{\underline{t}'})$, $cov(\overline{\phi}_{\underline{t}}, \overline{c}_{\underline{t}'})$) have been assumed to be zero. The present study has now included the correlation between $\overline{c}$ and $\overline{\phi}$ in the model and examines their effect.

Based on the above model, the value of a soil property ($\overline{c}$ or $\overline{\phi}$) at a point $\underline{t} = (x, y, z)$ is defined by $k(\underline{t})$. In general, $k(\underline{t})$ can be decomposed into a trend component $g(\underline{t})$ and a random component $\varepsilon(\underline{t})$ which has zero mean value, i.e.,

$$k(\underline{t}) = g(\underline{t}) + \varepsilon(\underline{t}) \quad (6)$$

where g(t) can be expressed as a polynomial estimated from the test results at various locations, $\underline{t}$, using the method of least squares. In this study, only the cohesion, c, and internal friction, ϕ are considered so that, from Eq. 6, these parameters can then be expressed as,

$$c(\underline{t}) = c_g(\underline{t}) + c_\varepsilon(\underline{t}) \quad (7)$$

and,

$$\phi(\underline{t}) = \phi_g(\underline{t}) + \phi_\varepsilon(\underline{t}) \quad (8)$$

The spatial local averages of cohesion and internal friction ($\bar{c}$ or $\bar{\phi}$) can thus be denoted as,

$$\bar{c}_v = \frac{1}{V}\int_V c(\underline{t})dt = \bar{c}_g|_v + \bar{c}_\varepsilon|_v \quad (9)$$

$$\bar{\phi}_v = \frac{1}{V}\int_V \phi(\underline{t})dt = \bar{\phi}_g|_v + \bar{\phi}_\varepsilon|_v \quad (10)$$

where V can be length L (line average), area A (areal average) or volume V (volumetric average). Under the simplest assumptions regarding the soil statistical properties: i.e. a constant mean component; constant statistical randomness (constant variance); and autocovariance and cross–covariance expressed as functions of distance, the statistics (Li & White, 1987b) are,

$$\begin{aligned} &E\{k(\underline{t})\}=g(\underline{t})=\text{constant} \\ &\text{var}\{k(\underline{t})\}=\text{var}\{\varepsilon(\underline{t})\}=\sigma^2=\text{constant} \\ &\text{cov}\{k(\underline{t}),k(\underline{t}')\}=\text{cov}\{\varepsilon(\underline{t}),\varepsilon(\underline{t}')\}=\sigma^2\cdot\varrho(\underline{v}) \\ &\text{cov}\{k(\underline{t}),k'(\underline{t}')\}=\text{cov}\{k_\varepsilon(\underline{t}),k'_\varepsilon(\underline{t}')\}=\sigma_k\cdot\sigma_{k'}\cdot\varrho'(\underline{v}) \end{aligned} \quad (11)$$

where $E\{\cdot\}$, var$\{\cdot\}$, cov$\{\cdot\}$, $\varrho(\underline{v})$, $\varrho'(\underline{v})$, σ and k (or k') are the expected value, variance, covariance, autocorrelation function, cross–correlation function, standard deviation and soil property respectively; $\underline{v} = (v_x, v_y, v_z) = |\underline{t}-\underline{t}'|$ is the lag distance between the position vectors $\underline{t}$ and $\underline{t}'$. The first three equations of Eq. 11 are collectively designated as a Type I soil profile by Li and White (1987b).

The cross–correlation function can be further simplified by assuming it has a form similar to the autocorrelation function, i.e.,

$$\varrho'(\underline{v}) = r\cdot\varrho(\underline{v}) \quad (12)$$

where r is the correlation coefficient ($-1 \le r \le +1$). Only a Type I autocorrelation function (Li & White, 1987b) will be used in present study. It is a simple exponential form, exp $\{-2v/\delta\}$ in 1–dimensional case or exp $\{-2(v_x/\delta_x+v_y/\delta_y)\}$ in 2–dimensions. The term δ is called the scale of fluctuation and measures correlation of the soil property in space. If δ is approximately zero, then the soil properties are independent and purely random, even though the property may be measured at quite close spacing. If δ is very large compared with the size of the problem, then the soil properties are effectively perfectly correlated in space. In this case, a soil property is the same throughout the soil mass for each realization and variability due to measurement noise. This is the situation that is assumed in most stochastic slope design. Some examples based on the Type I one dimensional autocorrelation function are shown in Table 1.

Table 1 Scale of Fluctuation (after Li & White, 1987b)

Material	Property	Scale of fluctuation, δ (m)	Source
Clay	shear strength	2.14	Wu (1974)
Marine clay	undrained shear strength	1.02	Lumb (1975)
Clay	undrained shear strength	0.32–0.67	Matsuo (1976)
Soft clay	undrained shear strength	0.61	Asaoka & A–Grivas (1982)
Soft clay	undrained shear strength	1.56	Asaoka & A–Grivas (1982)

Based on Eqs. 11 and 12 and using a Type I autocorrelation function, the covariance between the spatial local average of cohesion and that of internal friction will be as follows,

$$\begin{aligned} \text{cov}\{\bar{c}_V,\bar{\phi}_{V'}\} &= E\{(\bar{c}_V - E\{\bar{c}_V\})\cdot(\bar{\phi}_{V'} - E\{\bar{\phi}_{V'}\})\} \\ &= E\{\bar{c}_{\varepsilon_V}\cdot\bar{\phi}_{\varepsilon_{V'}}\} \\ &= \text{cov}\{\bar{c}_{\varepsilon_V},\bar{\phi}_{\varepsilon_{V'}}\} \\ &= \frac{1}{VV'}E\{\int_V c_\varepsilon(\underline{t})d\underline{t}\int_{V'}\phi_\varepsilon(\underline{t}')d\underline{t}'\} \\ &= \frac{1}{VV'}E\{\int_V\int_{V'} c_\varepsilon(\underline{t})\cdot\phi_\varepsilon(\underline{t}')\,d\underline{t}d\underline{t}'\} \\ &= \frac{1}{VV'}\int_V\int_{V'} E\{c_\varepsilon(\underline{t})\cdot\phi_\varepsilon(\underline{t}')\}d\underline{t}d\underline{t}' \\ &= \frac{1}{VV'}\int_V\int_{V'}\sigma_k\sigma_{k'}\varrho'(\underline{v})\,d\underline{t}d\underline{t}' \quad \text{from (11)} \\ &= \frac{\sigma_k\sigma_{k'}r}{VV'}\int_V\int_{V'}\varrho(\underline{v})\,d\underline{t}d\underline{t}' \quad \text{from (12)} \\ &= \sigma_k\sigma_{k'}r\cdot B(V,V') \end{aligned} \quad (13)$$

where the subscript ε_V or $\varepsilon_{V'}$ stands for the randomness of the related random variable with respect to the sample space of V or V' (length, area or volume) and $B(\cdot) = \frac{1}{VV'}\int_V\int_{V'}\varrho(\underline{v})\,d\underline{t}d\underline{t}'$ is called the covariance reduction factor. Since the autocorrelation function, $\varrho(\underline{v})$, is known or assumed, the integration can be completed using semi–analytical methods (Li & White, 1987b). If the standard deviations of the soil properties and the correlation coefficient can be determined from test samples, the covariance in Eq. 13 can be calculated. Then, the variance of the performance function, Var($G(\underline{X})$) can be evaluated by assuming cov$\{\bar{c}_V,\bar{\phi}_{V'}\}$ = cov$\{\bar{\phi}_{V'},\bar{c}_V\}$. This model thus includes consideration of both autocorrelation and spatial cross–correlation, and, as a result, the probability of failure of a slope can be modelled more realistically than even by those models that have adopted random field theory to date (e.g. Li & White, 1987b; Li & Lumb, 1987). This is a significant extension to existing work on reliability in geomechanics.

The effect of spatial cross–correlation can be visualized from Eqs. 1 and 4. From Eq. 4, the variance of $G(\underline{X})$ will increase if the cross correlation function between c and ϕ, $\varrho'_{c,\phi}$, is positive (positively correlated) or vice–versa. However, from Eq. 1, it can be observed that the standard deviation of $G(\underline{X})$ ($G(\underline{X})^{1/2}$) reflects most of the effect of cross–correlation. Thus, positive correlation will have less effect (i.e. a small decrease) on the calculation of the reliability of a slope with respect to that of zero cross–correlation. Meanwhile, for negative correlation, reliability will be profoundly increased due to a proportionally large reduction in the variance. These matters are shown in the illustrative examples.

5. PROGRAM PROBSN

A slope stability program, PROBSN, has been developed by Li (1987) based on the random field model. The performance function used in the program was formulated as the safety margin since it is likely to be more linear than the factor of safety. Li used the first order second moment (FOSM) method which included modelling autocorrelation, and a generalized procedure of slices (GPS) method of analysis (Li & White, 1987c) to calculate the probability of failure of a slope. This program has been modified to include the effect of spatial cross–correlation using a Type I autocorrelation function. It was assumed that the cross–correlation structure between c and $\tan\phi$ (μ) followed the same model as the autocorrelation of either c or $\tan\phi$. $\tan\phi$ or μ is called the coefficient of friction and is used instead of ϕ since ϕ always appears as $\tan\phi$ in the calculation. It should be noted that within PROBSN Eq. 5 is expanded to a 2n × 2n matrix with the spatial averages, $\bar{c}$ and $\bar{\mu}$, along the base of each slice considered to be the 2n random variables. In the original work by Li (1987) the upper right and lower left quadrants of this matrix are assumed to be zero. The current work has removed this assumption.

6. ILLUSTRATIVE EXAMPLES

In order to examine the effect of spatial cross–correlation on the values determined for the probability of failure, two slopes have been analyzed by the modified program, PROBSN.

6.1 Example 1: Waddell (1988)

Waddell (1988) analyzed a uniform soil slope with a face angle of 30° as shown in Fig. 1 which was comprised of materials with a mean cohesion of 50 kN/m^2 and a standard deviation of the cohesion of 15 kN/m^2, a mean angle of friction of 5° (standard deviation of 1.5°) that is equivalent to a mean coefficient of friction, $\bar{\mu}$, of 0.0875 with a standard of deviation (σ_μ) of 0.0262, and a fixed density 17kN/m^3 – see Fig. 1. $\bar{\mu}$ and the related standard deviation, σ_μ, are small so that μ varies linearly within one standard deviation but the actual values are not typical for realistic slope design. This slope has been re–analyzed using PROBSN with the simplified Bishop method with the critical circle for the minimum factor of safety under dry conditions (pore pressures, u, of zero everywhere). Only the local average cohesion, $\bar{c}$, and coefficient of friction, $\bar{\mu}$, are considered to be random variables. The sample size is assumed to be four because it is rather common to have only small support for the mean properties. The result is shown in Fig. 2. In all cases analyzed the mean factor of safety for this slope is 1.53.

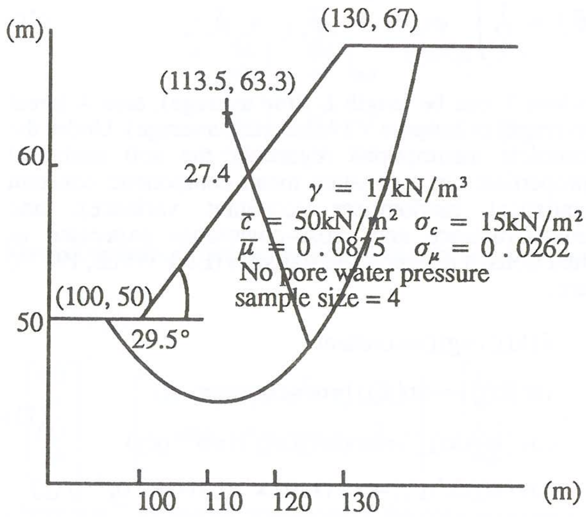

Figure 1 Slope analysed in example 1

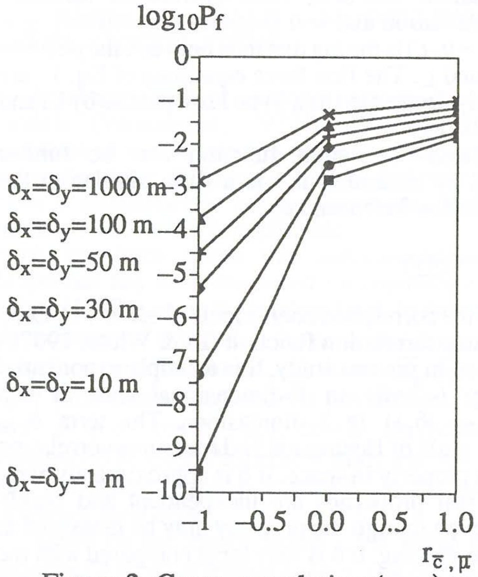

Figure 2 Cross–correlation ($r_{\bar{c},\bar{\mu}}$) against probability of failure (P_f) with different scales of fluctuation for the slope adopted in example 1

It can be seen that the reduction in the probability of failure is significant when $r_{\bar{c},\bar{\mu}}$ is varied from +1 to –1. It varies from 10^{-3} to less than 10^{-9} for $\delta_x = \delta_y = 1$m which is a common scale of fluctuation as shown in Table 1. Even when perfect correlation ($\delta_x=\delta_y=1000$ m) is assumed, as is commonly but erroneously done by many workers in this field (for examples, see Li 1992), the probability of failure is changed by more than an order of magnitude due to the effect of spatial cross–correlation.

6.2 Example 2: Chowdhury (1986)

The moment equilibrium method was used in the previous example. When a long weak plane exists in a slope, force equilibrium is considered for the failure mechanism. Chowdhury (1986) analyzed a soil slope with a face angle of 45° and a long weak failure plane with an angle of 35° as shown in Fig. 3. The soil properties along the failure plane are (mean & standard deviation): cohesion 20 kN/m^2 and 4 kN/m^2, angle of friction 38° and 21° (i.e. $\bar{\mu}$ = 0.78, σ_μ = 0.39), and density 23.58 kN/m^3 and 0 kN/m^3. σ_μ in this case is quite high which is not typical for realistic slope design. This slope has been re-analyzed assuming dry conditions. Sample size is again taken to be four. The mean factor of safety for this surface in all analyses is 1.71. The result is shown in Fig. 4.

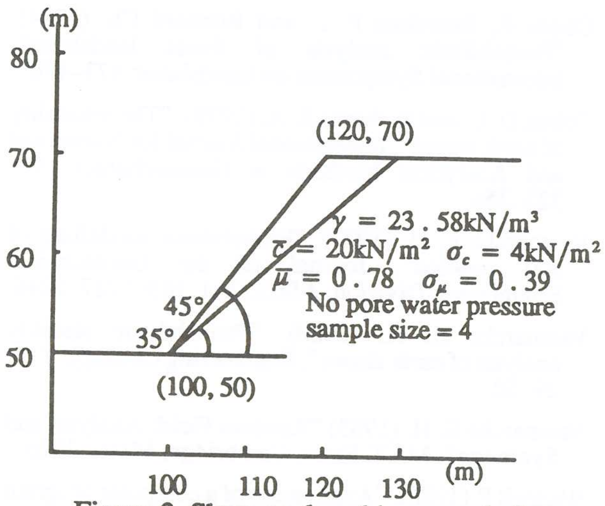

Figure 3 Slope analysed in example 2

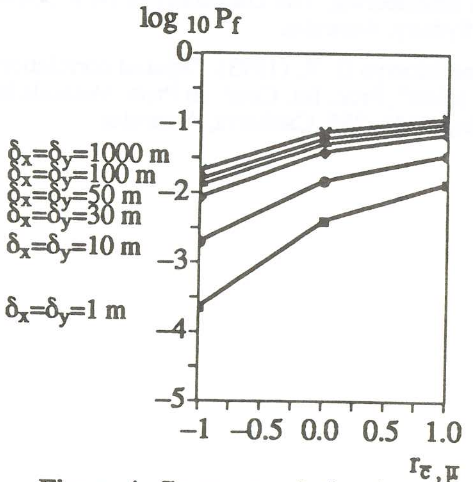

Figure 4 Cross–correlation ($r_{\bar{c}\bar{\mu}}$) against probability of failure (P_f) with different scales of fluctuation for the slope adopted in example 2

The figures show that a high factor of safety (1.71) does not guarantee high reliability. Based on the consideration of autocorrelation and spatial cross–correlation, the probability of failure will be lowered two orders of magnitude if perfect negative correlation and a realistic scale of fluctuation of 1m are assumed. However, the reduction of probability of failure in this example is not as great as in the previous example. This may be due to the very high standard deviation of the local average of the coefficient of friction, σ_μ, coupled with a relatively low standard deviation of cohesion, σ_c. In this case, one parameter dominates the reliability and thus we would expect the effects of spatial cross–correlation to be small. Hence, it can be seen that for this example the effect of autocorrelation at $r_{\bar{c}\bar{\mu}}$=0 is greater than the effect of cross–correlation.

6.3 Comments on the results

It can be seen that:

1. The reduction in the probability of failure will be great (i.e. a few orders of magnitude) if the soil properties ($\bar{c}$ and $\bar{\mu}$) are highly negatively cross–correlated.
2. The probability of failure is profoundly affected by the randomness of soil properties ($\bar{c}$ and $\bar{\mu}$). If they are perfectly correlated, i.e., a large value of scale of fluctuation, the variation of probability of failure with $r_{\bar{c}\bar{\mu}}$ will be small. If they are near to completely independent ($\delta \approx 0$), even weakly negative correlation (<–0.5) will cause a reduction in the probability of failure of more than 1 or 2 orders of magnitude.
3. The reduction in the probability of failure will be less if the stability is dominated by either c or μ. However, the boundary between small or large can only be determined by random field modelling.

7. CONCLUSIONS

The authors have modified the work of Li (1987) to include the effect of spatial cross–correlation of strength parameters in a random field (autocorrelated) probabilistic slope stability model. To the authors' knowledge this is the first time such a realistic model has been adopted for probabilistic geotechnical analysis. It is shown by a number of examples that not only autocorrelation but also spatial cross–correlation can, and often does, have a very large effect on the probability of failure determined for a slope. Therefore, the mere assumption of independent soil parameters ($\bar{c}$ and $\bar{\mu}$), can no longer be considered adequate for probabilistic analysis as the reliability of a slope is generally greatly under–estimated by such an approach. The importance of the actual value of scale of fluctuation and of the cross–correlation coefficient are both shown to be significant and both require further research which is currently being undertaken.

8. ACKNOWLEDGEMENT

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