LIQUEFACTION EVALUATION IN STRATIFIED SOILS

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ABSTRACT

The Canterbury Earthquake Sequence of 2010-2011 (CES) induced widespread liquefaction in many parts of Christchurch city. 55 case history sites around Christchurch have been studied to assess the liquefaction characteristics of Christchurch soils. Initial studies identified a correlation between soil stratification and liquefaction manifestation during the CES. Furthermore, comparisons with the simplified method show an under-prediction of liquefaction at sites that manifested liquefaction and an over-prediction at sites that did not. To further investigate these observations, an effective stress analysis was employed to study the effect of soil stratification on the development and manifestation of liquefaction, and to investigate the impact that neglecting these effects has on the predictive capabilities of current simplified methods. The analysis showed that the system response of soil deposits (i.e. the dynamic interaction between soil layers) is a key component in the development and manifestation of liquefaction.

1 INTRODUCTION

The Canterbury Earthquake Sequence 2010-2011 (CES) induced widespread liquefaction in many parts of Christchurch city. Observations of liquefaction manifestation show, to some extent, a geographic grouping of sites that manifested liquefaction during the CES and sites that did not. Liquefaction was more commonly observed in the eastern suburbs and along the Avon River (e.g. Avondale) where the soils are characterised by thick sandy deposits with a shallow water table. On the other hand, suburbs to the north, west and south of the CBD (e.g. Riccarton, Papanui) exhibited less severe to no liquefaction. These soils are more commonly characterised by inter-layered liquefiable and non-liquefiable deposits, also with a shallow water table.

As part of a large-scale study of the performance of Christchurch soils during the CES, detailed data including CPT, borehole logs, shear wave velocity (V_s) and compression wave velocity (V_p) has been collected for 55 sites in Christchurch. For this suite of Christchurch sites, predictions of liquefaction triggering using the simplified method indicated that liquefaction was over-predicted for 94% of sites that did not manifest liquefaction during the CES and under-predicted for 50% of sites that did manifest liquefaction (Cubrinovski et al., 2017).

The interaction between soil layers during cyclic loading, which is not considered in the simplified triggering procedures, can have a significant impact on liquefaction development and manifestation. This soil layer interaction may explain some of the aforementioned discrepancies between prediction and observation. This study aimed to assess the effect of soil layer interaction on liquefaction manifestation and evaluate how neglecting this effect impacts the accuracy of the simplified method as a tool for predicting liquefaction triggering.

2 METHODOLOGY

The effect of soil stratification on liquefaction development was studied using an effective stress analysis (ESA). ESA was chosen as it can effectively model water flow and pore water pressure dissipation, produces detailed time histories, and most importantly can capture the dynamic interaction between soil layers. For this study the soil deposit was modelled as a 1D soil column. The key inputs to the model were the soil profile, soil properties and input ground motion. These are outlined below:

1. *Soil Profile (Section 3)*: The soil profiles used in the ESA were chosen to capture key soil stratification characteristics of the Christchurch sites. The soil profiles were developed from the CPT, borehole and shear wave velocity data gathered for the 55 sites. These profiles were chosen to represent Christchurch soil deposits that both did and did not manifest liquefaction during the M_w7.1 4 September 2010 (SEP10) and M_w6.2 22 February 2011 (FEB11) earthquakes.
2. *Soil Properties (Section 4)*: The soil properties were key to accurately modelling the constitutive behaviour of the Christchurch soils. In this analysis, the elasto-plastic Stress-Density Model (SDM) (Cubrinovski, 1993; Cubrinovski & Ishihara, 1998a and 1998b) was employed as the constitutive model. To model the Christchurch soils the liquefaction resistance of key soil layers was determined using the simplified method (Boulanger & Idriss, 2014) and representative q_{c1Ncs} values. The SDM was then calibrated by adjusting the model parameters to accurately simulate the target liquefaction resistance.

3. *Input ground motion (Section 5)*: The input ground motion used in this study was obtained by deconvolution of surface ground motions recorded at strong motion stations in Christchurch. This method was chosen as there were no suitable ground motions recorded on bedrock during the CES.

3 SOIL PROFILES

To develop the characteristic profiles modelled in the ESA, the 55 Christchurch sites were first characterised based on typical soil layers, layer thicknesses, and location of layers in the deposit. For each site, this produced a simplified soil profile that characterised the soil stratification. In the second stage, specific soil layers were characterised based on their liquefaction properties. The above process was used to compare different Christchurch sites with regard to site and soil characteristics.

3.1 SIMPLIFIED PROFILES

The simplified profiles were developed with reference to the CPT tip resistance (q_c) and soil behaviour type index (I_c). In order to group the soils based on their overall composition, a generalised soil behaviour type classification based on I_c was adopted (Table 1) and layers were defined where nearly constant q_c and I_c values were observed. Each soil layer was then characterised by a q_{c1Ncs} value determined from a triggering analysis (Boulanger & Idriss, 2014) of the simplified profiles. Figure 1 gives an illustration of the simplified soil profile for one site at 1128 Avonside Drive in Avondale.

Table 1: I_c based characterisation of key soil types present at the Christchurch sites

Soil Type	I_c
GRAVEL	$I_c \leq 1.3$
Clean SAND	$1.3 < I_c \leq 1.8$
SAND with small amount of fines	$1.9 \leq I_c < 2.1$
Sandy SILT/ non-plastic SILT	$2.1 \leq I_c < 2.6$
Non-liquefiable silt/clayey soil	$I_c \geq 2.6$

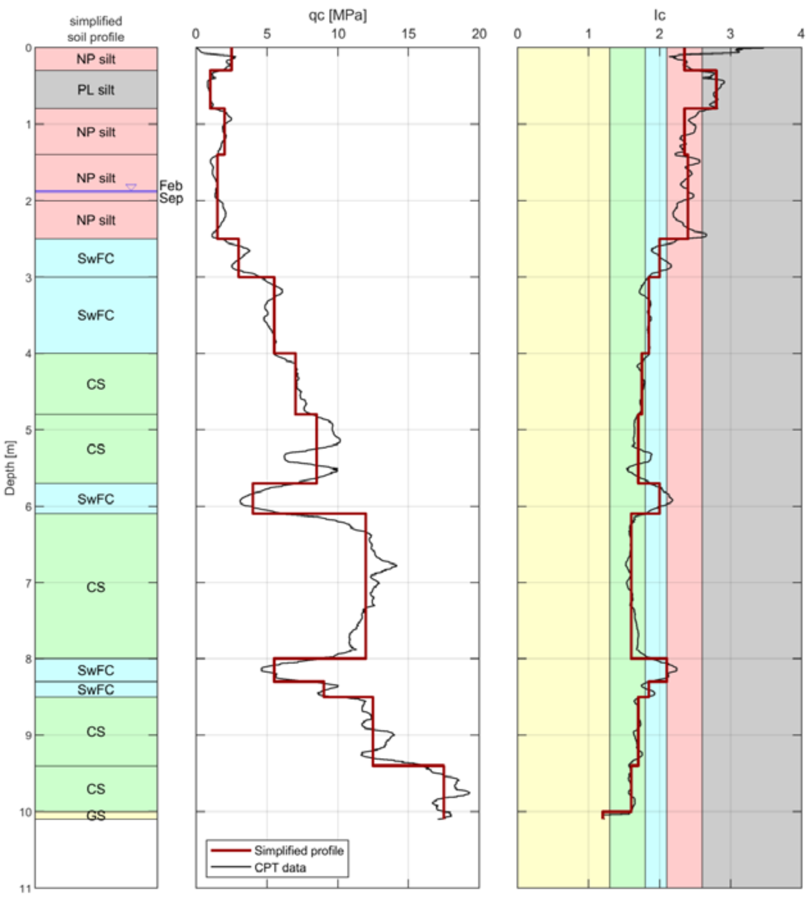

Figure 1: CPT q_c and I_c plots for 1128 Avonside Drive (blue line) showing the simplified profile interpretation (red line)

To investigate similarities and differences between sites that did and sites that did not manifest liquefaction during the CES, the simplified soil profiles were grouped into two categories based on observations of liquefaction manifestation. The first category contained sites where liquefaction was observed in both the SEP10 and FEB11 events (Yes/Yes, YY), and the second where no liquefaction was observed in either event (No/No, NN). Of the 55 sites, 15 fell into the YY category and 17 fell into the NN, the remaining 23 were sites where liquefaction was observed in FEB11 but not SEP10 (No/Yes, NY). In this initial study the NY sites were omitted. This was done for simplicity as it was expected that the YY and NN sites would represent the two extremes of soil layering characteristics and the NY cases would fall somewhere in between.

3.2 LIQUEFACTION CHARACTERISTICS OF CRITICAL LAYERS

Critical layer properties were compared between the YY and NN sites to investigate the liquefaction characteristics of the two categories. The critical layer was defined as the layer most likely to manifest liquefaction at a given site. As such, critical layers were those layers with the most critical combination of factor of safety, depth from the ground surface and thickness of the liquefied layer resulting from a triggering analysis (Robertson & Wride, 1998) of the simplified soil profiles.

A number of key critical layer properties were investigated. These included: q_{c1Ncs} (as a proxy for the soil density), depth to the top of the critical layer and thickness of the critical layer. As illustrated in Figure 2, the key result of this analysis was that no significant difference in q_{c1Ncs} (Figure 2a), critical layer depth (Figure 2b) or thickness (not shown) was discernible between the YY and NN sites. In order to determine what was causing some sites to manifest liquefaction whilst others did not a new classification of the critical layer, termed the ‘critical zone’, was developed. The critical zone was defined as any continuous layer of FS < 1 that was in contact with the critical layer. This was used to account for soil layer interaction through water flow and pore water pressure communication between the critical layer and any liquefiable layers in contact with it.

The depth to the top of the critical zone and thickness of the critical zone are illustrated in Figure 2b and 2c. Also plotted is the cumulative thickness of liquefiable material (Figure 2d). Contrary to the results for the critical layer, there was a notable difference in critical zone properties between the YY and NN sites. In fact, it was the thickness of the critical zone that best differentiated the two categories. This indicates that it is not the critical layer itself, but the thickness of a continuous zone of liquefiable material that influences liquefaction manifestation. This result is strong evidence for the role of soil layer interaction in liquefaction manifestation.

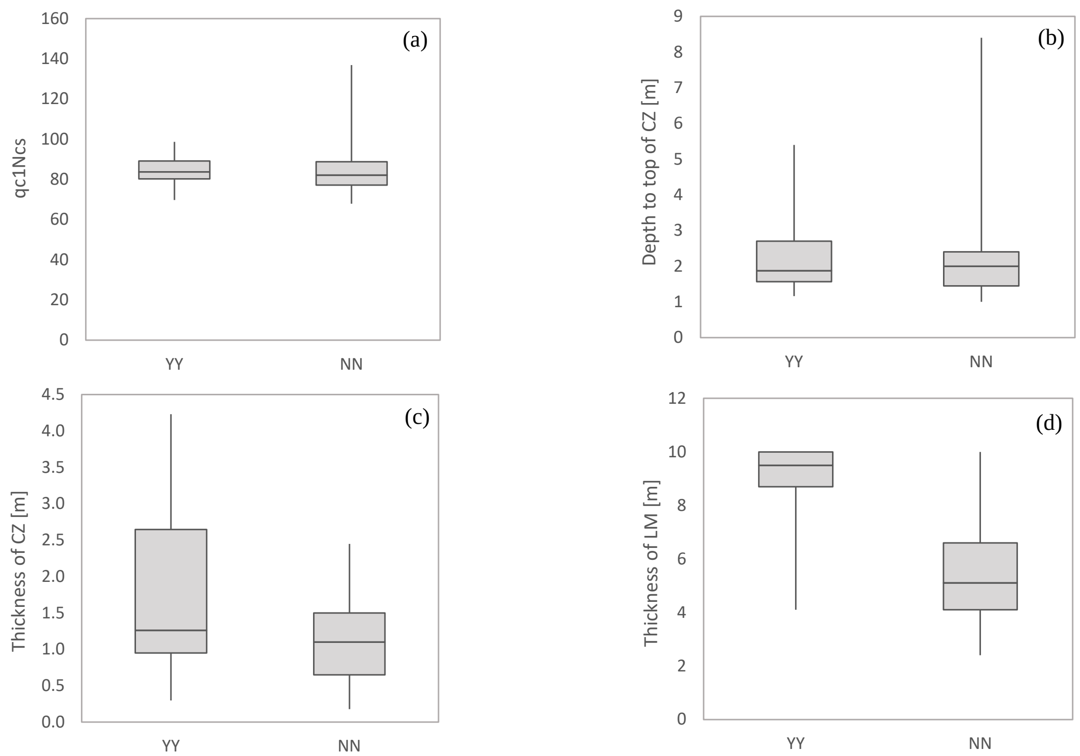

Figure 2: Statistical summary of critical layer analysis (median values shown as horizontal line, first and third quartiles shown as upper and lower limits of the ‘box’ and maximum and minimum values shown at the end of the ‘whiskers’): (a) q_{c1Ncs} calculated from Boulanger & Idriss (2014); (b) depth to top of the critical zone (CZ); (c) thickness of the critical zone; (d) cumulative thickness of liquefiable material (LM)

3.3 SOIL PROFILE CHARACTERISATION

Based on the simplified profiles (Section 3.1) and critical layer analysis (Section 3.2), characteristic soil profiles representative of Christchurch deposits were developed. Profiles were chosen to represent the YY sites that did manifest liquefaction during the CES and the NN sites that did not. The profiles were also designed to capture the findings of the critical layer analysis, hence the key difference between the YY and NN profiles was the thickness and vertical continuity of liquefiable layers. In accordance with the critical layer analysis, the liquefiable layers in the NN profiles were characterised by low liquefaction resistances similar to those of the critical layers in the YY profiles. Hence, if assessed in isolation (as done in current simplified triggering methods), liquefaction should be triggered in the critical layers of all profiles at approximately the same loading intensity. The characteristic profiles modelled in the ESA are outlined in the following sections.

3.3.1 YY Profiles

Two profiles were developed (YY1 and YY2) to represent the YY sites that manifested liquefaction in both the SEP10 and FEB11 earthquakes. The YY profiles were characterised by relatively thick vertically continuous sandy deposits. Both profiles had the critical layer located immediately below the water table followed by layers of low liquefaction resistance. The critical zone for these profiles was substantial and extended from ground water level at 1.8 m to 6 m for the YY1 profile and from 1.8 m to 4 m for the YY2 profile. The soil below the critical zone was generally denser than the overlying layers. The YY2 profile differed from YY1 in that it had a thinner critical layer and the density of the deposit increased rapidly below the critical layer. The YY1 and YY2 profiles are illustrated in Figure 3a and 3b respectively.

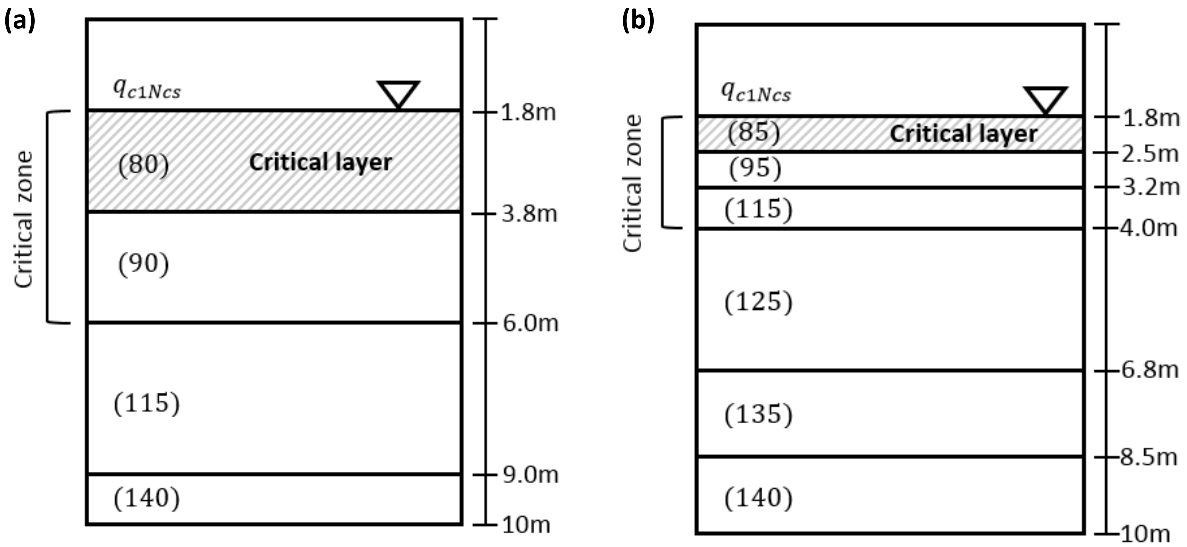

Figure 3: Characteristic soil profiles: (a) YY1; (b) YY2

3.3.2 NN Profiles

Two profiles were developed (NN1 and NN2) to represent the NN sites that did not manifest liquefaction in either the SEP10 or the FEB11 earthquakes. The NN profiles were highly non-uniform and were characterised by interbedded liquefiable and non-liquefiable layers. NN1 was characterised by two non-liquefiable layers and a single critical layer. The non-liquefiable layers consisted of a thin layer located directly below the ground water table (creating a thick non-liquefiable crust), and a thick layer of 4.5 m starting at 4.0 m below the ground surface.

The NN2 profile was characterised by highly interbedded and relatively thin non-liquefiable and liquefiable layers. The NN2 profile consisted of five non-liquefiable layers (with no soil layers thicker than 1.5 m) and three critical layers (of lowest liquefaction resistance) located throughout the top 10 m of the deposit. The NN1 and NN2 profiles are illustrated in Figures 4a and 4b respectively.

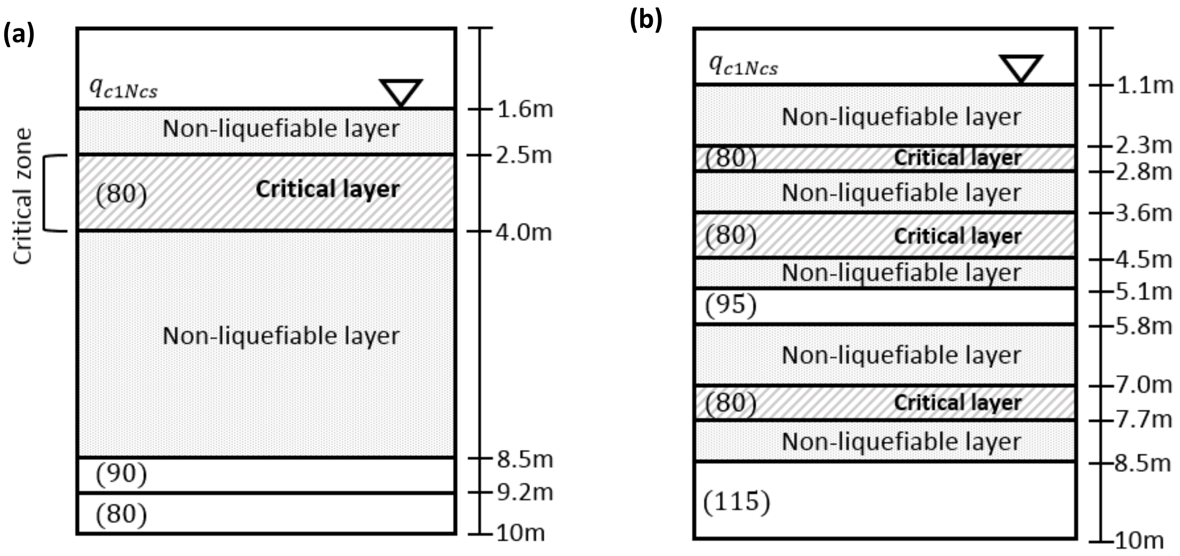

Figure 4: Characteristic soil profiles: (a) NN1; (b) NN2

4 SOIL PARAMETERS FOR ESA

Once the characteristic soil profiles and soil layers had been established, constitutive model parameters were developed for each soil layer. These parameters are usually developed from laboratory test data, however this data was not available (at the time of the study) for the Christchurch soils so a combination of empirical relationships and generic data was used. The soils were therefore modelled using model parameters established from laboratory tests on Toyoura Sand, and the model calibrated to the Christchurch soils by simulating target curves in element test simulations (ETS).

4.1 LIQUEFIABLE LAYERS

For the liquefiable layers, target liquefaction resistance curves were simulated in ETS. In this process a single soil element is subject to uniform amplitude cyclic loading and the number of cycles till liquefaction is recorded. This is repeated for a number of different loading magnitudes to give the simulated liquefaction resistance curve. This process is illustrated in Figure 5 and outlined below for the liquefiable soil layers:

1. Stress-Density Model (SDM) parameters already established for Toyoura Sand (Cubrinovski & Ishihara, 1998a; 1998b) were used as the initial set of model parameters;
2. Target liquefaction resistance curves (LRCs) were defined using the simplified method of Boulanger & Idriss (2014), as described in Rhodes (2017) and Cubrinovski et al. (2017);
3. The void ratio (soil density) for the ETS was selected to provide an appropriate soil density consistent with the target LRC;
4. Two SDM dilatancy parameters (controlling pore water pressure development) were modified to achieve the best fit with the target LRC.

Figure 6 illustrates the results of the SDM calibration.

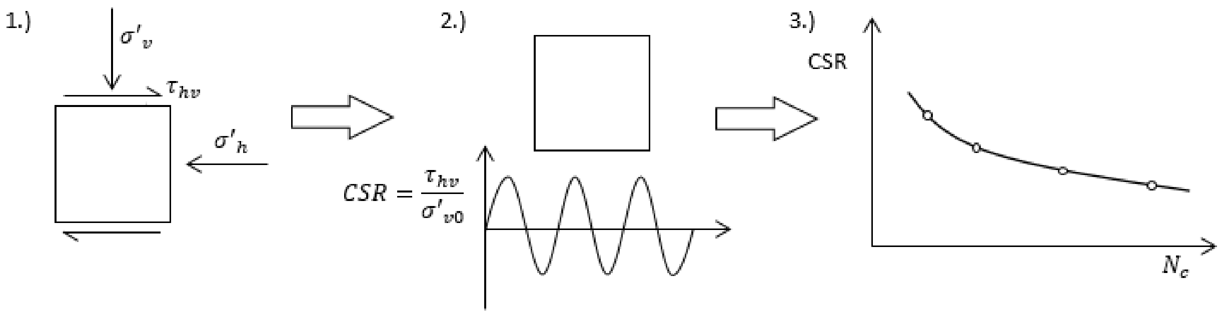

Figure 5: Schematic illustration of the element test simulations, where *CSR* is the cyclic stress ration and N_c is the number of cycles till liquefaction

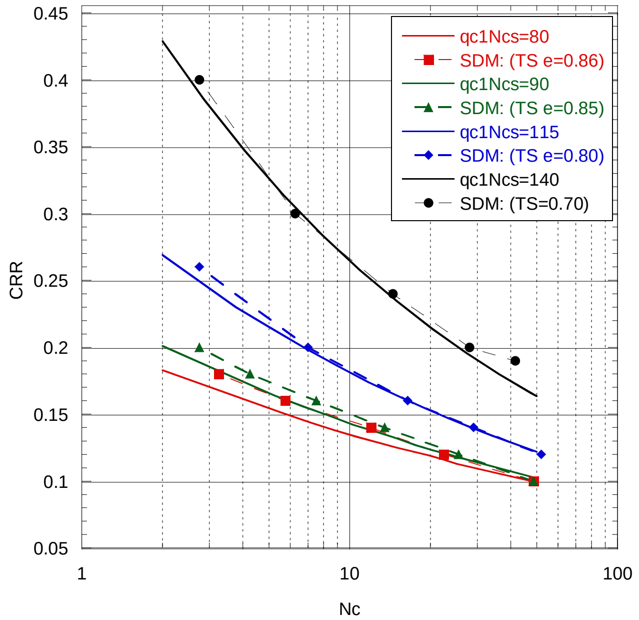

Figure 6: SDM LRCs (symbols and dashed lines) and target LRCs (solid lines) for YY1 liquefiable soils

4.2 NON-LIQUEFIABLE LAYERS

The constitutive model was calibrated for the non-liquefiable layers by simulating target degradation curves (G/G_o - γ curves) in monotonic ETS. The target curves were determined from the modified Darendeli formulation (Darendeli, 2001 and Yee et al., 2013). To model the target degradation curves the initial shear modulus was first determined using the shear-wave velocity measurements for the 55 sites, then SDM parameters were modified in total stress ETS under monotonic loading to simulate the correct shape of the target G/G_o - γ curve.

The soil deposits were modelled as non-liquefiable below 10 m depth. Liquefaction below 10 m depth rarely manifests at the ground surface, hence to simplify the analysis liquefaction at this depth was not considered. The non-liquefiable layers below 10 m depth were calibrated using the aforementioned method for non-liquefiable layers with the initial shear modulus determined using the empirical V_s-CPT relationship developed by McGann et al. (2015). The reader is referred to Rhodes (2017) for more detail on the establishment and calibration of the SDM.

5 GROUND MOTION

The final parameter required for the ESA was the input ground motion. Ideally, a ground motion recorded on rock would be used for this purpose, however as no such records were available for the CES the input ground motion was obtained through deconvolution of recorded surface ground motions. The motion recorded at the Canterbury Aero Club (CACS) strong motion site was chosen for deconvolution as it did not liquefy during the CES, had relatively low accelerations (so the level of non-linearity was low) and relatively shallow depth to the Riccarton Gravel. These were important factors as the equivalent-linear site response analysis used to deconvolute the ground motion cannot simulate highly non-linear soil behaviour.

The Riccarton Gravel was chosen as a firm stratum to represent the equivalent bedrock layer (i.e. the point where the seismic waves can propagate with no modification from soils) as the depth to bedrock in Christchurch is significant. The Riccarton Gravel was encountered at 6 m depth at the CACS site (Wotherspoon et al., 2013). The surface ground motion recorded at CACS during FEB11 was deconvoluted to this depth using the software STRATA (Rathje & Kottke, 2010) to give an equivalent outcrop motion. The deconvoluted motion was then given different scaling factors to represent an earthquake with 0.4 g PGA (analogous to FEB11), 0.3 g PGA (intermediate level shaking) and 0.2 g PGA (analogous to SEP10). These three motions were input at the base of the soil column model for the ESA.

6 RESULTS

The outputs from the ESA included time histories of accelerations, displacements, stresses, strains and pore water pressures throughout the soil deposit. Simulations were run for the four representative profiles under the three different loading intensities. An example of the YY and NN results are discussed herein to illustrate the effect that soil layer interaction (or 'system response') has on liquefaction development, how this effect differs between the YY and NN sites, and how the ESA results compare to results from the simplified method.

6.1 YY PROFILES

Under the highest intensity shaking the whole critical zone in the YY profiles liquefied. This was observed as an increase in pore water pressures (Figure 7, liquefaction has occurred when the pore water pressure is equal to the initial overburden pressure) and through the accumulation of significant strain (up to 4%) in the critical zone (Figure 8). Also notable was the accumulation of significant excess pore water pressure in the soil layers directly underlying the critical zone. These factors contributed to the system response of the YY profiles.

6.1.1 System Response

Soil layer interaction occurred in the YY profiles through pore water pressure dissipation and water flow. This occurred in three stages:

1. For both YY profiles, excess pore water pressures developed rapidly in the critical layer resulting in liquefaction after 10 s of strong ground motion.
2. Excess pore water pressures also developed in the deeper sandy layers underlying the critical layer. Although liquefaction was not induced in these layers, the excess pore water pressures exceeded those in the critical layer. Consequently, water flow from these deeper layers upwards into the critical layer exacerbated the liquefaction severity in the critical layer.
3. Excess pore water pressure dissipation resulted in water flow upwards from the critical layer. This was observed as a continued increase in excess pore water pressures in the crust layer even after the strong ground shaking had stopped at 20 s. This could have resulted in seepage induced liquefaction in the crust, further exacerbating the liquefaction manifestation at the surface.

The combined effect of the mechanisms described above resulted in an increased severity of liquefaction manifestation at the surface.

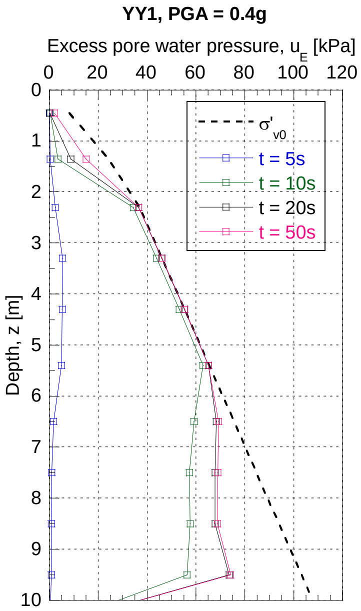

Figure 7: Distribution of excess pore water pressure with depth for YY1, PGA = 0.4 g

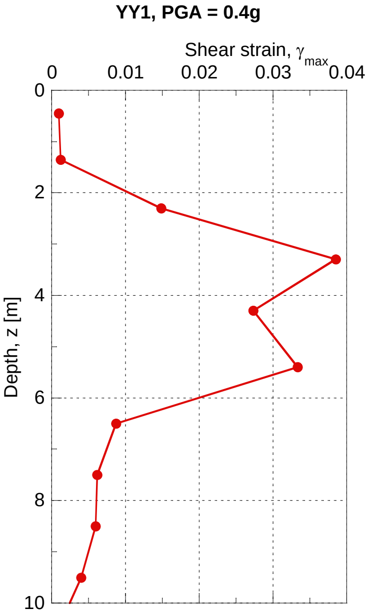

Figure 8: Distribution of maximum shear strain with depth for YY1, PGA = 0.4 g

6.2 NN PROFILES

Under the highest intensity shaking, liquefaction was observed in all liquefiable layers for both NN1 and NN2 profiles. However, under the lowest intensity shaking (PGA = 0.2 g), liquefaction occurred in only two of the five liquefiable layers in the NN2 profile (Figure 9). This change in response is attributed to ground motion modification.

Ground motion modification is the change in ground motion characteristics due to the presence of a liquefied layer. As seismic waves travel through a softened (i.e. liquefied) soil layer, high frequency motion is damped out and the period of motion elongated. As peak accelerations often occur at high frequencies, this damping is observed as a decrease in the amplitude of the ground motion. This reduction of accelerations is observed in Figure 10 for the NN2 profile and is a key characteristic of the dynamic response of liquefied deposits.

6.2.1 System Response

Soil layer interaction occurred in the NN profiles through ground motion modification. This occurred through the following processes:

1. The deepest critical layer in the NN2 profile (around 8-10 m depth) liquefied first after 10 s of strong ground motion (Figure 9);
2. The presence of this deep liquefied layer resulted in ground motion modification (expressed as a reduction in acceleration amplitude) as shown in Figure 10 for the NN2 profile;
3. As a result of this dynamic interaction, the soil layers closer to the ground surface were loaded by a motion with reduced acceleration (and consequent shear stress) amplitude;
4. For the lowest intensity motion, this damping was sufficient to prevent liquefaction from occurring in some of the critical layers in the NN2 profile. This effect is investigated in more depth in Section 6.4.

The system response of the NN profiles acted to reduce the severity of liquefaction manifestation at the ground surface. It should be noted that ground motion modification occurred for both the YY and NN sites, however the effect of this was more notable for the NN profiles due to the interlayered nature of these deposits and the first occurrence of liquefaction happening in the critical layer at depth.

6.3 SIMPLIFIED METHOD

The response of all four characteristic profiles was compared to triggering predictions by the simplified method. The results for the NN2 profile are presented herein to illustrate the key features of this comparison. An implicit assumption in the formulation of the simplified method is that soil layers act independently of surrounding layers. This means each soil layer is considered in isolation. The ESA illustrated that soil layer interaction is a key factor in liquefaction development and manifestation, so it was expected that the simplified method would be unable to accurately reproduce the results of the ESA.

This was particularly evident for the NN2 profile under the lowest intensity motion (where the system response was most evident). Figure 11 shows the excess pore water pressure ratio ($r_u = u_E / \sigma'_{v0}$) output from the ESA compared to that derived from the factor of safety calculation using the simplified method. The excess pore water pressure ratio was derived from the factor of safety (FS) by assuming a value of $r_u = 1.0$ for FS < 1.0 and inferring the pore water pressure ratio in the non-liquefied layers as the median value taken from Figure 27 in Marcuson et al. (1990). It is evident from Figure 11 that the simplified method predicted liquefaction in all liquefiable layers, however liquefaction was only induced in two of these layers in the ESA. In other words, the effect of ground motion modification in preventing liquefaction in some shallow soil layers was not captured by the simplified method.

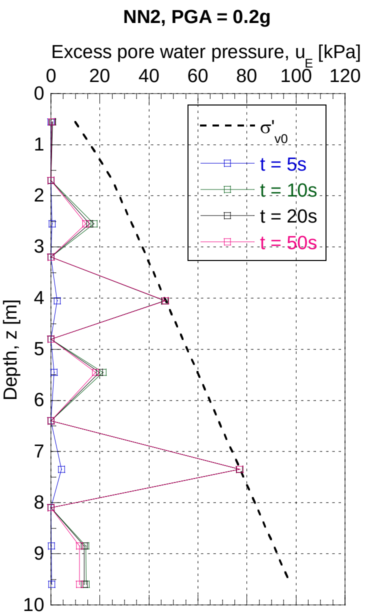

Figure 9: Distribution of excess pore water pressure with depth for NN2, PGA = 0.2 g

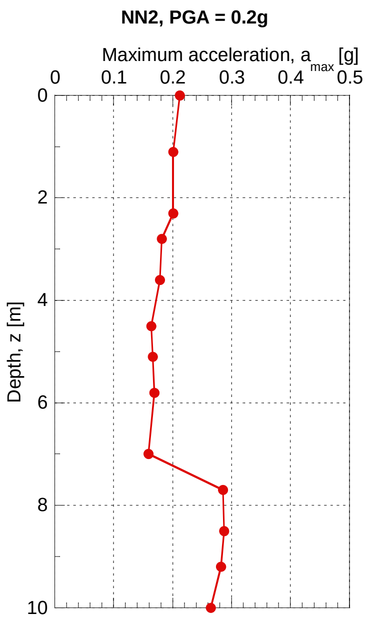

Figure 10: Distribution of maximum horizontal acceleration with depth for NN2, PGA = 0.2 g

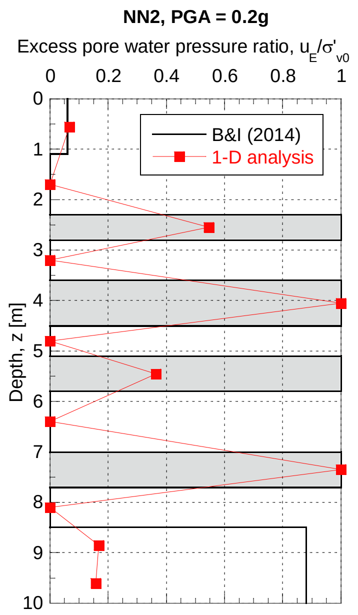

Figure 11: Pore water pressure ratio predicted by the simplified method and ESA for NN2, PGA = 0.2 g

6.4 SENSITIVITY OF GROUND RESPONSE AT SHALLOW DEPTHS

An additional investigation was conducted into the effect of ground motion modification on the NN1 profile. The ESA was repeated for the highest intensity shaking (PGA = 0.4 g) incrementally increasing the liquefaction resistance of the shallowest critical layer. The aim of this study was to determine a threshold liquefaction resistance in the critical layer when liquefaction is prevented due to soil layer interaction. The results are presented in Figure 12a (using pore water pressure ratio as a proxy for liquefaction triggering) and in Figure 12b (using single amplitude shear strain as a proxy for liquefaction triggering). These results indicate that liquefaction would be prevented in the shallow critical layer by an increase in liquefaction resistance of 25% - 30%.

A 25% - 30% increase in liquefaction resistance may reasonably occur due to partial saturation of shallow soil layers below the water table. In fact, partial saturation was consistently observed particularly in silty soils (such as those of the NN profiles) in the V_p measurements for the 55 reference sites. This effect, in combination with ground motion modification, could be the reason that liquefaction manifestation was not observed at some of the NN sites.

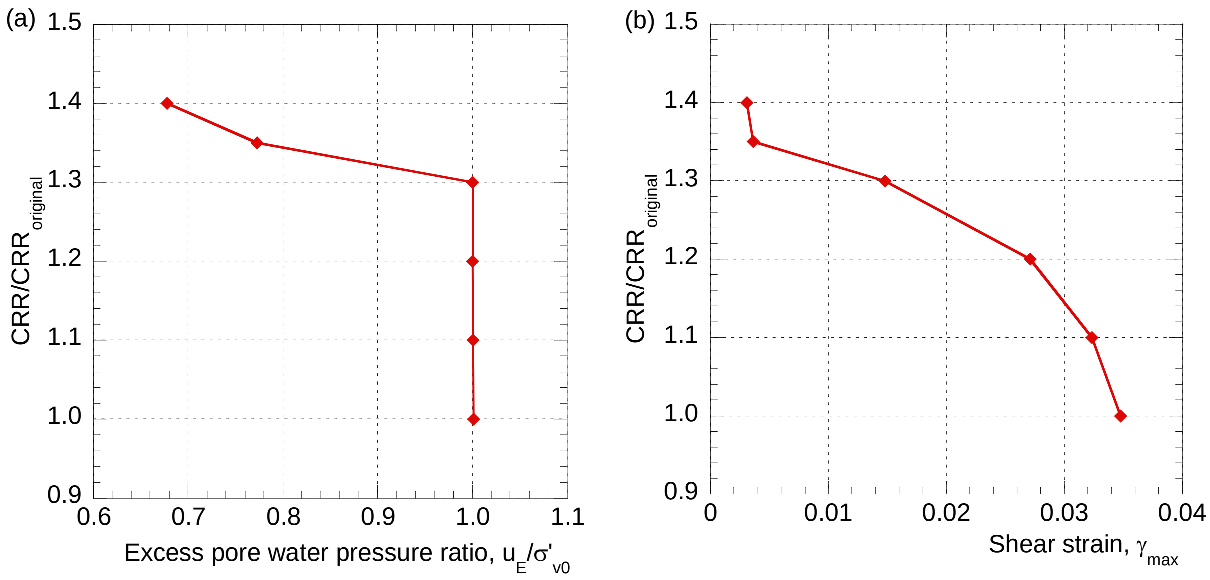

Figure 12: Percent increase in CRR required to prevent liquefaction in shallow soil layers of NN1 profile: (a) using pore water pressure ratio (threshold defined as ratio less than 1.0); (b) using single amplitude shear strain (threshold defined as a single amplitude strain less than 2%)

7 CONCLUSIONS

In this study, an effective stress analysis was used to assess the impact of soil stratification on liquefaction development and manifestation during the CES. The results were as follows:

1. The key difference between sites that manifested liquefaction in the SEP10 and FEB11 earthquakes and sites that did not was thickness and vertical continuity of liquefiable material, in other words the overall deposit characteristics of the sites.
2. Four soil profiles characteristic of Christchurch deposits were developed. Two profiles represented sites that liquefied in both SEP10 and FEB11 earthquakes (YY profiles), characterised by thick sandy deposits. Two profiles represented sites that did not liquefy in either event (NN profiles), characterised by interbedded liquefiable and non-liquefiable layers.
3. Soil layer interaction was the key factor that affected the severity and potential manifestation of liquefaction for the YY and NN profiles. For the YY profiles, soil layer interaction in the form of pore water pressure dissipation and water flow acted to increase the severity of liquefaction in the critical layer. On the other hand, for the NN profiles soil layer interaction in the form of ground motion modification acted to reduce the liquefaction susceptibility of the shallow critical layers.
4. Comparison of the ESA results with the simplified method showed the simplified method is incapable of accounting for the effects of soil layer interaction. This is because the simplified method assesses each soil layer as independent of those surrounding it.
5. Further investigation into soil layer interaction in the NN1 profile showed that an increase in the liquefaction resistance of the shallow critical layer (of the order of 25% - 30%) was sufficient to prevent liquefaction manifestation under the highest intensity shaking. Partial saturation of the soil layers directly below the ground

water table is one way in which the liquefaction resistance of shallow soil layers could increase. This is especially true for silty soils, such as those of the NN profiles.

6. As shown herein, soil layer interaction (i.e. the system response of soil deposits) can have a significant impact on liquefaction manifestation. Hence, efforts to incorporate such interaction effects in simplified triggering procedures is much needed. Accounting for these effects will enable a more reliable evaluation of liquefaction and its effects, and therefore enable engineering solutions to be designed that are appropriate for the site specific level of risk.

The research that has been presented herein was an initial study in a large body of work assessing liquefaction characteristics of Christchurch soils during the CES. Subsequent work has looked in depth at the mechanisms for intensification (YY sites) and mitigation (NN sites) of liquefaction manifestation and is presented in Cubrinovski et al. (2017). Further research is currently underway on the response of the NY sites which manifested liquefaction in FEB11 but not SEP10 (Ntritsos, 2016), and on a probabilistic assessment of the soil profile characterisation (Ntritsos & Cubrinovski, 2018).

8 ACKOWLEDGEMENTS

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