COMPARISON OF 2D AND 3D LIMIT EQUILIBRIUM SLOPE STABILITY ANALYSIS WHEN CONSIDERING VARIOUS LOADING CONDITIONS

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ABSTRACT

Two commercially available limit equilibrium slope stability analysis software packages (*Slide* and *Slide*^{3}) were used to compare the computed factors of safety (FoS) from two dimensional (2D) and three dimensional (3D) models when considering distributed and concentrated surcharges. Available published literature typically only considers the difference in model geometry from 2D to 3D, and generally suggests an approximately 10% increase in FoS when moving from 2D plane-strain to a 3D model. Experience indicates that this indicative increase in FoS is inaccurate when concentrated loads or surcharges are acting on the slope, and that for such cases the increase in FoS can be significantly higher.

A comparison is presented of the computed FoS resulting from 2D analyses and those obtained with 3D analyses when considering various lengths of surcharge. The study investigates the changes in FoS for a homogenous soil under undrained and drained conditions for varying slope heights and angles. The results indicate that the 3D effects are more significant for failure mass with smaller lengths of surcharge due to more pronounced end effects, and that these effects are most prominent in slopes made of cohesive soil.

For cohesive soils, up to a 50% increase in FoS is observed for surcharge lengths less than 1m. For granular soils, up to a 35% increase in FoS is observed for surcharge lengths less than 0.5m. However, for both cohesive and granular soils, the end effects are negligible when the length of the surcharge load exceeds 3m and no significant increase in FoS is observed.

1 INTRODUCTION

The stability of embankments, cut slopes, excavations and natural slopes is commonly analysed by limit equilibrium methods. This approach has been used for several decades in the geotechnical field and is considered to be reliable by most practitioners (Chen & Chameau, 1982). Various possible failure surfaces are assumed and a global FoS is calculated by comparing the maximum available shear strength with the shear strength mobilised by the disturbing mechanism.

Slope stability analyses are usually performed using 2D methods and assume plane strain conditions, despite the fact that most observed failure mechanisms in the field possess clear 3D characteristics. Auvinet and Gonzalez (2000) suggested that 3D analyses should be undertaken in situations such as the following:

- In the case of short slopes for which boundary conditions cannot be ignored such as earth dams built in a narrow valley,
- When soil properties vary significantly along the longitudinal direction of the slope,
- When the slope is submitted to concentrated loading,
- When the potential failure surface is irregular.

Slope stability modelling techniques have significantly improved over the years from basic kinematic analysis in the 1990's through to 2D limit equilibrium analysis and simple finite element modelling using PC's in the 2000's. The recent development and improvement of user friendly 3D limit equilibrium analysis software provides additional tools for developing more realistic slope stability models.

2D analyse methods, although useful for designing most slopes and embankments, are less appropriate for slopes with distinct local loads. Locally loaded slopes analysed by 2D methods incorporate loads of an infinite extent and may lead to a very conservative design.

Experience indicates that the increase in FoS between 3D and 2D analyses is especially significant when concentrated loads or surcharges are acting on the slope. Presented in this study is a comparison of the FoS resulting from 2D analyses and those obtained with 3D analyses when considering various lengths of surcharge. The analysis results are presented in the form of charts relating the geometrical parameters of the slope, surcharge and soil properties.

2 FORMULATION OF THREE-DIMENSIONAL LIMIT EQUILIBIRUM ANALYSIS

3D limit equilibrium slope stability analysis is directly analogous to 2D methods but includes computations that consider the third dimension. For a 2D analysis, the potential sliding mass is discretised into vertical slices; whereas for a 3D analysis, the potential failure mass of a slope is divided into a number of columns. Early numerical methods proposed for 3D limit equilibrium slope stability computation were subject to several constraints such as assumed sliding direction and failure to satisfy force and/or moment equilibrium. Significant improvements to 3D slope stability computation were proposed by Huang et al. (2002) and further extend by Cheng and Yip (2007).

Cheng and Yip's (2007) formulation of a 3D method based on Morgenstern-Price's theory assumes that the weight of soil and vertical load act at the centre of each column for simplicity. At the ultimate equilibrium condition, the internal and external forces acting on each soil column are shown in Figure 1. The assumptions required in the present 3D formulation are:

- Mohr-Coulomb failure criterion is valid,
- For Morgenstern-Price's method, the FoS is determined based on the sliding direction *a'* where FoS with respect to force and moment are equal; and
- Sliding direction is the same for all soil columns (Fig. 2).

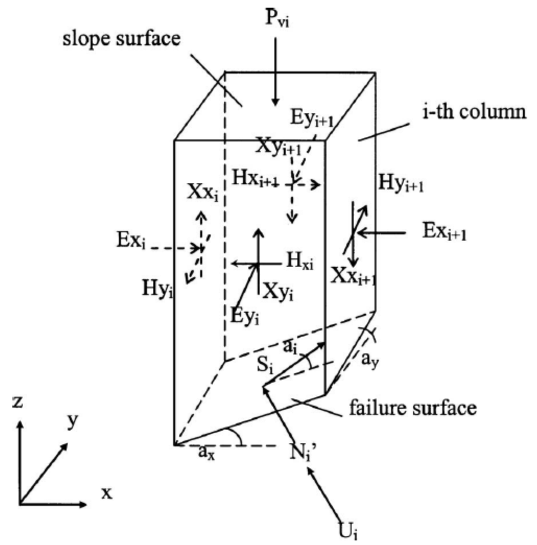


Figure 1. External and internal forces acting on a typical soil column. a_i = space sliding angle for sliding direction with respect to the direction of slide projected to x–y plane; a_x, a_y = base inclination along x- and y-directions measure at center of each column (shown at the edge of column for clarity); Ex_i, Ey_i = intercolumn normal forces in x- and y-directions, respectively; Hx_i, Hy_i = lateral intercolumn shear forces in x- and y-directions, respectively; N'_i, U_i = effective normal force and base pore water force, respectively; P_{vi}, S_i = vertical external force and base mobilised shear force, respectively; and Xx_i, Xy_i = vertical intercolumn shear force in plane perpendicular to x- and y-directions. (Cheng & Yip 2007)

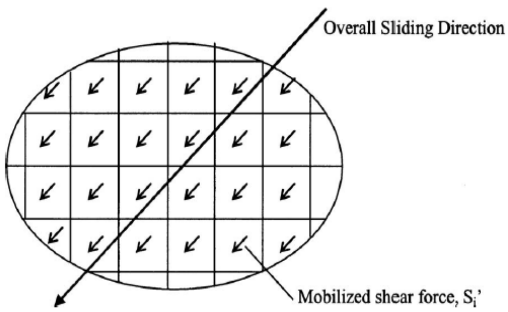


Figure 2. Unique sliding direction for all columns on plan view. (Cheng & Yip 2007)

3 PUBLISHED LITERATURE COMPARISONS

Several comparisons between 2D and 3D methods of analysis are documented in the literature. For 3D analyses, limit equilibrium methods involving columns are generally most popular and considered most feasible for practical engineering applications. Available published literature typically only considers the difference in model geometry from 2D to 3D, and ignores the potential impact of concentrated loads or surcharge.

Chen and Chameau (1982) considered a 3D failure comprising of a central cylinder attached to two semi-ellipsoids at the ends. They showed that, generally, as the width of the failure surface increases, the end effects are less and the ratio between the 3D and 2D factors of safety (F_3/F_2) decreases. As the width of the failure surface continues to increase, the problem tends towards plane strain condition and the F_3/F_2 ratio approaches unity. The end effects are caused by the geometry of the 3D ellipsoids and result in a more stable slope. Chen and Chameau (1982) noted that the steeper the slope, the lower the F_3/F_2 ratio. Since the volume of the failure mass is larger in a gentle slope, the end-effects are more pronounced.

Azzouz and Baligh (1983) compared the critical surcharge load between a 2D plane strain analysis corresponding to a strip load of infinite extent with a 3D stability analyses of slopes subjected to square loaded areas. The primary finding was that end effects can have an important effect on the magnitude of the critical surcharge load causing failure, especially in slopes which are close to failure due to gravity alone.

Leshchinsky and Baker (1986) considered homogenous and symmetrical slopes which are constrained in the longitudinal direction. The results show that 3D end effects are most pronounced in slopes made of highly cohesive soil. For cohesionless soil, no end effects exist since the critical slip surface coincides with the surface of the slope. Similar to the findings by Chen and Chameau (1982), Leshchinsky and Baker (1986) concluded that as the feasible width of failure increases, the ratio F_3/F_2 decreases; the end effect on stability decreases.

Cavoundis (1987) showed algebraically that the 3D FoS of a slope should always be greater than the 2D FoS of the same slope provided the central portion of the sliding mass is the same for the 2D and 3D analyses.

Michalowski (1989) found that for slopes where no external load is present, or if the load is small, the 3D mechanism of failure tends to a 2D plane-strain case. 3D analysis becomes important only when the slope is locally loaded, when the length of a slope is physically restrained or when local non-homogeneities in soil parameters occur.

In Lam and Fredlund's (1993) analysis of the Poplar River coal mine case study, it was concluded that the non-uniform geometry, irregular slip surface, and the external load cannot be adequately handled or realistically modelled using a 2D stability model. Results from the case history indicated that use of a 2D model considerably underestimated the factor of safety for the problem and, consequently, overestimate the shear strength parameter of the soils when used as a back analysis. This was attributed mainly to the nature of the 2D model to neglect the end effects and poor simulation of external loading.

Stark and Eid (1998) indicated that sliding masses failing in a translational mode exhibit the most pronounced difference between 2D and 3D FoS because of the large difference between the mobilised shear strength along the back scarp and sides of the slide mass and that along the base. Back-calculated shear strength of the materials involved in a slope failure using 2D methods can lead to gross overestimation of the shear strength. The case study shows that the difference between the 2D and 3D back-calculated friction angles can be as large as 30%.

Huang et al. (2002) found that for a number of case studies, the increase in FoS due to 3D effects was about 7.2% to 8.5% for semi-spherical failure surfaces. However, for composite failure surfaces with a weak layer, the increases in FoS due to 3D effects was 27% to 32%. The different 3D effects are attributable to the different influence of the end effects. At a location away from the symmetrical plane of failure, the failure surface consists of stronger soil stratum. Huang et al. (2002) demonstrated the importance of using 3D slope stability analysis methods, especially for cases of weak plane induced failures.

4 ASSESSMENT OF TWO-DIMENSIONAL AND THREE-DIMENSIONAL SLOPE STABILITY

4.1 AIM OF STUDY

The primary goal of this study is to present a comparison of the FoS resulting from 3D analyses and those obtained with 2D analyses when considering various lengths of surcharge, *L*, as shown in Figure 3 and 4.

4.2 ANALYTICAL METHODOLOGY

2D and 3D limit equilibrium slope stability analyses were carried out using commercially available software packages *Slide* (two-dimensional) and *Slide³* (three-dimensional) of RocScience Inc (2017). Circular failure surfaces have been considered in the 2D analysis, whereas ellipsoid failure surfaces have been considered in the 3D analysis. The Morgenstern Price method of analysis was selected for both 2D and 3D limit equilibrium analysis.

Locally loaded slopes analysed by 2D plane strain methods assume loads of an infinite length as illustrated in Figure 3. In many realistic cases, however, the extent of the load is finite as illustrated in Figure 4. 2D plane strain analysis cannot account for this finite length and ignores the end effects of the 3D failure surface. The 3D modelling was carried out to investigate the FoS resulting from 3D failure surfaces, which are considered to be more representative compared with a 2D plane strain assessment.

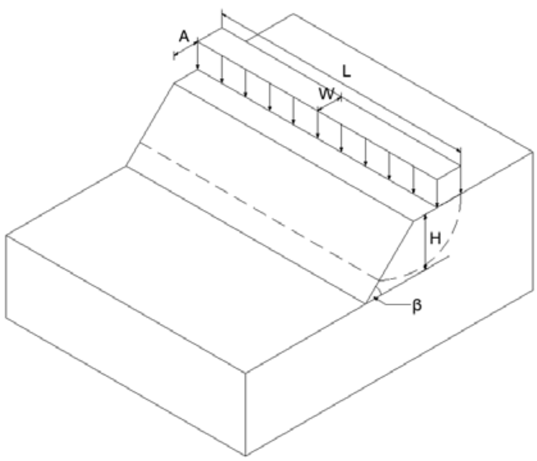


Figure 3. Plane Strain Analysis

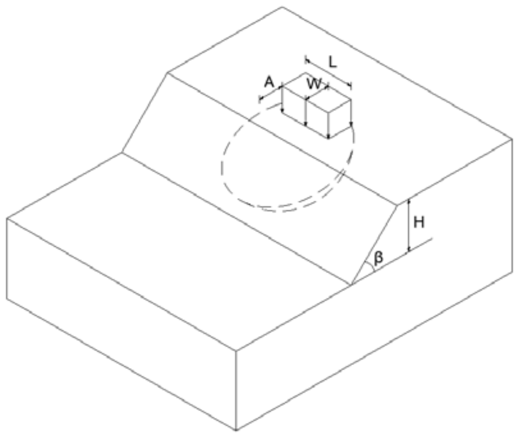


Figure 4. Three-Dimensional Analysis

4.3 INVESTIGATED PARAMETERS

Several slope geometries and material properties are investigated. The slope is assumed to be homogenous, and three different drained and undrained strength parameters are considered. The range of investigated parameters are summarised in Table 1. All combinations of strength parameters, slope angles and slope heights shown in Table 1 were considered for the drained and undrained analyses.

Table 1: Investigated Parameters

Material Type	Range of Drained Parameters c', φ'	Range of Undrained Parameters s_u	Range of Slope Angles β	Range of Slope Heights H
Drained	0 kPa, 30° 0 kPa, 35° 0 kPa, 40°	-	2H/1V 2.5H/1V 3H/1V	2m 5m 10m
Undrained	-	50 kPa 100 kPa 200 kPa	1H/1V 2H/1V 3H/1V	

In addition to gravity forces, the slope is subjected to a strip load of various lengths, *L*, and intensity of 300kPa. The surcharge acts at an edge distance, *A*, of 1.0m away from the crest of slope and has a loaded width, *W*, of 1.0m for simplicity. Pore-water pressure was not considered in any of the assessment.

The critical FoS resulting from the 3D ellipsoid analyses are then compared with those obtained with the 2D circular analyses.

5 RESULTS

5.1 HEIGHT OF SLOPE

The critical circular failures for the three different slope heights (H = 2m, 5m and 10m) considered are shown in Figure 5. It can be seen that all of the critical circular failures daylighted at the slope face within the top 2m of the slope crest. For the particular loading arrangements discussed in this paper, the failures are independent of the slope height.

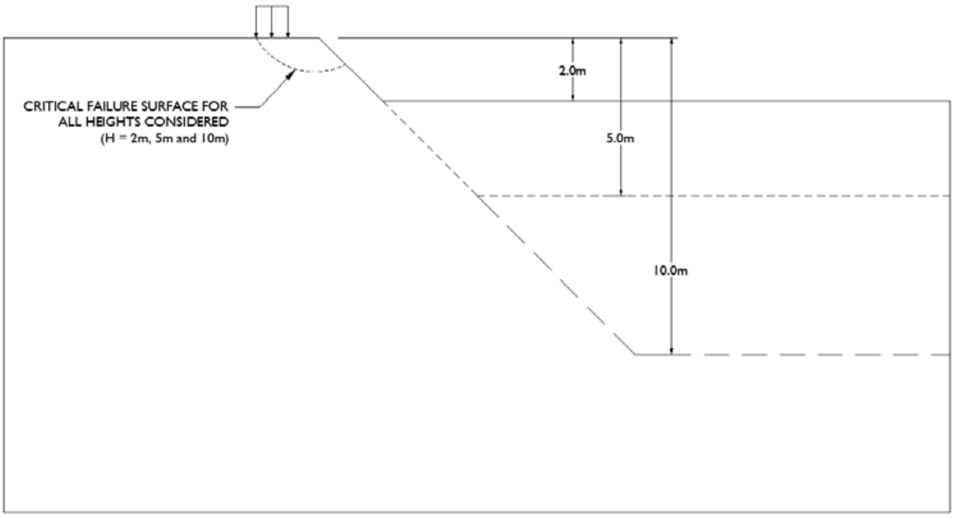


Figure 5. Critical Failure Surface for Various Slope Heights

5.2 COMPARISON CHARTS

Figure 6 shows the ratio of FoS obtained from the 3D and 2D analysis of a locally loaded slope as a function of *L* (where *L* is the length of the surcharge as shown in Figure 3 and 4) for several different slope angles and material strength properties.

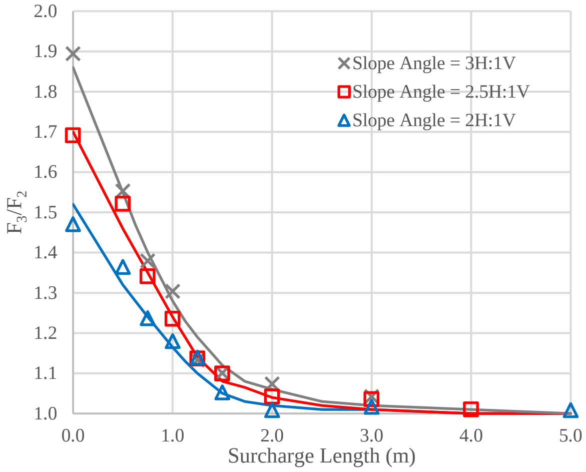
It can be seen that when *L* increases, the problem tends towards the 2D plane strain condition and the F_3/F_2 ratio approaches unity. This is expected; as *L* approaches infinity, the influence of the end effects on the 3D FoS becomes negligible. Another trend that can be recognised is that the steeper the slope, the lower the F_3/F_2 ratio. Since the volume of the failure mass is larger in a gentle slope, more end-effect is produced.

These trends generally agree with the observations made by Chen and Chameau (1982), Azzouz and Balign (1983), Leshchinsky and Baker (1986), and Cavoundis (1987)

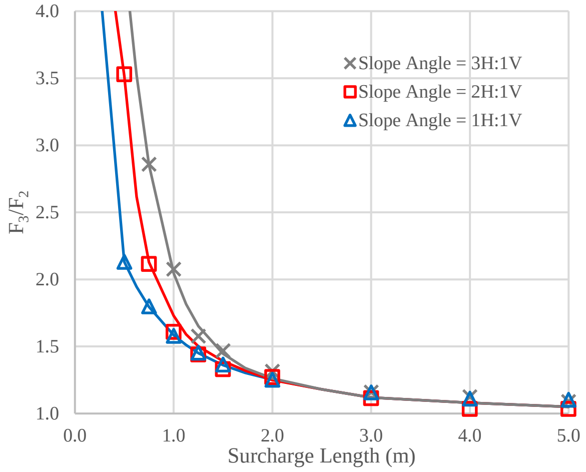
5.3 OBSERVATIONS FROM ANALYSES

The following general observations can be deduced from the graphs:

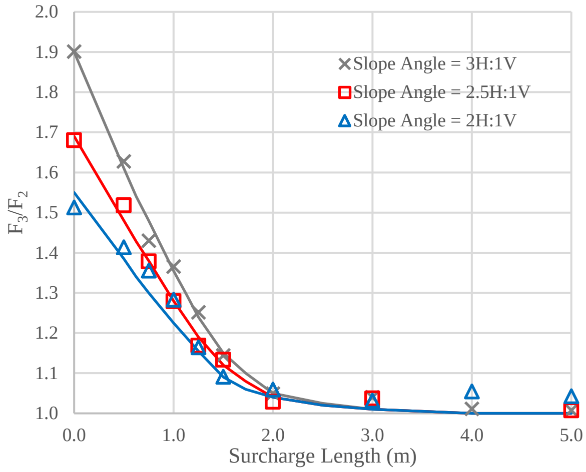
- For granular soils – when $L \leq 0.5$m, $F_3/F_2 \geq 1.35$
- For cohesive soils – when $L \leq 1$m, $F_3/F_2 \geq 1.5$
- For both granular and cohesive soils – when $L \geq 3$m, the end effects are negligible and $F_3/F_2 \approx 1.0$



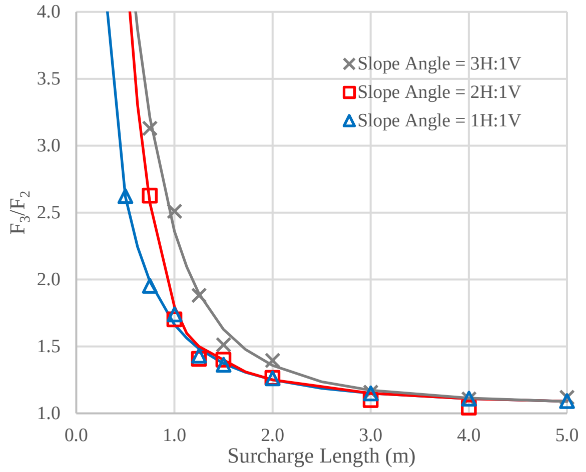
(a) c' = 0 kPa, φ' = 30°



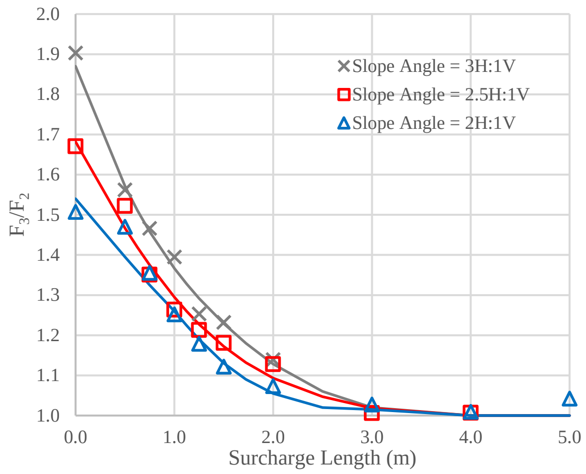
(d) s_u = 50kPa



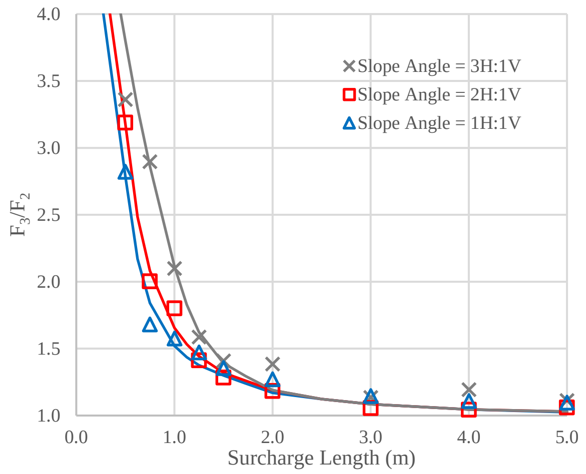
(b) c' = 0 kPa, φ' = 35°



(d) s_u = 100kPa



(c) c' = 0 kPa, φ' = 40°



(d) s_u = 200kPa

Figure 6. Comparison of FoS resulting from 3D and 2D Slope Stability Limit Equilibrium Analysis

6 CONCLUSIONS

This paper presented comparisons of the resulting FoS from 2D and 3D limit equilibrium modelling methods. The scope of this paper is limited to homogenous soil profiles and symmetrical slope stability problems, and a very specific loading arrangement has been considered. Therefore, it is important to note that the charts are only intended to provide the reader with an indicative increase in FoS when moving from 2D plane strain to 3D analysis. Readers are encouraged to consider the following points when dealing with concentrated sucharge in a slope stability analysis:

- The 3D effects are more significant for failures with shorter lengths of surcharge due to more pronounced end effects. For cohesive soils, up to a 50% increase in FoS is observed for surcharge lengths less than 1m and for granular soils, up to a 35% increase in FoS is observed for surcharge lengths less than 0.5m.
- 3D end effects are most prominent in slopes made of cohesive soil.
- For gentle slopes, the 3D effects are more significant.
- For both granular and cohesive soils, when the surcharge length exceeds 3m, the end effects are negligible and no significant increase in FoS is observed.

Only the computed FoS between 2D circular and 3D ellipsoid failures have been considered. The results and charts presented this paper should not be used for slopes governed by translational failures. For the loading arrangement considered in this paper, all of the critical failures daylighted in the slope face (above the toe failures). The charts presented in this paper are not applicable for base (below the toe) failures, which are typically associated with embankments overlying weaker materials.

The combined use of 2D and 3D analyses allow geotechnical engineers and engineering geologist to develop more realistic limit equilibrium models and provide better solutions for slope designs. However, constructing a realistic and representative 3D model is much more time consuming than producing a 2D model. Due to this reason, it is essential for geotechnical engineers and engineering geologists to approach the design in an efficient and economical manner. With the charts presented in this paper, one can predict the outcome of a 3D slope stability analysis simply from the 2D result and determine whether the invested time and resource will produce a viable design.

7 ACKNOWLEDGEMENTS

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