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Using Bender Elements to Measure Soil Stiffness at Small Strains in Laboratory Samples

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USING BENDER ELEMENTS TO MEASURE SOIL STIFFNESS AT SMALL STRAINS IN LABORATORY SAMPLES

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ABSTRACT

Over the last two decades improved laboratory techniques have shown that soil stress-strain responses are highly non-linear over the range of strains and stresses that are relevant around foundations and excavations. Accurate predictions of settlement and of the stresses in the ground depend on knowing this non-linear stress-strain response. One of the parameters needed to describe this response is the stiffness at very small strains. A variety of techniques are available to measure this stiffness. In this paper the bender element method is described. One of the attractions of using bender elements is that they can be easily incorporated into existing equipment, and at least in principle, the test is very simple to conduct. A high frequency pulse is applied to one bender element and the time for this to be detected at the other end of the test specimen is recorded. It is shown how bender elements can be incorporated into existing tests with minimal hardware requirements. A method of automating the test interpretation is presented which should enable reliable stiffness measurements to be obtained from commercial laboratory environments. This method makes use of cross correlation between input and output signals. It is argued that the approach is easy to integrate into existing test control programs, and it is shown how the technique may be implemented at relatively low cost.

1. INTRODUCTION

Accurate prediction of ground deformations around soil constructions of all kinds remains problematic. Over the last two decades it has become apparent that accurate predictions require a better understanding of soil's stress-strain response, which has been found to be highly non-linear over the range of strains (and stresses) that develop under working conditions. Many authors (e.g. Burland, 1989, Atkinson, 2000) have shown that the majority of the ground beneath and around foundations and excavations experiences direct strains of less than 0.1%, and if successful predictions of ground movement are to be made the behaviour at strains of 0.01% or better is necessary. A typical variation of secant modulus with strain is shown in Figure 1. The modulus is approximately constant for strains of less than 0.001%, and then drops significantly over the range of interest in the ground, between 0.001% and 0.1%. As pointed out by Burland (1989) these observations have far reaching practical and fundamental consequences. The non-linearity of the stress-strain response can have significant effects on soil-structure interaction, stress distributions, and displacement profiles around loaded areas and excavations. Also the interpretation of field measurements and in-situ tests needs to consider this non-linearity.

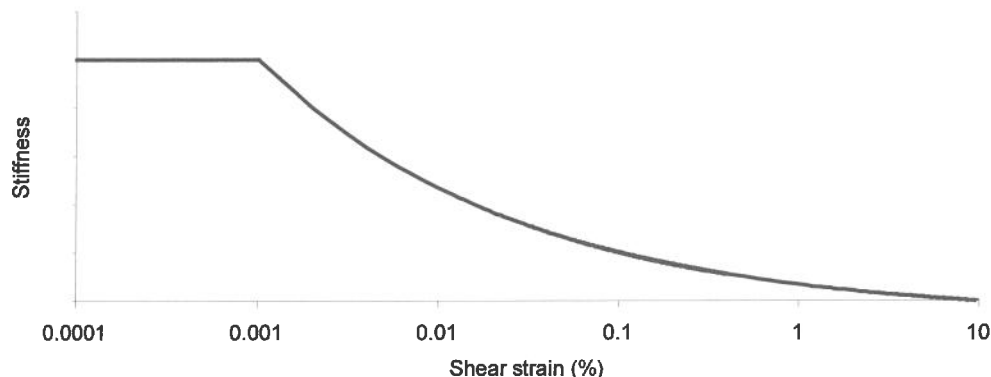


Figure 1 Typical tangent stiffness- shear strain response

Detailed design and analysis of major geotechnical structures taking account of soil non-linearity and other important features of soil behaviour is a complex task, but if this is done good predictions are possible (e.g. Jardine et al, 1991). For routine design Atkinson (2000) has suggested that the parameters needed are the stiffness at very small strains ($<.001\%$), E_o , the failure strength, and the strain at failure. From these parameters it is possible to estimate the variation of modulus with strain and hence obtain compatible loads and settlements. Atkinson (2000) also suggested that the modulus at small strains could be obtained using bender elements. He stated that, "in principle, these tests are sufficiently simple to perform and the results are sufficiently reliable for routine analysis". Since 1997 we have been using bender elements at Sydney University for tests on cemented sands. The tests are simple to perform, but obtaining reliable interpretation has been the subject of considerable research and development (Mohsin and Airey, 2003). We believe that we now understand how to obtain reliable results, and that it is possible for these to be obtained during routine tests. This paper gives a brief overview of bender elements: what they are, how they work, and how to interpret the results.

2. BENDER ELEMENTS

The concept of using piezo-ceramic bender elements to measure the shear wave velocity in soils is not new. It was first suggested by Shirley & Hampton (1978) and closer to home was reported by Meyer et al. (1996) at the 7th ANZ geomechanics conference. Over the last two decades many researchers have incorporated bender elements into triaxial, oedometer and direct shear apparatus. The increasing popularity of this procedure is because of the increasing importance attached to knowing the shear modulus at very small strains, its ease of implementation, and practical difficulties with the use of alternative on-sample measurements.

The principle behind the operation of the bender elements is that certain "piezo-electric" crystals will become polarized when stress is applied to them, or conversely when placed in an electric field the crystals will change their shape slightly. The bender elements used in most geotechnical laboratory applications, known as bimorphs, are comprised of two thin strips of piezo-ceramic material bonded together and typically have dimensions of $13 \times 10 \times 0.5$ mm. They are mounted in a slot in the end platens of the triaxial or other device, as shown schematically in Figure 2a. The bender elements must be encapsulated in an epoxy for waterproofing, and located in the slots in the end platens so that the bender elements extend about 3 mm into the sample. The gap between the slot and bender is filled with silicone sealant. The final arrangement is shown in Figure 2b.

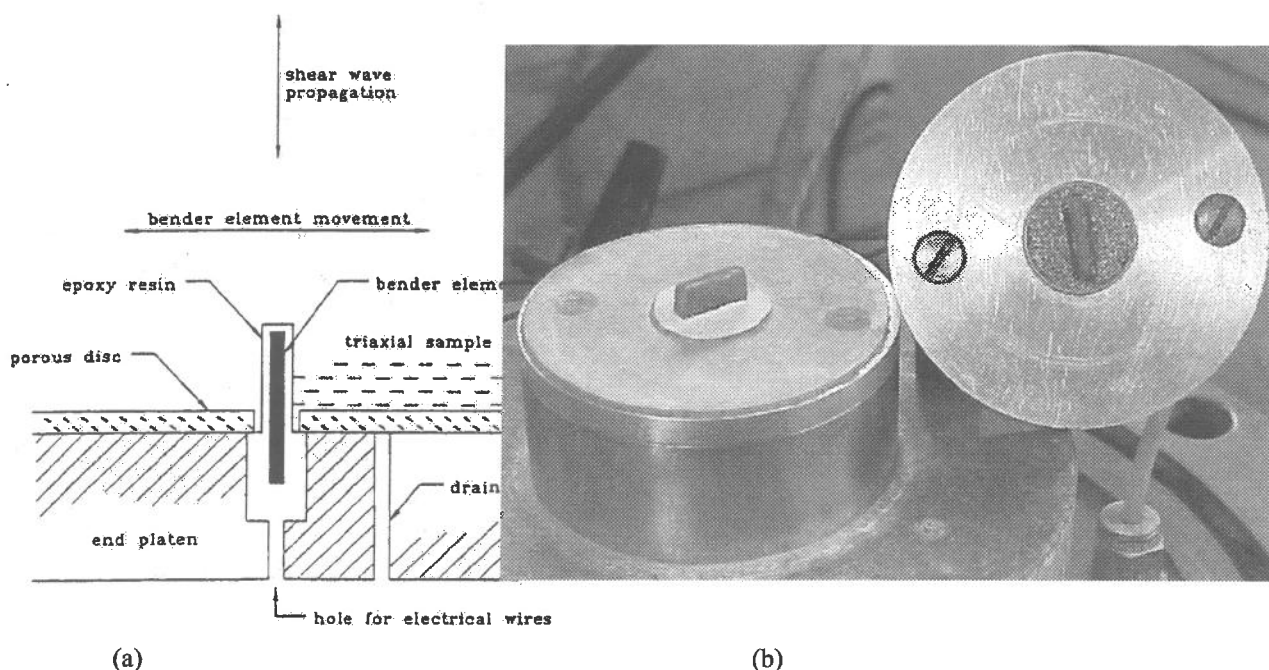


Figure 2 (a) section through end platen showing details of bender location (after Meyer et al, 1996), and (b) photograph showing bender elements mounted in 50 mm diameter end platens

The principle behind the interpretation of the bender element test is very simple. An electrical pulse is applied to one bender element, which is converted to mechanical energy and causes waves to propagate through the soil. A second bender element detects the wave and converts the mechanical wave to an electrical pulse. By measuring the time delay, T , between the applied and received pulses a shear wave velocity, v_s can be calculated knowing the length, L , between the tips of the bender elements. Hence the shear modulus, G , can be determined knowing the bulk density, ρ , from

$$G = \rho v_s^2 \quad (1)$$

Unfortunately, detection of the time delay is not straightforward and this has resulted in different researchers using a range of methods to obtain the “correct” arrival time. The following methods have been reported:

- Direct observation of the first arrival of the shear wave
- Direct observation from characteristic points (peaks, troughs) in the input and output signals
- Direct observation of the second arrival of the output signal
- Cross-correlation of input and output signals
- Frequency domain analysis to indirectly determine the phase angle, which is related to the time delay
- Frequency domain analysis using phase-sensitive detection techniques

Experimental and numerical results presented by Arulnathan et al. (1998) suggest that none of these methods are totally reliable, but under the right conditions all these methods are capable of giving the correct time delay (Mohsin and Airey, 2003). For all methods the factors that have the most impact on the accuracy of the estimated time delay are the frequency and shape of the input waveform, but there is no accepted procedure for selecting appropriate frequencies and waveforms.

Of the methods of estimating the time only those that analyse the complete waveform are suitable for automation, and hence of providing reliable and operator independent results. The method that has been developed at Sydney University and is discussed below is based on the cross-correlation of input and output signals.

3. EQUIPMENT AND PROCEDURE

The equipment required to implement bender element tests is shown schematically in Figure 3. Two arrangements are shown. Figure 3a is the conventional arrangement, which comprises a function generator to provide an input pulse and an oscilloscope to view both input and output traces. This is all that is required if the time is to be estimated from the traces on the oscilloscope. If any waveform analysis is to be carried out, as recommended here, then the information from the oscilloscope must be downloaded to a computer. To avoid the cost associated with a function generator and oscilloscope an alternative arrangement has been developed as shown in Figure 3b. This makes use of a computer sound card to generate the pulses and a computer analogue to digital (A-D) card to record the waveforms.

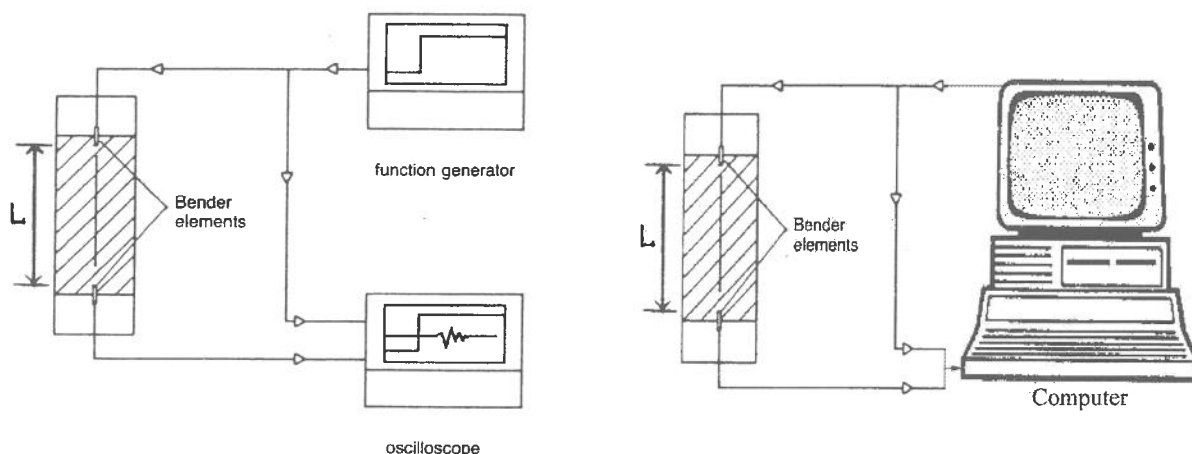


Figure 3 (a) Conventional equipment arrangement, and (b) Alternate low cost arrangement

The equipment we have been using consists of a 15 MHz HP33120A function/arbitrary waveform generator to provide excitation of the bender elements, and a Yokogawa DL1520L digital oscilloscope to monitor the input and received waveforms from the bender elements. An input signal amplitude of ± 10 V has been used. After passing through the sample the output signal has been amplified to give a peak-to-peak amplitude of approximately 8 mV. The pulses, typically a sine wave with a frequency of 10 kHz, have been sent repetitively from the function generator at a rate of 100Hz. To remove the effects of random noise the oscilloscope is set up to display the average of the last 256 signals. During a test the averaged responses on the oscilloscope can be automatically downloaded to the computer at regular intervals. If a sound card is to replace the function generator it needs to provide pulse frequencies of up to 20 kHz and this requires the latest generation of (96kHz) sound cards. We are using a SoundBlaster Audigy 2 card although other sound cards have similar technical specifications. The sound cards can be programmed to output any desired waveform, however, the approach we have followed is to make use of a freeware program "Soundarb" that emulates all the functions of the function generator that are needed for the bender elements. A 16 bit A-D card with a 200kHz sampling rate is used to record the waveforms. The same A-D card can also be used to monitor and record the load, displacement and pressure data in the triaxial tests. To overcome noise some averaging of the signals is required and this requires accurate timing, which can be provided by the sync output from the sound card. It has been found that satisfactory traces can be obtained averaging 25 records of the output signal, and taking a single record of the input.

For both equipment arrangements after the computer receives the data the cross correlation of the input and received signals is computed. This is achieved by obtaining the fast fourier transforms, $G(f)$ of the input signal $(X(t))$, and $H(f)$ of the response signal $(Y(t))$. The cross-correlation $CC(\tau)$ can then be calculated using

$$CC(\tau) = IFFT(H^*(f)G(f)) \quad (2)$$

where IFFT indicates the inverse fast fourier transform, and $H^*(f)$ is the complex conjugate of $H(f)$. The fast fourier transforms have been performed using freely available software routines (Press et al. 1986) that can be easily linked into existing data-logging and control programs. The times of the positive peaks in the cross correlation signal are then recorded. There is no need to save the waveforms, although as discussed below it is desirable to view the waveforms to assist in selecting the correct time delay and for quality control purposes.

When implementing bender elements it is important that the elements are aligned so that they bend in the same plane, and with the same polarity so that positive displacement gives a positive signal for both source and receiver. The polarity can easily be checked by placing the bender elements in contact, and this also enables a check to be made for any the time lag in the electronic equipment.

4. RESULTS

For two similar waves shifted in time the peak in the cross correlation signal gives the time delay. However, due to resonance of the soil-bender element system, wave dispersion, and wave reflections the peak in the cross correlation is rarely found to coincide with the correct time delay. Some authors have cited this, along with the dissimilar input and output waveforms, as a reason for not using cross correlation. Nevertheless, when dispersion is minimal and the soil is behaving as a linear system, as is assumed in interpreting the results, one of the peaks should correspond to the correct time delay (Oppenheim & Willsky, 1997). Therefore the procedure adopted has been to record the times of all the significant maxima in the cross correlation signal occurring at or before the peak correlation. Typical waveforms showing the input and output signals at a confining stress of 100 kPa are shown in Figure 4. The cross correlation is also included on this figure. The peak in the correlation is labeled 1, and the preceding maximum is labeled 2. In this case it can be seen that maximum 2 corresponds approximately to the time of the first arrival of the shear wave. In Figure 5 shown below it will be seen that there are many maxima in the cross correlation and the correct time delay can correspond to the third and occasionally fourth maximum preceding the peak. How the correct time delay can be determined is discussed in the next section.

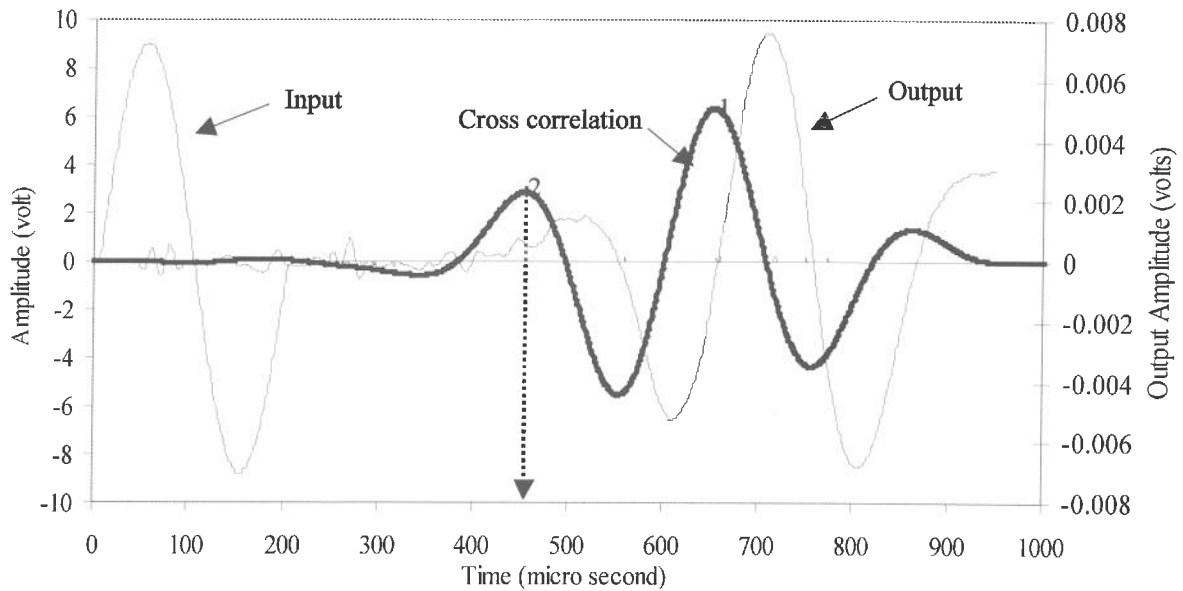


Figure 4 Typical bender element signals for Toyura sand, $p' = 100$ kPa, $L = 102$ mm

To demonstrate that the low cost arrangement using the sound card and computer A-D card perform satisfactorily a comparison is presented in Figure 5 of the input and output waveforms together with their cross correlation produced by: (a) the function generator and oscilloscope; and (b) the sound card and A-D card. Both plots show very similar input and output signals, and the times of the maxima, marked 1, 2, 3, from the cross-correlations are identical. The waveforms in Figure 5b are noisier than in Figure 5a because the output signal has only been averaged 25 times using the A-D card compared to 256 times using the oscilloscope. This appears to have no effect on the timing of the correlation peaks.

The output voltage in Figure 5b is shown to be similar to that produced using the function generator, but to achieve this some additional amplification of the signal was required. For a linear system, such as provided by the soil sample, the output amplitude is directly proportional to the input and the lower voltage from the sound card, ± 2.25 V, compared to ± 10 V for the function generator inevitably leads to a lower output signal. However, it should be noted that using a 16 bit A-D card and a sound card there is no need to have signal amplification despite the lower signal level. Amplification was only used here because the signals were being displayed on the oscilloscope, which had a lower resolution.

5. DETERMINATION OF CORRECT TIME DELAY

As already noted reliably determining the correct time is the biggest challenge when using bender elements. When using cross correlation there are two main factors to consider:

- The frequency and wave form provided by the function generator/sound card
- How to determine which is the right peak in the cross correlation

Other studies (e.g. Jovicic et al, 1996, Arulnathan et al. 1998) have reported that both the waveform and frequency can influence the identification of the correct time, particularly if using the first arrival of the shear wave. When using cross correlation it has been found (Mohsin and Airey, 2003) that if the frequency is greater than some critical value the timings of the peaks in the cross correlation do not change. The critical frequency depends on the shear modulus, and increases as the modulus increases. For sands subjected to a range of confining stresses between 20 kPa and 2000 kPa, corresponding approximately to shear moduli between 20 MPa and 400 MPa, the critical frequency varied from 8 kHz to 15 kHz. From these observations it appears that using as high a frequency as possible is desirable. However, the maximum frequency that can be used is limited by increasing attenuation in the soil, by the bender elements, which

cannot transmit pulses accurately at high frequencies (>25kHz but depends on bender element), and by the response becoming noisier and the first arrival becoming increasingly difficult to detect.

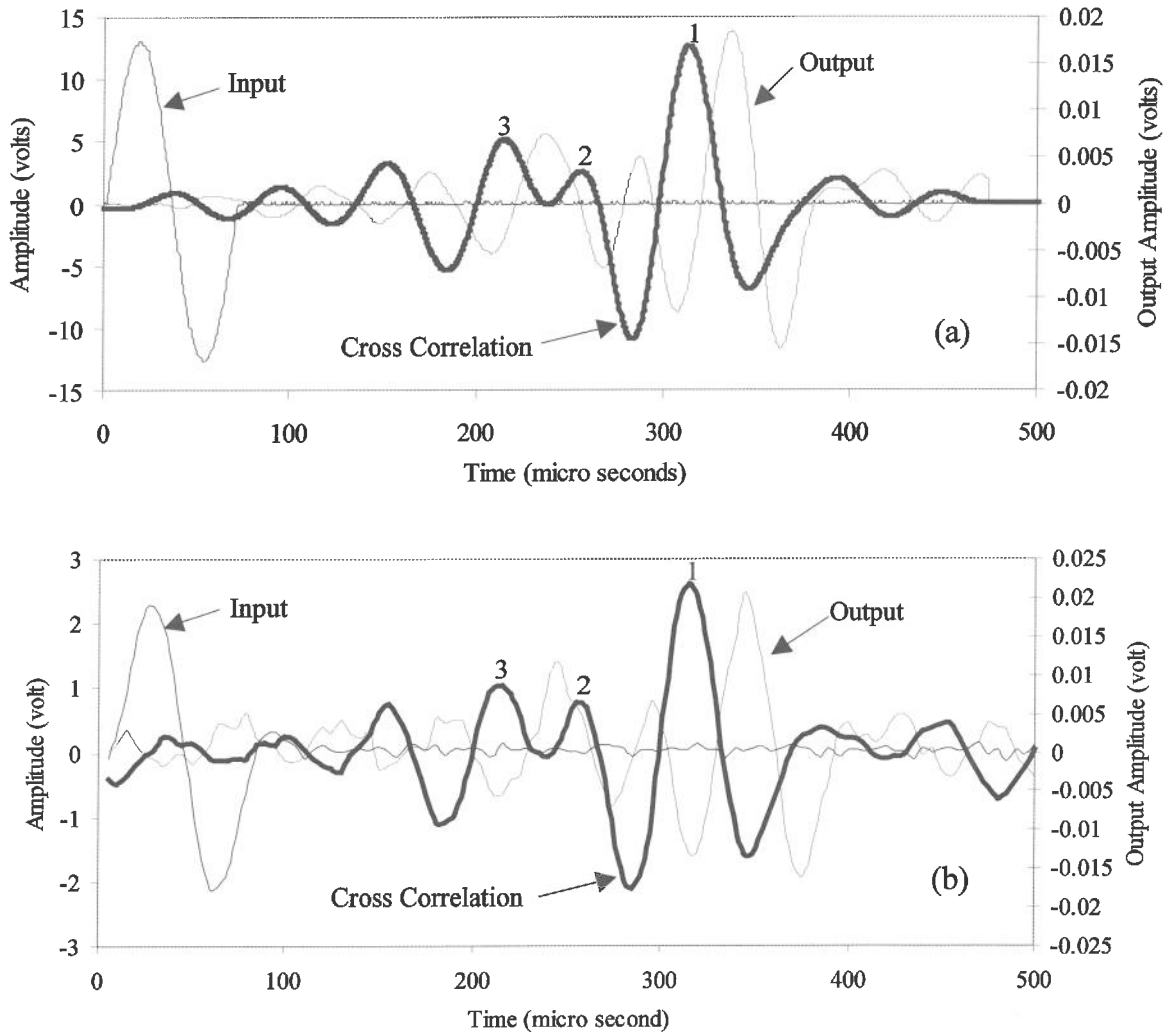


Figure 5 Traces from (a) Function generator/Oscilloscope, (b) sound card/A-D

A variety of waveforms have been investigated including single sine pulses, sine pulses with two or three waves, a triangular pulse and a chirp comprised of three sine waves of different frequency (Mohsin and Airey, 2003). All the pulsed waveforms give essentially identical travel times when the frequency is greater than the critical value. The best waveforms have been found to be a triangular input wave, which gave the least frequency dependent travel time, and a chirp waveform comprising a packet of sinusoidal waves with frequencies of approximately 20 kHz, 13 kHz and 8 kHz. Having a range of frequencies in the input signal is an advantage as this results in less variability in the estimated times and avoids the need to adjust the input pulse frequency during a test.

The major limitation when using cross correlation is that the times of several peaks in the cross correlation must be recorded because there is no simple way of determining which peak is giving the correct time. This is particularly a problem when testing sand as the first arrival can be difficult to detect, as for example in Figure 5. It has also been seen in Figure 4 that the maximum in the cross correlation giving the correct time is not necessarily associated with the greatest correlation. To resolve this problem the following approach has been used. The signals are correlated at regular intervals so that a continuous variation of the times of all the maxima is obtained. Figure 6 shows a typical result during isotropic compression of Toyura sand. At each confining stress 3 or more points are shown corresponding to the times of

Figure 6 Influence of confining stress on the timing of the maxima in the cross correlation signal

the various maxima in the cross correlation signal. The different symbols on the plot refer to the different peaks in the cross correlation. In general it has been found that the lowest monotonically varying value gives the correct arrival time. But, if only a small range of confining stress is considered this may not be reliable. To overcome this uncertainty it is recommended that the correct time at low confining stress be identified by adjusting the frequency until the arrival time can be confidently estimated. As the input frequency is changed it is possible to observe a small range of frequencies where the response signal has at least one peak-trough cycle at the same frequency as the input. In this "optimum" frequency range the time delay can be determined between corresponding peaks or troughs in the input and output signals. This can then be used to indicate which curve in Figure 6 is the correct one. At later points in the test only with this continuous variation is it possible to confidently detect the correct peak from the cross correlation data.

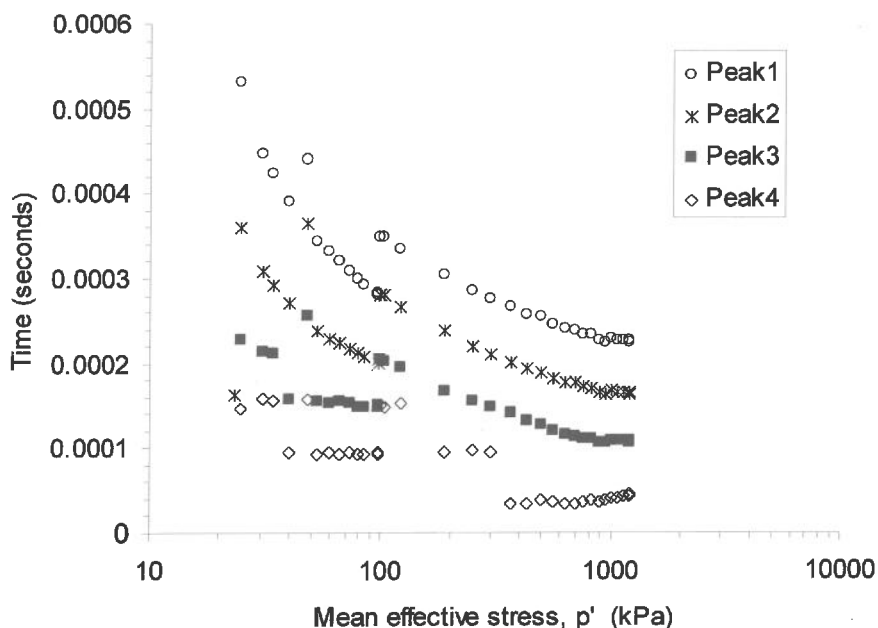
The cross correlation technique has been criticised because it is computationally intensive (e.g. Blewett et al. 1999) requiring the capture and analysis of complete waveforms and the use of fast Fourier transforms. However, with modern fast computers the time to capture the data and complete the calculations is not significant, and data storage is minimal as only the times of the peaks in the cross correlation need to be saved. It is also probably desirable to save a few waveforms to confirm the identification of the correct peak in the cross correlation and for quality control purposes.

Mohsin and Airey (2003) have shown that this method is capable of giving values of G_{max} that agree with values obtained by torsional shear and resonant column tests. As the method gives reasonable results and removes any operator dependence it is believed that the method outlined here is suitable for use in commercial laboratories.

6. SUMMARY

The stress-strain response of most soils is highly non-linear over the range of stresses and strains relevant to working conditions around foundations and excavations. The stiffness at very small strains is one of the parameters needed to define the stress-strain response. Using bender elements it is possible to measure the small strain stiffness during standard soil mechanics tests such as triaxial, oedometer, and simple shear. These devices are relatively cheap and are easy to install in the end platens of the test apparatus. It is possible to perform the tests making use of already existing hardware such as computer sound cards and existing data-logging equipment.

Obtaining reliable measurements from the method requires some care in the selection of frequency and waveform of the input pulse, and in the interpretation of the wave travel time. It has been shown that cross correlation can give reliable and operator independent results. However, it is important to realize that the time is obtained from one of the maxima in the cross correlation signal, and not necessarily the peak correlation as would be expected if the signals were simply



shifted in time. It has been found that the frequency of the input pulse must be above some critical value that increases with effective confining stress. To avoid having to adjust the frequency during a test a Chirp waveform comprised of sine waves with frequencies of 20, 13 and 8kHz has been found to work satisfactorily for sands with confining stresses from 20kPa to 2MPa.

The major limitation of this technique is that it is difficult to determine which is the correct peak in the cross correlation signal. The data show that the arrival times predicted by several peaks in the cross correlation vary monotonically during the test. To select the correct peak a visual check of the waveforms may be required at some stage of the test.

The technique is relatively cheap to implement and should be within the capabilities of commercial testing laboratories.

ACKNOWLEDGEMENT

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