

Reliability-based geotechnical design in an Australian context

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ABSTRACT

The design philosophy adopted by the international geo-engineering community over the last few decades has largely evolved from working stress design (WSD), in which a single or lumped factor of safety (FoS) is adopted, to a load and resistance factor design (LRFD) approach. In LRFD, partial factors are applied to actions (i.e. loads), soil parameters and / or resistances. These partial factors vary in magnitude depending on the relative uncertainty of each parameter to which they are applied. The approach adopted in Australia has differed, with both WSD and LRFD approaches being widely used. In the authors' experience, there is limited awareness in the Australian geo-engineering community of the relationship between the concept of reliability and the partial factors adopted in LRFD, and therefore of the potential benefits of undertaking reliability-based design (RBD). The outcome of this is that RBD, in which the uncertainty of the variables which may affect the design is individually assessed, is rarely undertaken. This paper discusses the concept of RBD and its place within the framework of Australian Standards and presents practical means of adopting RBD with accompanying examples from the literature. The intention of the paper is to encourage practitioners to consider uncertainty in geotechnical design more rigorously, whilst acknowledging the importance of maintaining engineering judgement in design.

Keywords: geotechnical design, working stress, limit state, reliability.

1 INTRODUCTION

At its simplest, the purpose of any geotechnical design is to ensure that the risk of failure is sufficiently low to be deemed acceptable. Failure may be the catastrophic collapse of a retaining wall for example, but could also be an unacceptable degree of cracking in a building due to the excessive movement of the footing system.

Historically, the geo-engineering professional has considered this risk by applying a lumped FoS to a calculated ultimate resistance, i.e. WSD. This methodology has largely been replaced internationally by LRFD, in which various factors are applied to actions (including loads), soil parameters and / or resistances, the magnitude of which are typically defined in national design standards and are a function of the expected relative uncertainty of the variable under consideration. The introduction of LRFD brought with it a lexicon which was new (and in some cases, potentially unnecessary), and which remains frustrating to many geo-engineering professionals.

If LRFD is considered to be an evolution of WSD, RBD may be considered to be an evolution of LRFD. RBD involves the rigorous assessment of the uncertainty of the variables on which a design is dependent. RBD has been discussed in the literature for over 40 years, but its adoption as a design methodology has been slow. However, it is now included in some international design standards, including the Canadian Highway Bridge Design Code (Canadian Standards Association, 2014).

This paper sets out the current state of geotechnical design in Australia, describes the relationship between LRFD and RBD, and presents ways in which RBD can be adopted in practice.

2 THE STATE OF GEOTECHNICAL DESIGN IN AUSTRALIA

National Standards exist to ensure that the balance between cost and safe / serviceable structures is appropriately achieved, and that a similar level of reliability is achieved in design undertaken by different practitioners. The state of geotechnical design in Australia, within the framework of national design standards, is somewhat unusual for a number of related reasons.

Firstly, whereas the UK, previously through BS 8004 (BSI, 1986) which encompassed all types of foundations and now through Eurocode 7 (BSI, 2004), and the United States through AASHTO (2017), have complete and overarching design standards, there are a limited number of Australian Standards for the design of various geotechnical elements, for example AS2159 (Standards Australia, 2009) for piled foundations, AS2870 (Standards Australia, 2011) for residential slabs and footings, and AS4678 (Standards Australia, 2002) for (predominantly gravity) retaining walls.

Secondly, relevant Australian Standards have not been published for commonly encountered design scenarios such as slopes and shallow footings including raft foundations for high rise buildings (other than residential slabs and footings, as described above), although AS5100.3 (Standards Australia, 2017a) is sometimes adopted for shallow footing design for infrastructure projects.

Thirdly, whilst most international design standards have moved from WSD to LRFD, Australia has effectively maintained both approaches. For example, pile foundations are typically designed using a limit state approach to AS2159, whereas slopes are typically designed using a lumped FoS approach.

To further complicate matters, the analysis tools commonly adopted in geotechnical design have changed over the last number of years such that parts of the national design standards have lost some of their relevance. For example, embedded retaining walls are now commonly designed using tools such as WALLAP and PLAXIS 2D. However, both AS4678 (Standards Australia, 2002) and AS5100.3 (Standards Australia, 2017a) provide insufficient guidance on how such soil-structure interaction analyses should be undertaken. Although beyond the scope of this paper, the reader is directed to Haberfield (2017), for example.

3 THE RELATIONSHIP BETWEEN RBD AND LRFD

3.1 Introduction

As stated in Section 2, design must balance cost with the safety / serviceability of a structure. Regardless of the various design methodologies which may be adopted, or whether the design is a structural or a geotechnical one, an appropriate design must establish that the design resistance is not less than the design actions (applied loads).

The degree of safety of a structure has traditionally been assessed in a deterministic manner by engineering professionals in terms of a calculated FoS. The engineering professional undertakes this assessment using either what he / she considers to be appropriate values for applied loads and resistances to take account of their potential variability, or what the relevant design standard mandates. This is illustrated in Figure 1, in which the margin of safety is defined as the difference between the resistance and the loads. However, this methodology neglects to robustly consider the potential variability of both the applied loads and the resistances.

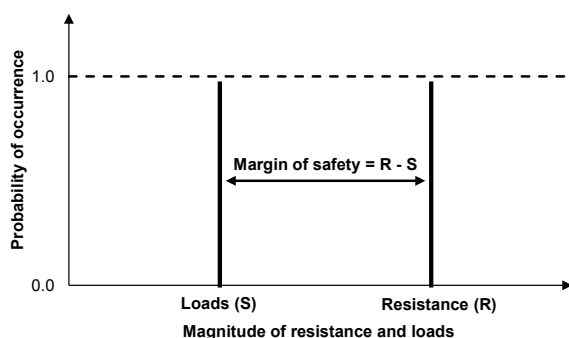


Figure 1. Graphical illustration of margin of safety for WSD (after Becker, 1996)

3.2 Load and resistance factor design

LRFD requires the engineering professional to demonstrate that serviceability and ultimate limit states (SLS and ULS, respectively) are not exceeded. In simple terms, the purpose of the SLS check is to assess if movement beyond an allowable limit occurs, whilst the purpose of the ULS is to ensure that collapse will not occur.

As set out in Becker (1996b), the partial factors which are applied in LRFD may be based on calibration using one or a combination of:

- Previous experience.
- WSD approach.
- Reliability theory (as discussed in Section 3.2).

One of the benefits of LRFD over WSD is that the engineering professional is required to apply partial factors to individual components of load and resistance, thereby allowing the higher risk of variation in (say) live loads compared to (say) the self-weight of concrete to be considered in the design. This is not possible in WSD, as a single lumped FoS is applied.

LRFD has been subject to criticism by Australian practitioners, for example Day (2001), Pells (2011), Day *et al.* (2007), and Wong *et al.* (2007). It is acknowledged that many of the arguments made in these references are valid, but note that a discussion of these arguments lies outside the scope of this paper. It is noted that despite the criticisms set out in these references, Wong *et al.* (2007) state that "...despite these difficulties, there is definitely a place for limit state design in geotechnical engineering" and "the use of limit state design...has benefits as long as the underlying design principles and soil-structure interaction effects are properly understood and communicated".

3.3 Reliability based design

Reliability theory was introduced initially as a design concept for structural engineering, and subsequently for geotechnical engineering, primarily to counter this lack of robust consideration of the potential variability. The use of reliability theory in the field of geotechnical engineering is not new – it was the subject of the 17th Terzaghi Lecture in 1981 (Whitman, 1984).

The requirement for this more robust consideration is discussed below with reference to Figure 2. This has previously been presented in the literature (including Becker, 1996a). Loads and resistances are variables and are not defined by a single value as considered in the deterministic approach described in Section 3.1. Sources of geotechnical uncertainty include spatial variation, measurement error, transformation error (for example, the calculation of undrained shear strength of a cohesive material from Cone Penetration Test data), and model calculation uncertainty (i.e. the degree to which the analytical model represents the true behaviour of the system).

By plotting both the load and resistance frequency distributions on a single plot, the relatively low probability of a combination of high loads and low resistance which would result in failure is represented by the intersection of the load and resistance frequency distributions.

In Figure 2:

- Cases (a) and (b) are two curves representing the frequency distributions of resistance which have the same design value, but differing standard deviations, noting that the standard deviation for Case (a) is less than that for Case (b).
- Case (a) represents a condition whereby the resistance is well defined (i.e. the frequency distribution curve is relatively narrow). This could be the case where a significant amount of geotechnical investigation has been undertaken at a site and where there is relatively small variability in the geotechnical data. The area of intersection (in green) of the load and resistance curves is relatively small and therefore the probability of failure is relatively low.
- Case (b) represents a condition whereby the resistance is not well defined (i.e. the frequency

distribution curve is relatively broad). This could be the case where little geotechnical investigation has been undertaken at a site and where there is relatively high variability in the geotechnical data. The area of intersection of the load and resistance curves (the combined green and red areas) is relatively large and therefore so is the probability of failure.

- The higher probability of failure of Case (b) compared to Case (a) is not captured by the traditional FoS approach, as the mean margin of safety (and therefore FoS) for both Cases (a) and (b) is the same.

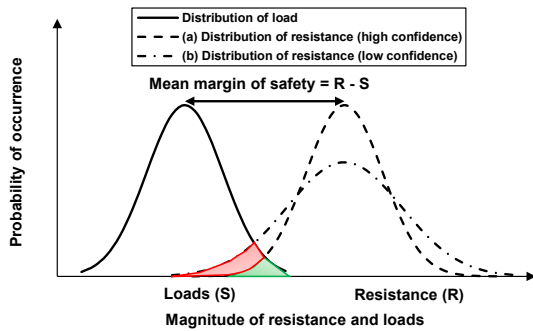


Figure 2. Variation of loads and resistance (after Becker, 1996a and Green, 1989)

The intention of RBD is to provide adequate confidence that the probability of failure (P_f) is sufficiently low. RBD allows a more consistent assessment of reliability to be calculated compared to either WSD or LRFD. Although the theory of reliability analysis lie beyond the scope of this paper, reliability analysis requires assumptions to be made in relation to the frequency distributions of the loads and resistances (for example the normal distributions shown in Figure 2), and therefore the P_f calculated should not be considered to be the “true” probability of failure, but a reasonable estimate. Furthermore, as with any design, reliability analysis requires the engineering professional to use appropriate means of calculation, interpretation of geotechnical data, etc..

As an alternative to P_f , the likelihood of failure may be expressed in terms of the reliability index (β), which is a description of the safety of the structure normalised by its uncertainty. A higher β value indicates a more robust design. The target β for a specific structure should be based on a number of considerations, but primarily the consequences of failure (for example, risk to life and economic consequences). The target β for Australian structures for various considerations is set out in AS5104 (Standards Australia, 2017b), and which includes an appendix focussed on geotechnical engineering.

The concept of β is explained in Low (2005) with reference to Figure 3 and the First Order Reliability Method (FORM, also known as the Hasofer-Lind method, Hasofer and Lind, 1974), demonstrated using the example of the rotational failure of a gravity wall, as follows:

- The ellipses in Figure 3 represent combinations of the effective angle of friction (ϕ') of the retained material and of the soil-wall interface friction angle (δ) which have the same probability of occurring concurrently.

- The ellipses are centred about characteristic values of ϕ' and δ .
- The “critical combination” is the most probable combination of ϕ' and δ which would result in failure, and is the shortest distance on the plot from the characteristic values of ϕ' and δ to the failure surface.
- β is defined as the quotient of the shortest distance from the centre of the ellipse to the failure surface (R in Figure 3) and one standard deviation (r in Figure 3).

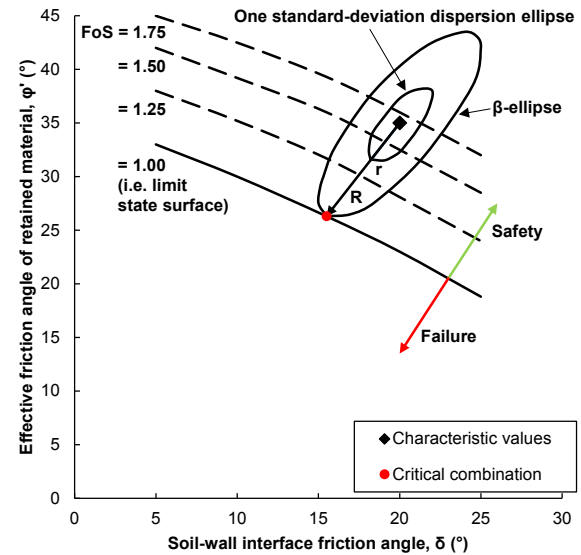


Figure 3. Definition of β (after Low, 2005)

More than two variables can be considered in reliability analyses, the first step being the replacement of the ellipses by ellipsoids in three dimensions. However, to allow graphical representation in two-dimensional space, the example above has been limited to two variables only.

If for some design scenario the calculated β value was too low, the engineering professional could increase the strength and size of structural members while leaving the uncertainty changed. This would result in an increase in the distance from the characteristic points to the failure surface (R) and leave r unchanged, thereby increasing the calculated β value. An alternative solution would be to leave the strength and size of structural members unchanged and decrease the uncertainty, for example by undertaking more and / or higher quality geotechnical investigations. This would cause a reduction in the radius of the one standard-deviation dispersion ellipse (r), also resulting in an increase in the calculated β value.

The partial factors presented in national design standards and which are required to be adopted in LRFD may have been (but are not always) calculated based on reliability analysis, as discussed in Section 3.2. The benefit of the presentation of partial factors within national design standards is that the engineering professional is not required to undertake reliability analyses for a design, and can instead use the provided partial factors to achieve the required level of reliability. However, in deriving partial factors to be applied for all potential scenarios to achieve adequate reliability, reasonable ranges of uncertainty must be considered. This means that in some design situations, the partial factors presented in national design standards will result

in a β for a structure which is greater than that required, i.e. the resulting design will be overly conservative. A site-specific RBD therefore presents the engineering professional with the potential opportunity to adopt lower partial factors, if:

- The engineering professional has the expertise to undertake such an analysis,
- Sufficient geotechnical investigation data are available to undertake a valid statistical assessment of the data, and
- The relevant national design standard allows it.

4 UNDERTAKING RBD IN PRACTICE

4.1 Introduction

The purpose of Section 4 is to demonstrate the way in which RBD may be adopted in practice. Practical examples of RBD for various commonly encountered geotechnical design scenarios are summarised.

Whilst there are many examples in the literature of reliability assessments which have been undertaken using both non-commercially available finite element software (for example, Fenton and Griffiths, 2003), and bespoke software coupled with commercially available software (for example, Schweckendiek *et al.*, 2007), it is the authors' opinion that these are unlikely to be practical for the majority of consulting geo-engineering professionals considering the current level of maturity of RBD in both Victoria and Australia more widely. The focus has therefore been on more routine examples.

4.2 Design approach and methodologies

Fenton *et al.* (2016) present a general methodology for the adoption of RBD which is common to many geotechnical design scenarios. This methodology includes the following (albeit high-level) steps:

- Identify the limit state under consideration, for example bearing capacity of a shallow footing.
- Describe the relevant parameters (for example effective cohesion, c' and ϕ') statistically by using appropriate frequency distribution curves.
- Use a numerical model to assess for many scenarios if the limit state is exceeded.
- Assess the probability of failure based on the number of cases which result in the limit state being exceeded.

Of the above steps, a typical geo-engineering professional will likely be least familiar with the assessment of frequency distribution curves and with the identification of a suitable numerical model to assess the limit states for the purposes of a reliability analysis.

An assessment of an appropriate frequency distribution requires a working knowledge of statistics. Ideally, the geo-engineering professional would have a thorough understanding of the site conditions from an appropriate site investigation, with many data points to allow an appropriate frequency distribution to be identified (e.g. normal, log-normal, etc.). However, this is not always practical and so the geo-engineering professional may use information presented in the literature in relation to the mean and coefficient of variation of various soil properties (refer Uzielli *et al.*, 2006 for example). It is noted that the coefficient of variation (COV) is the

quotient of the standard deviation and mean of the sample, and is a description of its relative dispersion.

Baecher and Christian (2003) present a summary of various numerical methodologies which can be adopted for the practical adoption of RBD, some of which are summarised below.

- First Order Second Moment (FOSM) method.
- Point Estimate method.
- FORM.
- Monte Carlo simulation.

Many geo-engineering professionals will reasonably endeavour to adopt reliability based methodologies within the software with which they are familiar and which is commonly adopted in the industry. The SLOPE/W software, for example, allows the user to undertake Monte Carlo simulations. Whilst the application PROBANA has been used in the past to undertake FORM and Monte Carlo analysis coupled with PLAXIS 2D, the authors understand that this application is neither commercially available nor supported by more recent versions of PLAXIS 2D (Bentley Systems, personal communication, 10 August 2021).

4.3 Design examples – shallow footing

Cherubini (1990) presents a closed-form solution for the probabilistic evaluation of the bearing capacity of a shallow foundation for a cohesionless soil with $\phi' \leq 35^\circ$. ϕ' only is considered as an independent variable. The frequency distribution of the ultimate bearing capacity, calculated using numerical integration, is presented for various standard deviations of ϕ' .

Low and Tang (1997) present a more sophisticated reliability analysis of a shallow footing. As the methodology is similar to that set out in Low (2005) and discussed in Section 4.5, this is not discussed further herein.

4.4 Design example – pile group

Randolph and Buttlng (2022) demonstrate how the Monte Carlo method, scripted using Python, can be applied to undertake a probabilistic analysis of a pile group analysed using the spreadsheet program PIGLET. In the example presented by Randolph and Buttlng (2022), a number of inputs to the pile group analysis are treated as random variables, including the shear modulus (both axial and lateral) of the soil in which the piles are constructed, the axial capacity of the piles, and the normalised lateral pile head displacement at which the secant stiffness is 50% of the initial tangent value. As the analysis undertaken was a test case (and not a design to be constructed), the range of the random variables used as inputs to the probabilistic analysis was expressed through the use of published COV data, rather than a statistical assessment of site-specific geotechnical investigation data.

The Python subroutine was used to carry out 100,000 analyses for 38 load cases, which took approximately 150 minutes on a standard computer. It was found that the pile group lateral displacement limit of 50 mm (considered as a ULS case) was exceeded by 0.33% of the simulations, giving a β value of 2.72. This compares with a value of 3.1 recommended in AS5104 (Standards Australia, 2017b) for Class 2 structures with a high cost of safety measures. A typical bridge such as a highway

bridge would probably be defined as a Class 3 structure in which case β would be required to be 3.3. As noted above, for the test case of the subroutine the site specific soil data was not used, but it was considered that β could relatively easily be raised to 3.3 if required, such as by a better site investigation, for example using a dilatometer or pressuremeter to measure the in situ modulus and its variability.

4.5 Design examples – retaining walls

Low (2005) presents reliability analyses of two types of retaining walls which were undertaken using FORM within Microsoft Excel and its built-in optimisation program application Solver. The reliability analyses were undertaken on both a gravity retaining wall (Figure 5) to assess the ULS failure modes of rotation and sliding, and an anchored retaining wall (Figure 6) to assess the ULS failure mode of rotation. Low (2005) provides detailed information on how the methodology was undertaken and includes a link from which the relevant Microsoft Excel files may be downloaded.

For the case of the gravity retaining wall, limiting equations for rotation and sliding (referred to as performance functions) are derived from basic physics and geo-mechanics. Three variables are considered: ϕ' , δ and base adhesion. The degree of correlation between ϕ' and δ is also considered, and the degree to which the reliability is sensitive to each variable is assessed. β is calculated using the Solver application described above.

For the case of the anchored retaining wall, nine variables were considered in the assessment of the required wall embedment to achieve a target β . These variables included soil unit weight, surcharge pressure, depth to groundwater and the yield force of the anchor. In a similar manner to the gravity retaining wall case, the equations for force and moment equilibrium are used as performance functions, and a reliability assessment undertaken.

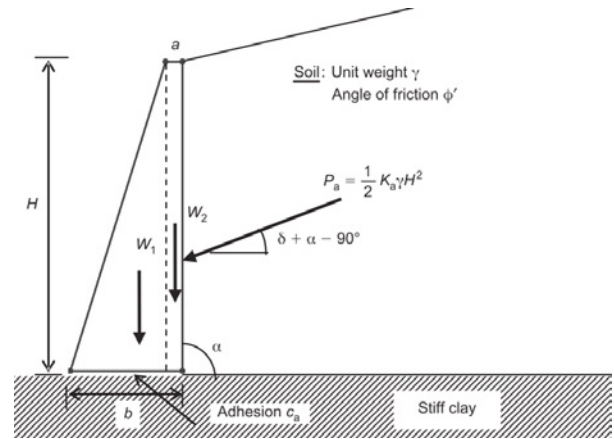


Figure 5. Reliability analysis of gravity retaining wall (from Low, 2005)

Low (2005) concludes that the calculated β values using FORM are in good agreement with those calculated from an alternative Monte Carlo simulation, but with significantly less computational effort.

5 DISCUSSION AND SUMMARY

If RBD is to be adopted more widely in the geotechnical industry in Australia, a number of issues need to be addressed. These include:

- Increased focus on appropriate statistics in undergraduate geo-engineering courses.
- Upskilling of the knowledge of relevant statistics of currently practising geo-engineering professionals.
- Further research into the benefits that RBD has over WSD and LRFD, and clear communication of these benefits to clients such that the additional high-quality site investigations required for RBD are approved.

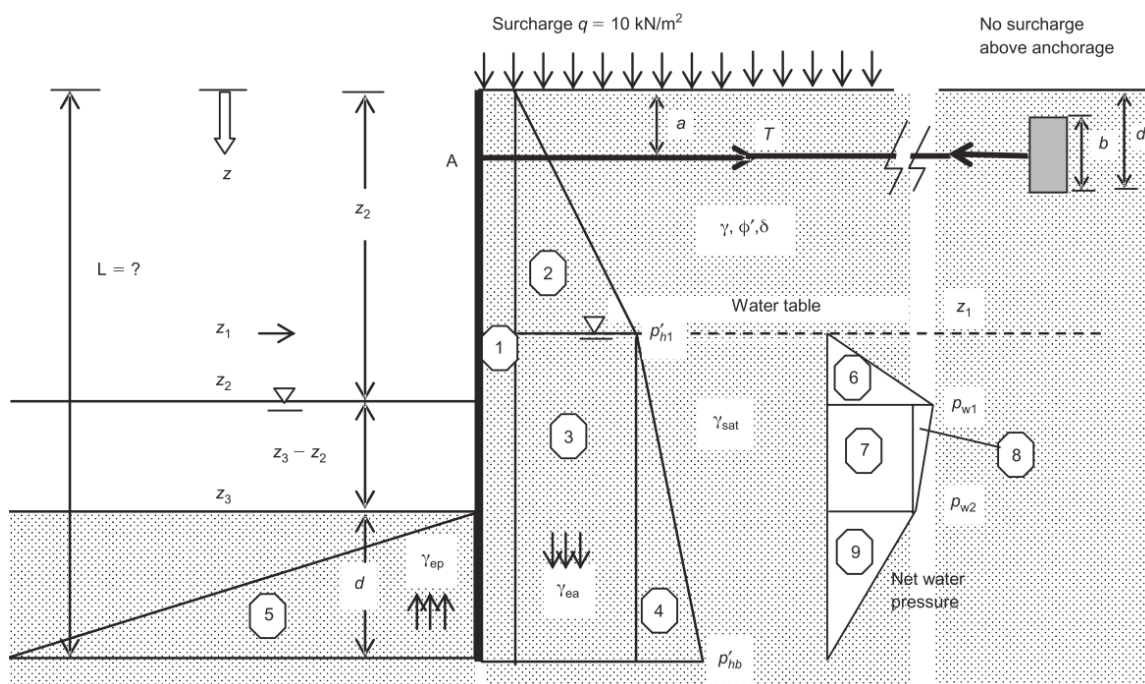


Figure 6. Reliability analysis of anchored retaining wall (from Low, 2005)

- The improvements in recent years in the way in which electronic site investigation data are gathered and manipulated has allowed significant efficiencies to be gained in the management of geotechnical data. The time and budget gained through these improved efficiencies needs to be re-invested in more sophisticated interpretation methodologies to allow RBD to be undertaken.
- Greater engagement with RBD by the Australian geo-engineering community.
- Increased co-operation between industry and academia in relation to how RBD may be adopted in practice, e.g. the Recent Trends in Geotechnical and Geo-Environmental Engineering and Education (RTGEE) workshops.
- Development of an overarching Australian Standard for geotechnical design, similar to that which exists in Europe and the USA, which addresses the concept of RBD.

Subject to the above, the authors consider that RBD has the potential to be a highly useful design tool for the geo-engineering professional. Notwithstanding this, the authors consider that, as described by Becker (1996a):

- Whilst probability and statistics are useful tools if properly applied, they must not become a substitute for trying to understand the behaviour of geotechnical materials.
- Geotechnical design must not become overly distracted by levels of safety and their quantification, but instead focus on the understanding of the basic failure mechanisms and fundamental material behaviour.

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