

FINITE AND BOUNDARY ELEMENT (FAB) METHOD : THEORY & APPLICATION

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ABSTRACT

Several methods for analysis of the effect of tunnelling-induced ground movements on pile foundations are available in the literature. The methods range from empirical, two-dimensional (plane strain), semi-analytical to purely three-dimensional analysis. A recent addition in the literature is the combined 3-D FE and Boundary Element (FAB) numerical method proposed by Surjadinata *et al.* (2004). This paper briefly describes this method and illustrates the degree of accuracy of the FAB method. Furthermore, the recent experience of using the FAB method for a tunnelling project in Melbourne is discussed. This discussion demonstrates the efficient and practical use of the FAB method for examining the impact of tunnelling on pile foundations.

1 INTRODUCTION

One of the main concerns that arise from tunnelling in an urban setting is the effect of the induced ground movements on surrounding structural foundations along the proposed tunnel alignment. The foundation types considered in this paper are pile foundations, which are commonly used to support high rise buildings and bridges in many major cities.

Several methods have been suggested for predicting the effect of tunnelling-induced ground movements on pile foundations, ranging from those that are two dimensional (2-D) and semi-analytical (e.g. Loganathan, 1998) to purely three dimensional (3-D) numerical solutions (e.g. Mhroueh and Shahrour, 2002).

Although they have proven useful, the limitation of most of the available 2-D prediction methods is that the problem is essentially 3-D. However, it is common knowledge that a full 3-D analysis, even with the current state of computing power, still is relatively time consuming.

There is a need for an efficient, yet rigorous, method for predicting the influence of tunnelling on existing single piles and pile groups. The aim of this paper is to demonstrate a method of analysis that can overcome the need for a full 3-D numerical analysis for every scenario in a tunnelling project and yet still take into account the 3D nature of the problem. To achieve this, a summary of the Finite and Boundary (FAB) element numerical method proposed by Surjadinata *et al.* (2006) is presented. A brief demonstration of the degree of accuracy that can be achieved with this method is presented by comparing predictions from the proposed method with published field measurements and conventional 3-D FE solutions. The efficiency of the method will be demonstrated by briefly examining the use of the FAB method in a recent \$230 million dollar project in Melbourne, Victoria.

2 OVERVIEW OF FAB METHOD

The finite element (FE) method has become virtually a standard analytical tool for practicing engineers to carry out analyses of tunnelling problems. However, the cost may be very high if a conventional 3-D FE analysis is conducted for each and every possible scenario of interest. If non-linear behaviour of the soil surrounding the tunnel and complicated construction sequences are to be taken into account the cost may become prohibitively high.

By combining finite element predictions of tunnelling-induced free-field ground movements from a model without any pile, with a separate boundary element analysis of a single pile foundation or a pile group, a large number of cases can be analysed efficiently. Chen *et al.* (1999, 2000) previously demonstrated the efficacy of this type of approach, but the ground movements input to their boundary element analysis were empirically derived. In the method proposed here, for relatively uniform ground conditions only a single 3-D FE analysis is required for each tunnel configuration, independent of the multitude of configurations of pile foundations that may be of interest. This method, described previously in detail by Surjadinata *et al.* (2004) and designated as FAB (combined finite and boundary element method), therefore has the potential to generate economical predictions for a very large number of cases of practical interest, enabling practicing engineers to investigate many more cases than are viable at present by conventional 3-D FE analyses.

2.1 FINITE-ELEMENT ANALYSIS OF SOIL MOVEMENT

The first step in the analysis of tunnelling interaction with piles is to predict the free-field soil displacements induced by a tunnelling operation. As a first approximation, this prediction can be achieved using a linear elastic 3-D finite element analysis. However, more sophisticated models of soil response could also be used with this approach. In this paper, the AFENA finite element package (Carter and Balaam, 1995) was used for this purpose. Since the linear elastic finite element method is well known, it will not be discussed here. The finite element model does not include a pile adjacent to the tunnel, but rather it calculates the tunnelling induced soil displacement (where the pile would be) but without the pile in place. A typical 3-D FE mesh for this purpose includes from 2,000 to 4,000 twenty node isoparametric hexahedral elements.

Simulation of tunnel excavation can be achieved in broadly two ways. In the first, each excavation stage is simulated by removal of elements from the finite element mesh to create the appropriate increment of tunnel void. Removal of these elements means their stiffness no longer contributes to the global stiffness of the soil. Appropriate force increments are then calculated and applied to the new tunnel boundary to simulate complete removal of stress from the surfaces created by excavation. Thereafter, in the incremental analysis this surface remains stress free. It should be noted that the number of steps used to excavate does not affect the results in a linear elastic body (Brown and Booker, 1985). In the second way, it is sometimes possible to simulate excavation of the tunnel by specifying the displacements of the tunnel boundary corresponding to a pre-defined ground loss. The ground loss is defined as the reduction of tunnel volume as a percentage of the initial excavated volume.

For the calculations presented in this paper the second of these methods was used, i.e. a pre-defined ground loss value was specified in the finite element model. Ground loss values quoted for the cases described below correspond to the value when plane strain conditions are achieved, usually corresponding to the end-of-excavation condition.

2.2 BOUNDARY-ELEMENT ANALYSIS OF PILE RESPONSE

The free-field soil displacements predicted by the finite-element model are used as input to a special boundary-element analysis of a single pile or pile group surrounded by elastic soil in order to predict the response of the pile to those soil movements and thus also in response to the tunnel excavation. The boundary element computer programs PALLAS (Hull, 1998) and GAPAX have been used to obtain the pile response prediction in the 3 orthogonal directions. PALLAS provides pile response prediction in the lateral and longitudinal directions, while GAPAX provides prediction in the vertical direction. The boundary element programs are independent from one another, thus no interactions of pile response in the three directions are considered.

Both programs utilise the integration solution of Mindlin (1936) for displacements due to a point load embedded in a semi-infinite elastic half space, as published by Douglas and Davis (1964). In order to carry out an elastic analysis of the movements of the soil and the pile, each pile is discretized along its length to have N elements and the finite difference solution of the beam bending equation is applied at discrete nodal locations along the pile. It is found that typically $N=30-40$ provides an accurate boundary element representation of most single piles.

4 ACCURACY OF FAB METHOD

In order to establish confidence with the FAB method, comparisons have been made with predictions obtained using full three-dimensional finite element analyses, in which the pile is explicitly included in the mesh. Furthermore, a full-scale reported field measurement case study was back analysed and compared with prediction from the FAB method.

3.1 FULL 3D-FEM COMPARISON

The comparison involves a tunnel with diameter (D) of 7 m adjacent to a pile with a diameter (d) of 1.2 m. The ground loss value adopted is 3%. The pile has a square cross section with its length $L_p = 6D (= 42 \text{ m})$, width $d = 1.2 \text{ m}$ and Young's modulus $E_p = 35,000 \text{ MPa}$. The distance between the tunnel axis and the pile axis, W , is one tunnel diameter ($W = D$), i.e. the minimum clear distance between the side of the pile axis and the tunnel wall is 2.9 m.

The finite element mesh includes 4,186 twenty node isoparametric hexahedral elements with 18,550 nodes. The computing time (excluding pre and post processing) for a typical single excavation step in this mesh is about 15 minutes. The analysis was computed on a machine with an Intel Pentium 4 processor running at 3.07 GHz.

On the same machine, the FAB method still requires 15 minutes for a single FE mesh solution and an additional 2 seconds for the boundary element method computing time. However, if more than one pile arrangement were required, the FAB method only needs additional 2 seconds of computing time for each additional pile arrangement. In contrast, the full 3-D FEM requires additional 15 minutes computing time for each additional pile arrangement.

Figures 1 and 2 compare the 3-D FE and the FAB predictions of distribution of lateral displacements of the pile and the bending moments, respectively. Both figures show the lateral pile responses that correspond to three tunnel face positions, i.e. as the tunnel face approaches the pile axis position ($S = -5D$), when it is parallel to the pile axis position ($S = 0D$) and after the tunnel face has advanced a significant distance beyond the pile axis position (Plane Strain).

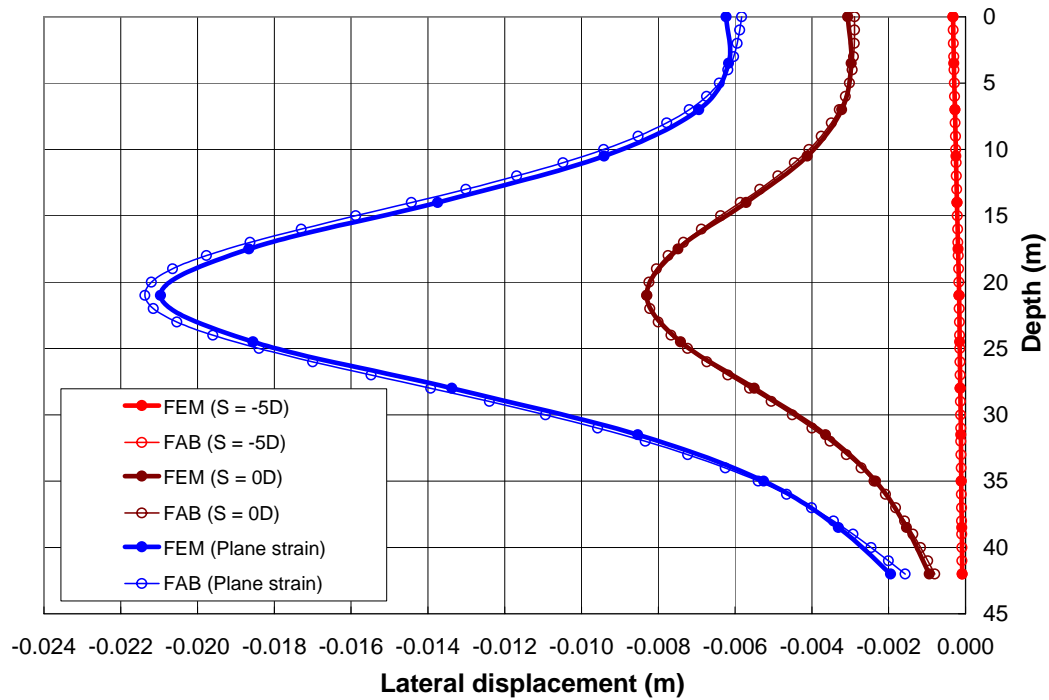


Figure 1: Comparison of full 3-D FE and FAB predictions of lateral pile displacement

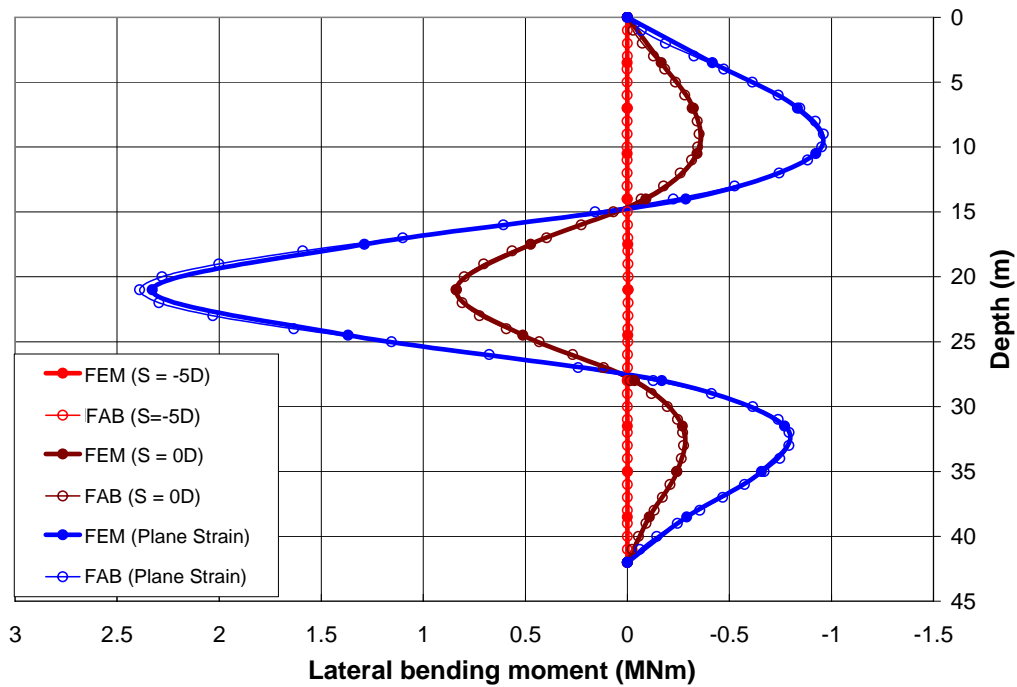


Figure 2: Comparison of full 3-D FE and FAB predictions of lateral pile bending moment

Figures 3 and 4 show respectively comparisons of the longitudinal displacements of the pile and the corresponding bending moments. Both figures show the longitudinal pile responses that correspond to two tunnel face positions, i.e. as the tunnel face approaches the pile axis position ($S = -5D$) and when it is parallel to the pile axis position ($S = 0D$). No longitudinal pile responses corresponding to the plane strain condition are presented since these responses are theoretically zero and were found to be so in the FE analysis.

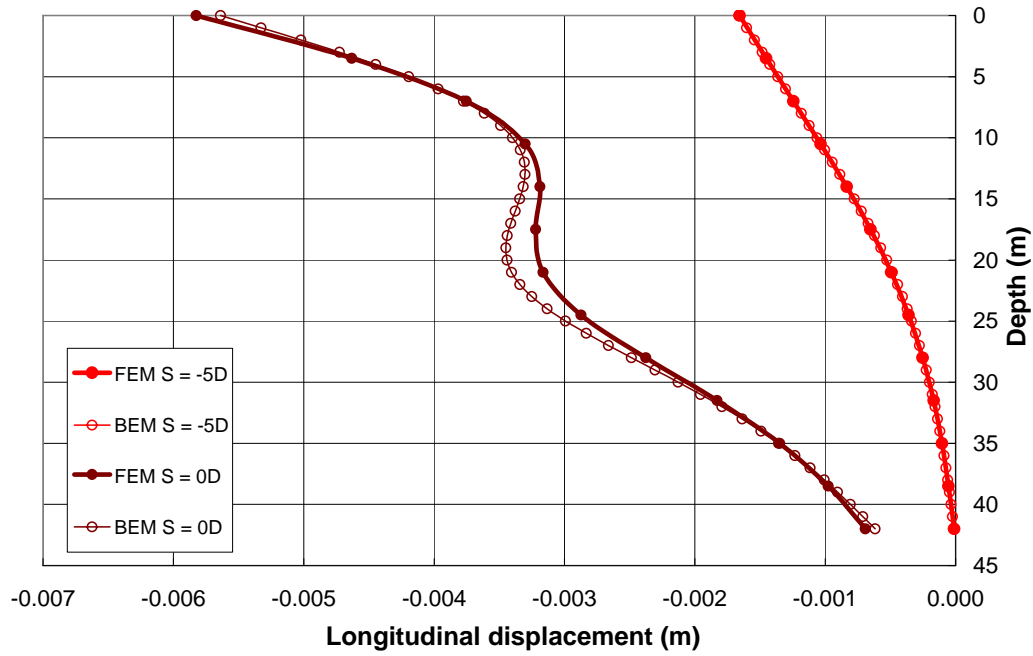


Figure 3: Comparison of full 3-D FE and FAB predictions of longitudinal pile displacement

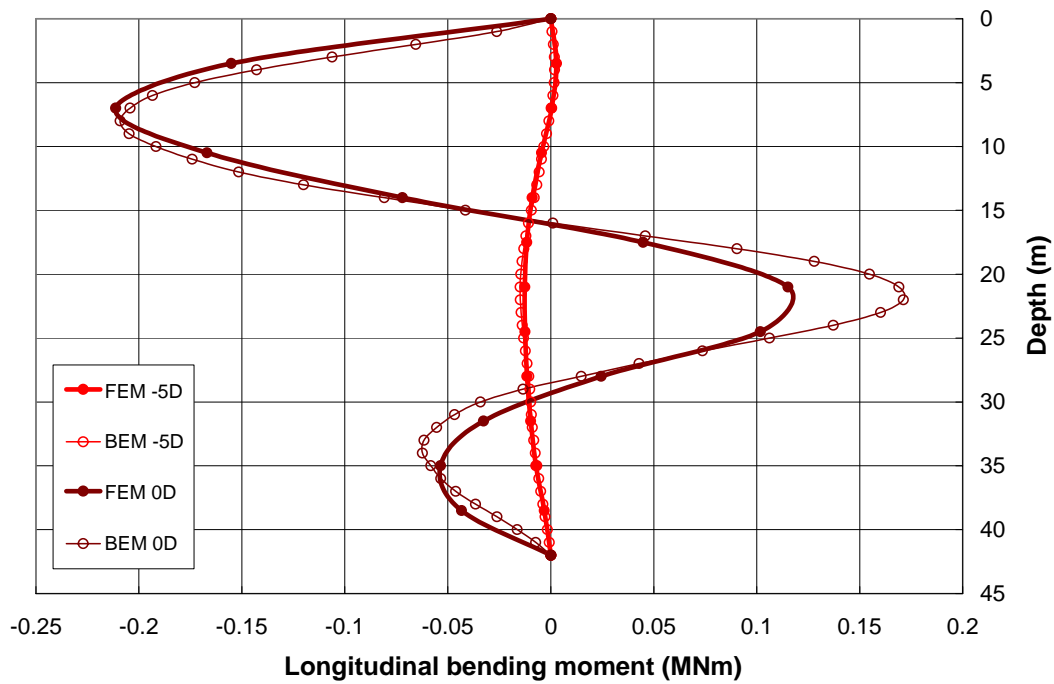


Figure 4: Comparison of full 3-D FE and FAB predictions of longitudinal pile bending moment

Figures 5 and 6 show the comparisons of the vertical displacements and the corresponding axial forces in the pile respectively. Only the axial pile responses that correspond to the tunnel at the plane strain condition are presented in both figures, since this is the condition when the maximum pile axial response is observed.

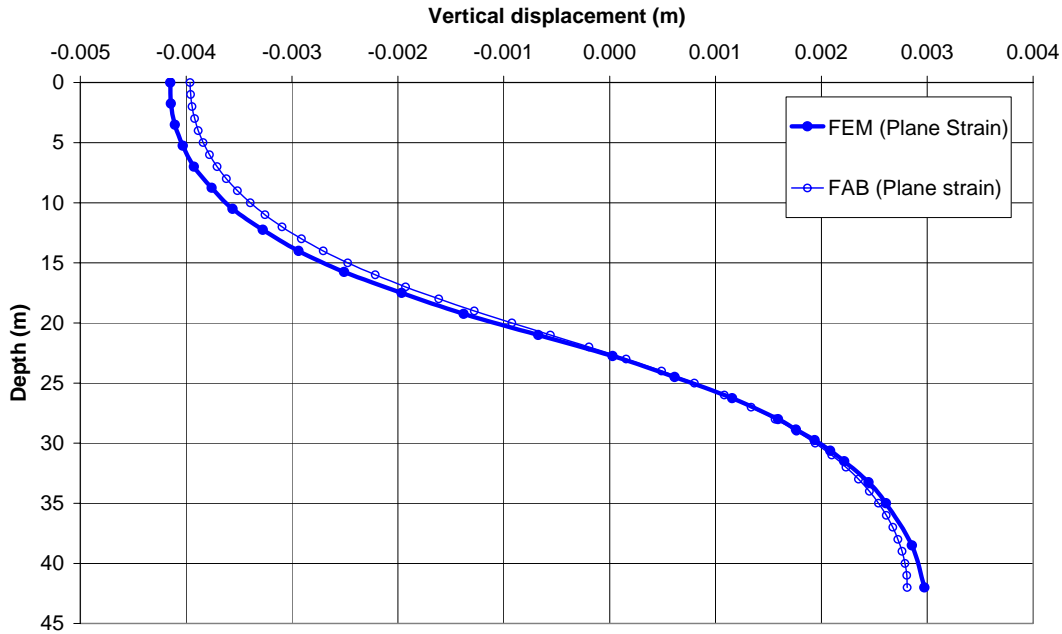


Figure 5: Comparison of full 3-D FE and FAB predictions of pile settlement

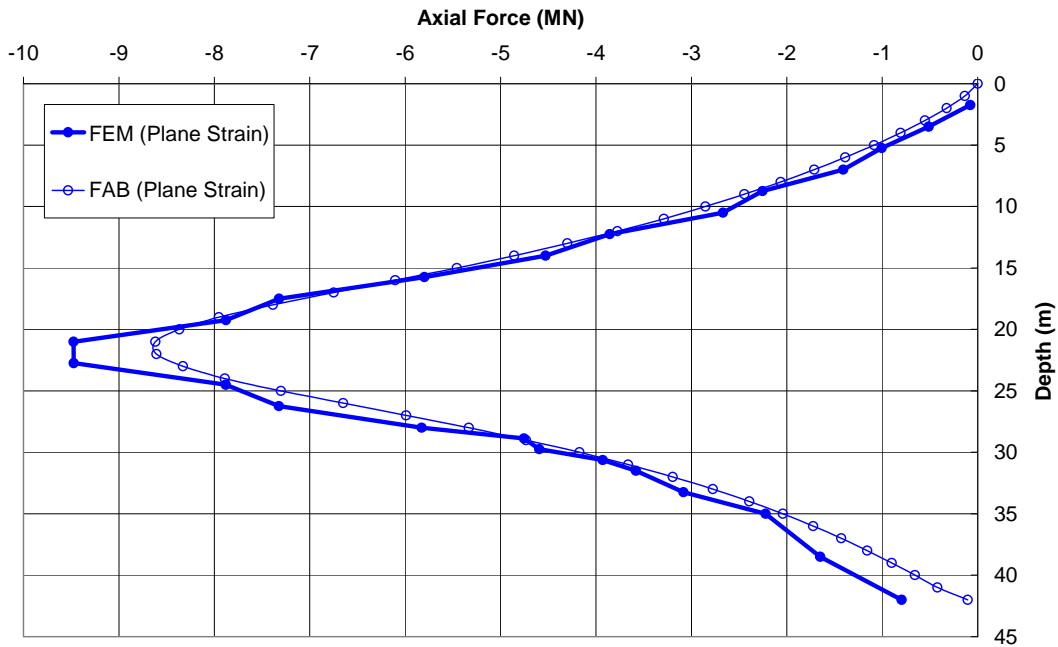


Figure 6: Comparison of full 3-D FE and FAB predictions of pile axial force

Figures 1 to 6 suggest that the agreement between the two approaches is satisfactory.

3.2 MASS RAPID TRANSPORT NORTH EAST LINE C704, SINGAPORE BACK ANALYSIS

The case history published by Pang *et al.* (2006) includes data of instrumented piles during the construction of a tunnel (South-bound) for the MRT (Mass Rapid Transport) North East line C704 in Singapore. The tunnel was excavated at a depth of 21 m by 6.5 m diameter EPB (Earth Pressure Balance) machines. Before excavation of the tunnels commenced, a viaduct bridge was constructed with each of its piers resting on a pile group. The pile group was situated in between the twin tunnels mentioned above. Each pile in the pile group has a diameter of 1.2 m. A Young’s modulus of 28 GPa was used to represent the pile stiffness. The geometry of this problem including sectional and plan views of the pile group is shown in Figure 7. Further detail of this problem can be found in the paper by Pang *et al.* (2006).

The pile group of interest reported by Pang *et al.* (2006) is Pier 20. This pier rests upon a group of four piles, as can be seen in Figure 7. The pile in this group used for comparison purposes was designated P2 by Pang *et al.* (2006) and its position can be seen in Figure 7. It is located furthest away from the South bound tunnel with 4.6 m clear distance between the side of the tunnel and the side of the pile. The reported pile P2 response corresponds to the sole influence of South bound tunnel excavation.

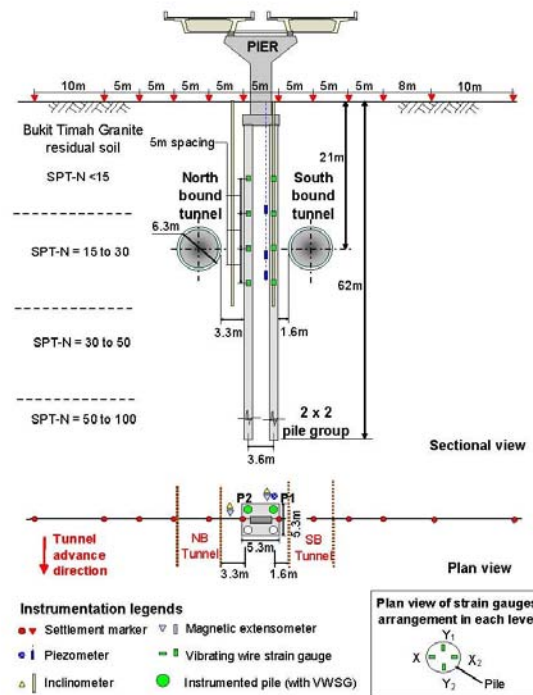


Figure 7: MRT NE Line C704 pile-tunnel arrangement (after Pang *et al.*, 2006)

Figure 8 shows the FAB predictions of axial force in pile P2 and the measured axial pile response reported by Pang *et al.* (2006). Figure 8 indicates that the FAB method provides a slightly conservative prediction relative to the measured values.

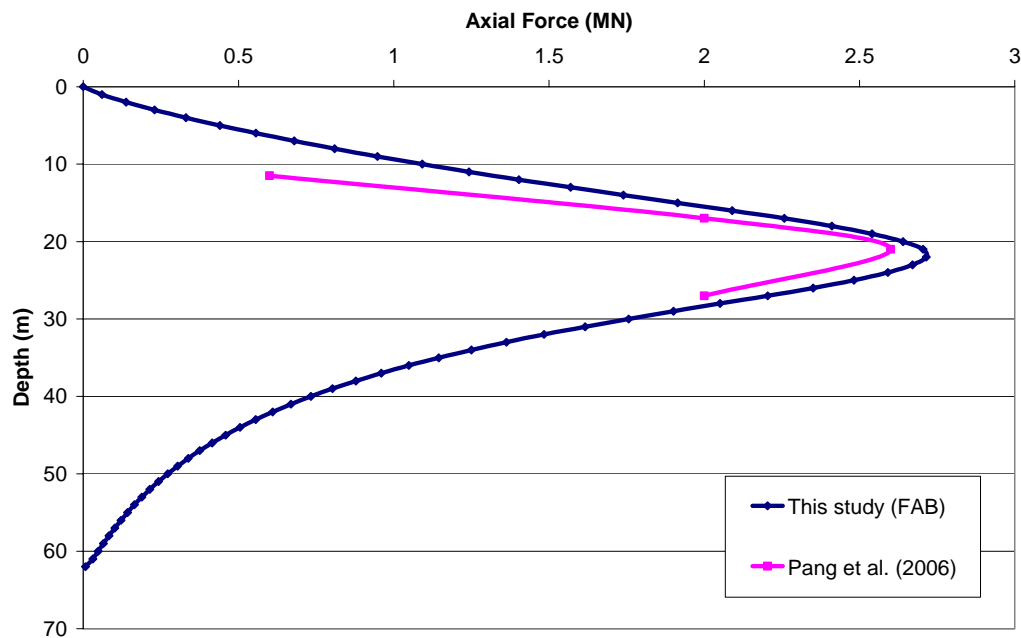


Figure 8 : Measured and predicted pile axial force of pile P2 located 8.45 m from the centreline of the South Bound tunnel for the MRT North-Eastern Line C704, Singapore

The degree of accuracy shown above provides confidence in the FAB method adopted for a recent project in Melbourne, Victoria.

4 MELBOURNE MAIN SEWER REPLACEMENT CASE STUDY

4.1 BACKGROUND

The Melbourne Main Sewer Replacement (MMSR) Project will replace the current 2.2 km long gravity sewer servicing a corridor between Melbourne CBD and Port Melbourne to the existing Hobson's Bay Main Sewer. This project involves construction of a 2.5 km long 3 m diameter circular tunnel under densely built urban areas. To assist with Third Party Assets Impact Assessment requirements, GHD Geotechnics were asked by GHD Melbourne to assess the induced loads in piles close to the tunnel caused by tunnel excavation.

The originally proposed tunnel alignment identified 3 pile group foundations within the tunnel zone of influence and these were modelled. Subsequently a new tunnel alignment was selected and additional 3 pile group foundations were identified within the zone. Among the 6 pile foundations (total of 97 piles) identified, the Melbourne Convention Centre (MCC) pile foundation is chosen to provide a brief illustration of the FAB efficiency.

There are 18 piles in the proposed MCC pile foundation that are identified to be within the tunnelling zone of influence, as indicated in Figure 9. The standard pile size for the MCC foundation is 0.75 m in diameter and 34 m in length.

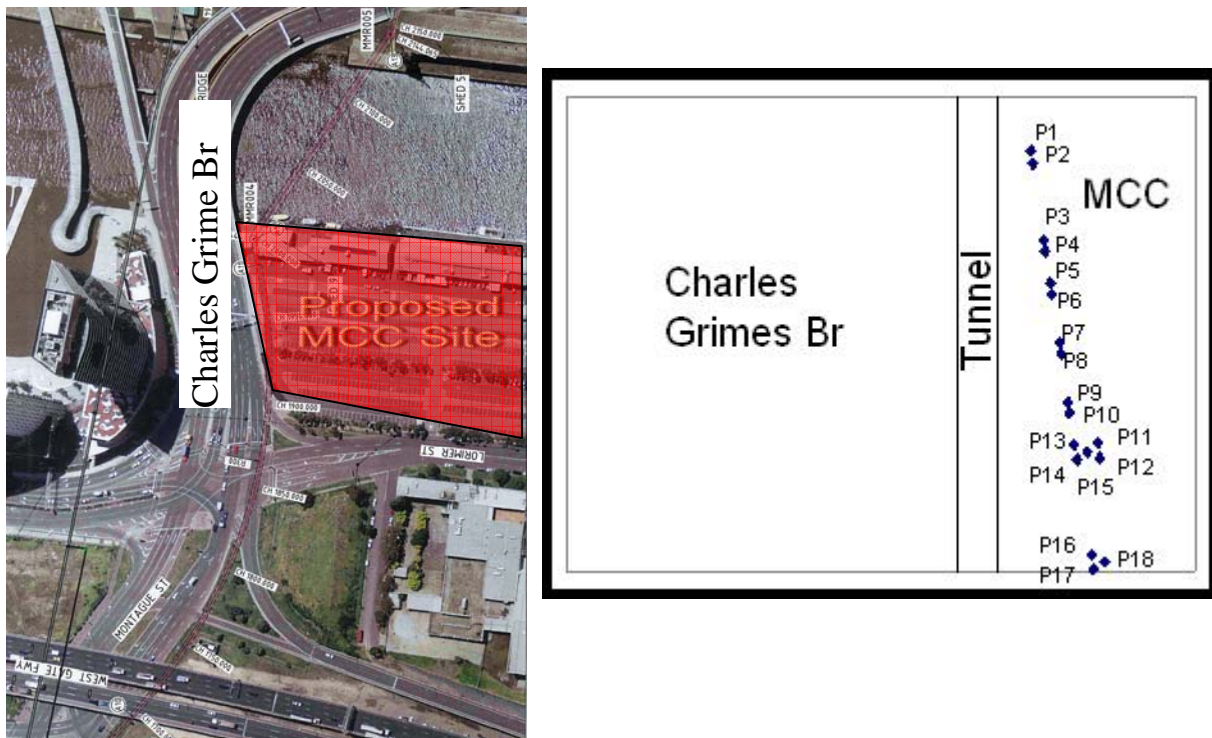


Figure 9: Eighteen piles of the proposed MCC pile foundation that are within the tunnelling zone of influence

A complete staged 3-D tunnel excavation model of the situation is possible. However, it is commonly recognised that the most severe condition is associated with completion of the tunnel, that is plane strain conditions are appropriate. Hence only the soil movement corresponding to the plane strain condition is applied to the BEM. Furthermore, the soil model chosen is linear elastic.

The detailed soil profile at the proposed MCC site and the idealised soil profile equivalent can be found in Figure 10. This idealised profile was adopted in the analysis and it consists of seven idealised distinct layers modelled in the FE analysis. The depth of each soil layer modelled can also be found in Figure 10 and the corresponding undrained parameters can be found in Table 1.

Both drained and undrained parameters were adopted in analyses. It is noted that the drained problem generally has less curvature for the lateral free field soil movements, which should result in less severe lateral pile loading. Hence only the undrained parameters and results are presented in this paper.

Two boundary conditions at the pile head were modelled, i.e. fixed and free to represent respectively a rigid and flexible pile cap.

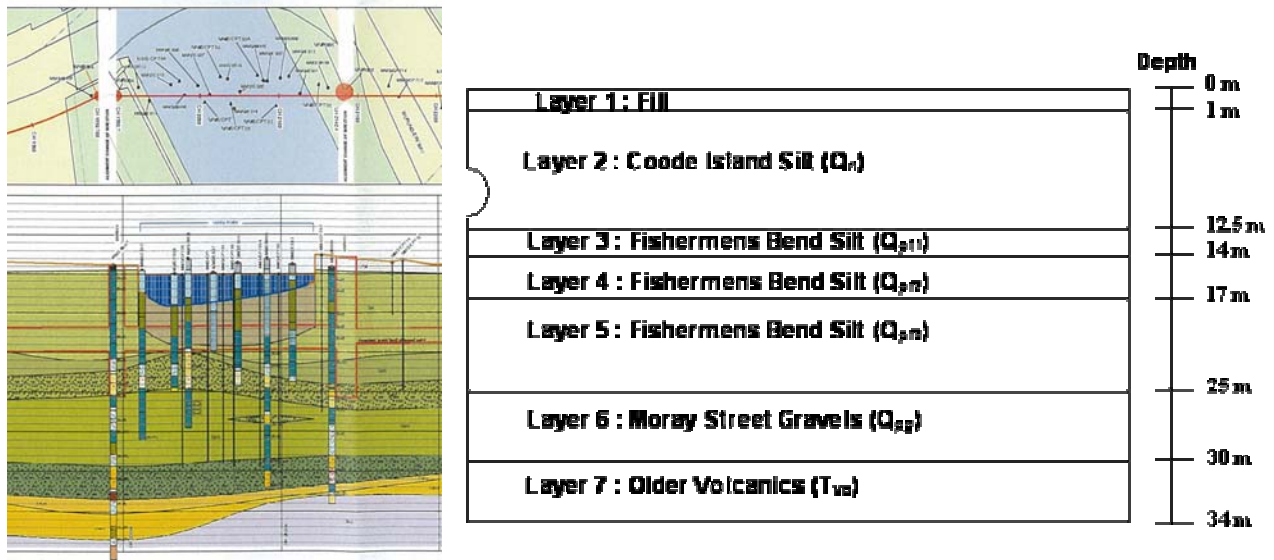


Figure 10: MCC - Idealised soil profile adopted in the analysis

Table 1: MCC - Soil undrained parameters adopted in the analysis

Soil Layer	Undrained Properties
1. Fill	E_u : 11.5 MPa ; ν : 0.49
2. Coode Island Silt (Q_r)	E_u : 8.25 MPa ; ν : 0.49
3. Fishermens Bend Silt (Q_{pf1})	E_u : 11.0 MPa ; ν : 0.49
4. Fishermens Bend Silt (Q_{pf2})	E_u : 15.0 MPa ; ν : 0.49
5. Fishermens Bend Silt (Q_{pf3})	E_u : 12.0 MPa ; ν : 0.49
6. Moray Street Gravels (Q_{pg})	E_u : 40.0 MPa ; ν : 0.49
7. Older Volcanics (T_{vo})	E_u : 10000 MPa ; ν : 0.49

4.2 RESULTS

The prediction of the tunnelling induced lateral and vertical pile displacement for the proposed MCC piles can be found in Figures 11 and 12, respectively. The pile responses presented are for a ground loss value of one per cent.

The time it took from FE mesh generation to the end of the Boundary Element analysis for the presented results was less than 12 working hours. For this case, the finite element and boundary element computation times were 6 seconds (6882 d.o.f) and 2 seconds (1224 d.o.f) respectively. In both cases the analysis was computed on a machine with an Intel Pentium Dual core processor running at 2.13 GHz and the times quoted are for the solution of only one load step (one tunnel excavation stage by 2D finite element analysis and one corresponding pile analysis by PALLAS).

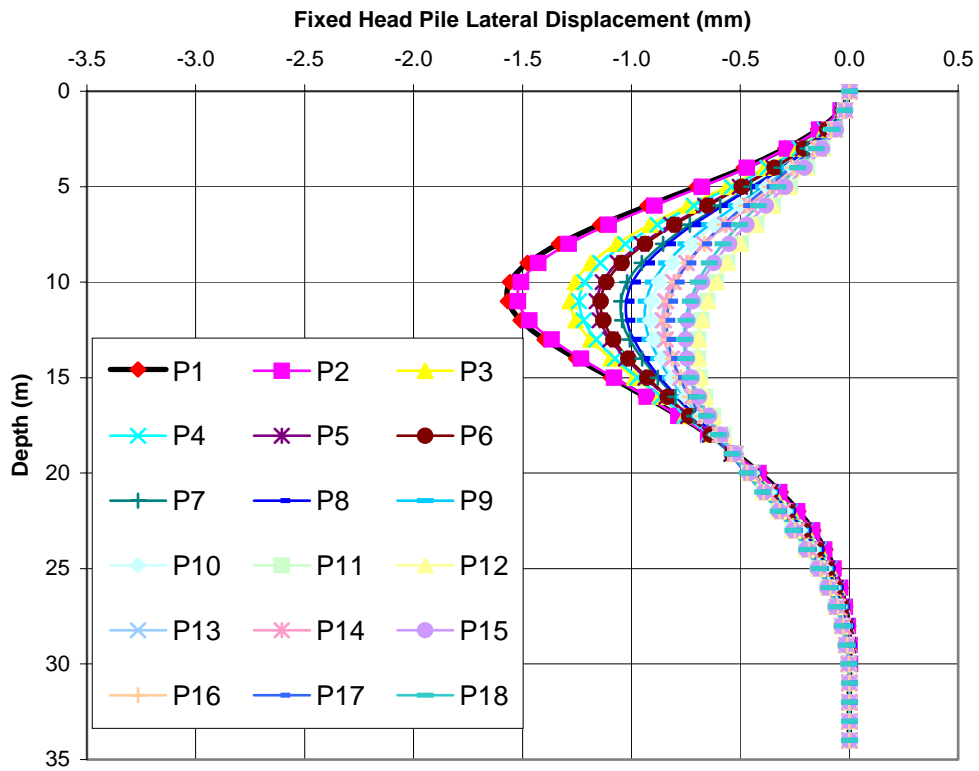


Figure 11: MCC – Prediction of tunnelling induced lateral pile displacement.

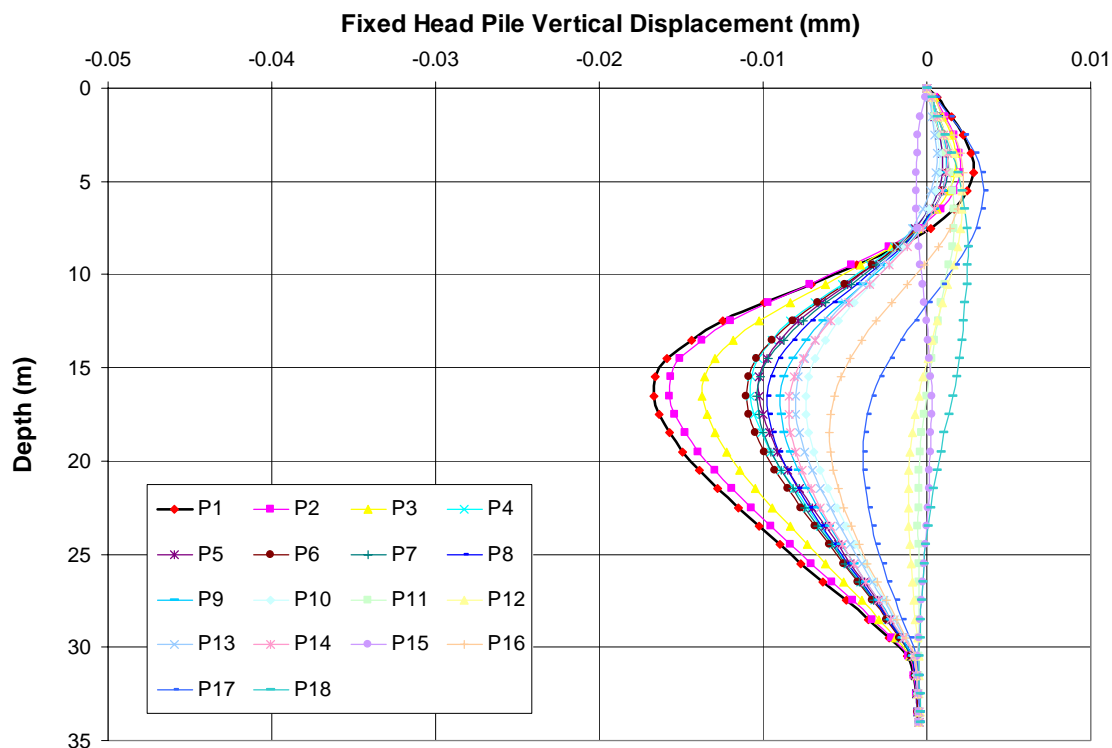


Figure 12 MCC – Prediction of tunnelling induced vertical pile displacements

5 ADVANTAGES OF FAB METHOD

It may be seen from the above that in terms of execution time, the FAB solution time is a mere fraction of the time needed for a typical full 3-D finite element analysis to analyse a number of pile and tunnel arrangements. Hence the FAB represents a major saving over the cost of conducting the analysis of each pile and tunnel arrangement using the 3-D finite element approach. This suggests that the FAB method is an economically viable method to be used where a significant number of piles and pile-tunnel arrangements are involved.

Due to the expediency of the FAB method, GHD Geotechnics was able to provide results to GHD Melbourne in time to liaise with the MCC piling contractor to identify if any of the proposed piles to be installed for the MCC need improvement due to the load induced by the adjacent tunnelling.

The practicality of the FAB method combined with the satisfactory degree of accuracy that can be achieved, shows the FAB method is a valuable method for a geotechnical engineer to assess the pile foundation response due to tunnelling.

6 CONCLUSION

The major advantage of the combined finite and boundary element method (FAB) when used for tunnelling problems is that only one 3-D finite element analysis is required for each tunnel and soil configuration, independent of the multitude of configurations of pile foundations that may be of interest. The displacement field generated by the finite element analysis, corresponding to tunnel excavation, is input as the free-field soil displacements in a separate boundary element analysis of the pile foundation. 3-D finite element predictions are relatively expensive in terms of data preparation (pre- and post-processing) and computer execution times, while boundary element analysis of a pile foundation is relatively inexpensive, thus combination of the two provides a good, practical compromise for design situations where a significant amount of pile arrangements are involved.

The satisfactory agreement between the prediction of the FAB method with full 3-D FEM and reported observed field measurement has been demonstrated. This demonstration showed the possible degree of accuracy that can be achieved with the method and establishes some confidence in the method.

Once confidence was established, the FAB method was adopted in the recent MMSR project, where the FAB was used to generate economical predictions for a very large number of cases of practical interest. A total of 97 piles was included in the assessment. The efficiency of the FAB method allowed the time for GHD Melbourne to liaise with the piling contractor to identify if any of the piles in the tunnel influence zones would need improvement.

7 ACKNOWLEDGEMENTS

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