COMPUTER MODELLING OF A PILE GROUP

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ABSTRACT

Conventional analysis of pile groups involves simplifications that are a response to the difficulty experienced in the past with employing advanced modelling techniques. Such simplifications include the model employed for the soil, the geometry restrictions considered in the analysis and the experience(s) of the modeller or available software. Historically, methods gained acceptance for their ease of use and development of an empirical database of performance. Whilst past experience with limited modelling and computing power should no longer be considered a restriction to the use of more advanced modelling techniques, the checking of these methods against experience often remains as an impediment to their use. The “Catch 22” in this situation is that the very use of these “newer” methods is necessary before their use is seen to be attractive. A newer method based upon current analysis capabilities is presented as a viable approach to improved pile group modelling. This paper attempts to provide an example of the use of more advanced techniques of pile group analysis than are commonly employed, and suggest some benefits, from a design point of view.

1 INTRODUCTION

Often, pile group design has relied upon the accepted ways of analysis that have developed in response to the perceived needs of good foundation design, and have taken the form dictated by the existing level of knowledge and current computational capabilities. For some time now, it has been possible to make use of solutions for axial and lateral pile responses that require the knowledge of basic material properties (based on an elastic continuum model) considered appropriate for modelling the founding soil or rock. The work of Poulos and Davis (1980), Randolph and Wroth (1979), Poulos and Hull (1989), Carter and Kulhawy (1992), etc. demonstrate the significant input of Australia in the development of this elastic continuum approach to pile modelling for the purpose of design.

In North America considerable time and effort has been given to the development of an alternative approach to pile analysis that finds its basis rooted in empirically derived models. These model the pile structural stiffness in combination with soil response models that cater for a range of soil types for which full scale tests have been conducted; Matlock and Reese (1960), Reese, Cox and Koop (1974 and 1975), Coyle and Reese (1966), etc. These are generically referred to as ‘p-y analyses’. The soil response model is a modification of the work of Winkler (1867) in which he related any surface settlement (here a lateral displacement, y) to the ‘pressure’ applied on the surface (here a lateral distributed load) by a spring constant.

The shortcomings of the p-y system of representing the soil include its inability to predict the interaction that occurs between piles through the soil. Interaction is the result of the proximity of one pile to another in terms of the transfer of load through the soil between piles. There is also a corresponding effect in the stiffening of the soil by each pile, in a similar manner to the effect of reinforcement in concrete or reinforced soil and rock. There have been considerable efforts made to include the effects of interaction in the ‘p-y’ based analyses, for example Brown, Reese and O’Neill (1987) and Ochoa and O’Neill (1989). To date, however, only a simplistic use of multiplier factors with no predictive capability is available.

By contrast, the elastic continuum based analyses can be naturally extended to consider interaction between piles in a group. Despite the fact that proponents of ‘p-y’ analyses, and others criticise the continuum approach because two-pile derived interaction factors have been shown to overestimate group displacements, there is no suggested alternative predictive model for ‘p-y’ analyses, only empirically based multipliers.

This paper provides an example of the use of more advanced techniques of pile group analysis than are perhaps commonly employed, and suggests some benefits of this, from a design point of view. The example is a two by eight bored pile group with a large number of design load combinations. The analysis considers axial response of the piles, including axial interaction between piles and lateral response of the piles, including lateral interaction between piles. The analysis largely follows conventional methods for considering the combined axial and lateral group response resulting from the pile cap, and as such, does not include individual pile torsion or the cross-influence effect of axial loading of piles on the lateral response, and vice versa. However, the analysis does include the effects of lateral loading in two directions and the formulation involves a group load with three force components and three moment components, that is, six degrees of freedom for the pile cap.

2 PROBLEM AND EXAMPLE DEFINITION

The analysis will be employed to consider a hypothetical pile foundation consisting of 800 mm diameter piles of length 20 m in a series of 2 by 8 pile groups is analysed. The supported structure, to be placed on the pile groups, is assumed to have been designed after the adopted external loading cases have been applied to an analytical model of the structure in which each of the 2 by 8 pile groups is assumed to not move under the loading. By this means the resultant loads to be sustained by each pile group can be determined. This "rigid" assumption for the response of the pile foundation is typically assumed to lead to over estimation of the reaction loads. This assumption is seldom questioned. The layout of the pile group is shown in Figure 1.

The magnitude of the resulting movements predicted by the pile group analysis due to the above determined structure reaction loads may then allow assessment of the validity of the assumption of a rigid non-moving pile group. The pile group having a finite stiffness would be expected to result in some form of reduced reaction loads if the true stiffness of the pile group were employed in the structural analysis.

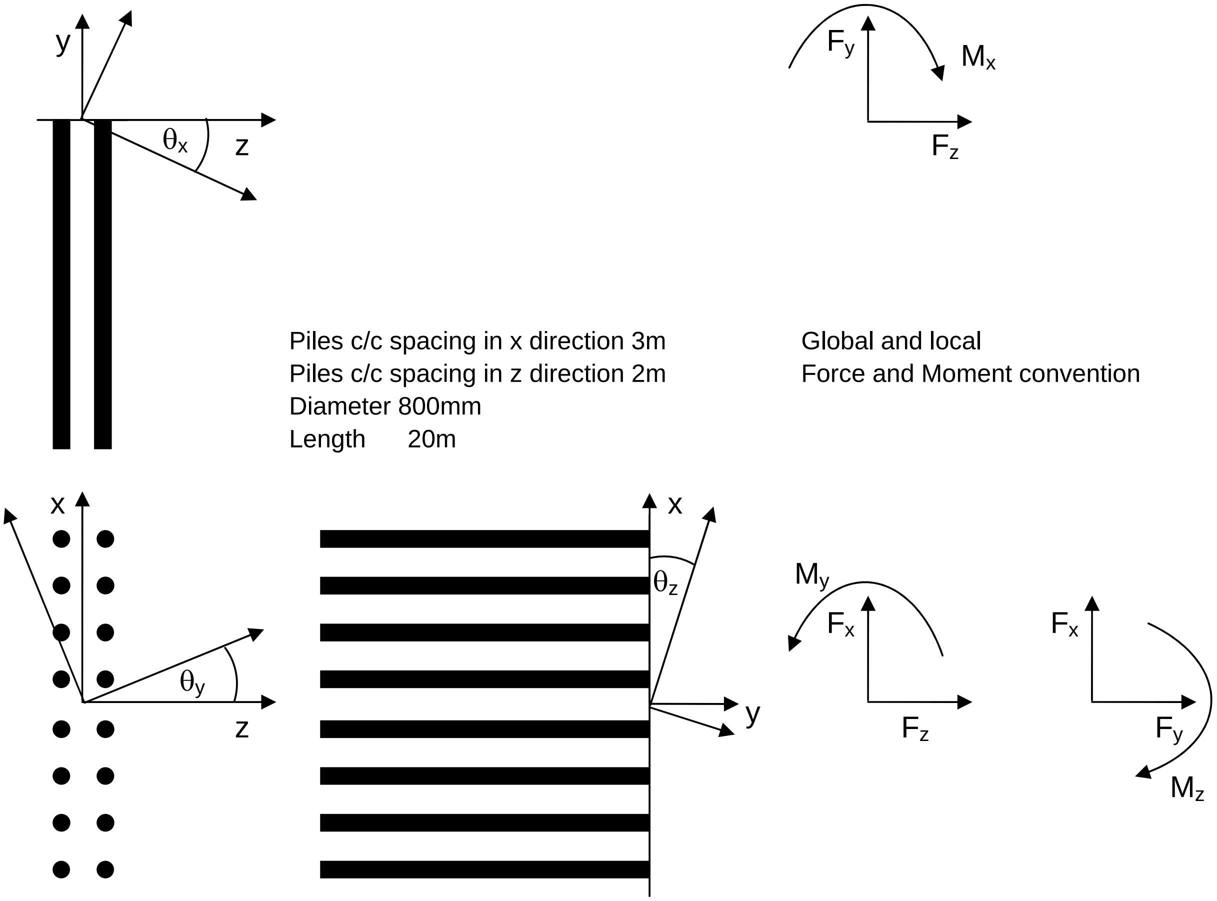
The assumed geotechnical model is detailed in Table 1 and consists of two layers. The values of Young's modulus, Poisson's ratio, ultimate skin friction, end bearing and lateral bearing pressures are given. The analysis will determine whether the limiting strengths are achieved and will indicate if the pile group stiffness becomes load dependant and require iteration since the analysis has then become non linear.

At present the analysis proceeds with manual intervention for this non-linear procedure, but the automation of the non-linear group geotechnical analysis is possible and would avoid this limitation. However, currently it is not possible to directly link the geotechnical analysis to the structural stiffness analysis and so this limitation is not detrimental to the overall analysis.

Table 1 Assumed geotechnical model for example problem.

Depth below pile cap	*Material*	*E' (kPa)*	*ν'*	*f_s (kPa)*	*f_b (kPa)*	*p_u (kPa)*
0 – 10m	Fill - silty sand	20,000	0.2	30	150	100
10 – 25m	Sandstone	200,000	0.2	100	1,900	1,900

More complicated models are possible; with perhaps parameters for axial and lateral soil response chosen to be different if the available data from say pile tests were to indicate this were true at a particular site. The effect of Poisson's ratio is normally considered to be of secondary importance (especially for lateral loading).



Piles c/c spacing in x direction 3m
Piles c/c spacing in z direction 2m
Diameter 800mm
Length 20m

Global and local
Force and Moment convention

Figure 1 Plan and elevations of pile problem, coordinate system, displacement, rotation and load convention.

3 METHOD OF ANALYSIS

The pile analysis has been carried out by first estimating the settlement, the two horizontal displacements and the three rotations of the pile cap under the effect of loading and moments applied in the three coordinate directions. The pile cap is assumed to be rigid here, but consideration of its flexibility could, in theory, be included.

The program PALLAS, Hull (1998), has been used to calculate the stiffness of the pile cap under horizontal and moment loading and the program GAPAX has been used to calculate the axial stiffness. Both programs were developed at The University of Sydney, Centre for Geotechnical Research and rely upon the techniques of pile analysis pioneered by Professor Harry Poulos. Both analyses incorporate the ability to-

1 model piles of varying length and diameter within a group,

2 take account of interaction between all piles and

3 not rely upon interaction factors determined from analysis and loading of only two piles.

The cross-interaction between axial, torsional and lateral response of individual piles is not included and conventionally this has been assumed to be of minor importance. However, the axial pile response contribution to lateral rotational group stiffness is incorporated, as is the contribution of horizontal loading in both lateral directions to the torsional response (vertical axis rotation) of the pile group.

3.1 CALCULATION OF PILE RESULTANTS DUE TO GROUP MOVEMENTS

In separate analyses, the piles in the group are assigned distributions of displacements that are consistent with three rigid body displacements and three rigid body rotations (from a proportional displacement with no displacement of the centroid); based on the three Cartesian coordinate directions (x, y and z) and centred about the geometric centroid of the pile group at the level of the soil surface. This is achieved by load cases of-

1. a uniform group displacement (u_{GX}) in an x-direction PALLAS analysis,
2. a uniform group settlement (u_{GY}) in a y-direction GAPAX analysis,
3. a uniform group displacement (u_{GZ}) in a z-direction PALLAS analysis,
4. a proportional settlement based pure group rotation (θ_{GX}) in a y-direction GAPAX analysis,
5. a proportional displacement based pure group rotation (θ_{GY}) in an x-direction PALLAS analysis,
6. a proportional displacement based pure group rotation (θ_{GY}) in z-direction PALLAS analysis and
7. a proportional settlement based pure group rotation (θ_{GZ}) in a y-direction GAPAX analysis.

Note that two lateral PALLAS analyses are required to model the vertical twist components of the pile group and each individual torsional pile response (M_Y) has been ignored. Therefore, only three analyses (two PALLAS with two 'load' cases each and one GAPAX with three 'load' cases) are required to determine the rigid body deformations of the pile cap

The important results from these analyses are the different axial loads (F_Y), shear forces (F_X and F_Z) and bending moments (M_X and M_Z) generated at the head of each pile in the group. From the consideration of equilibrium between the applied group loads (F_{GX}, F_{GY}, F_{GZ}, M_{GX}, M_{GY} and M_{GZ}) and summation of the effects of the generated individual pile head loads the solution proceeds.

3.2 METHOD OF ANALYSIS

The analysis is based upon two sets of matrices governing the equilibrium of the pile group and the rigid body deformations that have been assumed for the pile cap. These are presented in Equation 1.

$$\begin{bmatrix} F_{GX} \\ F_{GY} \\ F_{GZ} \\ M_{GX} \\ M_{GY} \\ M_{GZ} \end{bmatrix} = \sum_{i=1}^{Numberof\ piles} [A]_i^T \begin{bmatrix} F_X \\ F_Y \\ F_Z \\ M_X \\ M_Y \\ M_Z \end{bmatrix}_i \quad and \quad \begin{bmatrix} u_X \\ u_Y \\ u_Z \\ \theta_X \\ \theta_Y \\ \theta_Z \end{bmatrix}_i = [A]_i \begin{bmatrix} u_{GX} \\ u_{GY} \\ u_{GZ} \\ \theta_{GX} \\ \theta_{GY} \\ \theta_{GZ} \end{bmatrix} \quad where\,[A]_i = \begin{bmatrix} 1 & 0 & 0 & 0 & \bar{z}_i & 0 \\ 0 & 1 & 0 & -\bar{z}_i & 0 & \bar{x}_i \\ 0 & 0 & 1 & 0 & -\bar{x}_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad [1]$$

with $\bar{x}_i$ and $\bar{z}_i$ the distances from the centroid of the pile group to the pile centre.

With the resulting relationship between the applied rigid body movements and the generated group loads, a six by six pile cap stiffness matrix is generated that can be used in the structural analysis.

This relationship can be inverted and used to estimate the group response for a large number of group load cases. The rigid body deformations may then be applied directly as head defined deformation load cases in the original seven deformation driven pile analyses and the results assembled to determine the distributions of axial load, as well as the two sets of shear force and bending moment along each pile.

4 EXAMPLE RESULTS

Table 2 details the group's rigid body displacements and rotations, at the geometric centroid of the pile group at the level of the soil surface, and a particular applied group load case set of forces and moments, together with the corresponding pile head horizontal displacements, settlements and rotations. Note all the piles have the same rotations as the cap and because M_{GX} is zero so too is θ_{GX}. And although the centroid of the group does not displace in the z direction because the load F_Z is zero, the piles have a z direction displacement.

Table 2 Pile group response and individual pile responses to a particular ultimate limit state load case.

u_{GX}	-0.00591426	m	F_{GX}	-2034.26	kN
u_{GY}	-0.00245831	m	F_{GY}	-16788.25	kN
u_{GZ}	0	m	F_{GZ}	0	kN
θ_{GX}	0	radian	M_{GX}	0	kNm
θ_{GY}	-7.6973E-05	radian	M_{GY}	-2313.635	kNm
θ_{GZ}	2.62664E-05	radian	M_{GZ}	6613.543	kNm

Pile	coordinates		Horizontal	Horizontal	Vertical	Horizontal	Horizontal	Torsional
No.	z	x	u_z	u_x	u_y	θ_z	θ_x	θ_y
	(m)	(m)	(m)	(m)	(m)	(radian)	(radian)	(radian)
1	-1	2	-8.08213E-04	-5.83728E-03	-2.73411E-03	2.62664E-05	0.00000E+00	-7.6973E-05
2	-1	5	-5.77295E-04	-5.83728E-03	-2.65531E-03	2.62664E-05	0.00000E+00	-7.6973E-05
3	-1	8	-3.46377E-04	-5.83728E-03	-2.57651E-03	2.62664E-05	0.00000E+00	-7.6973E-05
4	-1	11	-1.15459E-04	-5.83728E-03	-2.49771E-03	2.62664E-05	0.00000E+00	-7.6973E-05
5	-1	14	1.15459E-04	-5.83728E-03	-2.41891E-03	2.62664E-05	0.00000E+00	-7.6973E-05
6	-1	17	3.46377E-04	-5.83728E-03	-2.34011E-03	2.62664E-05	0.00000E+00	-7.6973E-05
7	-1	20	5.77295E-04	-5.83728E-03	-2.26131E-03	2.62664E-05	0.00000E+00	-7.6973E-05
8	-1	23	8.08213E-04	-5.83728E-03	-2.18251E-03	2.62664E-05	0.00000E+00	-7.6973E-05
9	1	2.	-8.08213E-04	-5.99123E-03	-2.73411E-03	2.62664E-05	0.00000E+00	-7.6973E-05
10	1	5	-5.77295E-04	-5.99123E-03	-2.65531E-03	2.62664E-05	0.00000E+00	-7.6973E-05
11	1	8	-3.46377E-04	-5.99123E-03	-2.57651E-03	2.62664E-05	0.00000E+00	-7.6973E-05
12	1	11	-1.15459E-04	-5.99123E-03	-2.49771E-03	2.62664E-05	0.00000E+00	-7.6973E-05
13	1	14	1.15459E-04	-5.99123E-03	-2.41891E-03	2.62664E-05	0.00000E+00	-7.6973E-05
14	1	17	3.46377E-04	-5.99123E-03	-2.34011E-03	2.62664E-05	0.00000E+00	-7.6973E-05
15	1	20	5.77295E-04	-5.99123E-03	-2.26131E-03	2.62664E-05	0.00000E+00	-7.6973E-05
16	1	23	8.08213E-04	-5.99123E-03	-2.18251E-03	2.62664E-05	0.00000E+00	-7.6973E-05

Figure 2 presents some results of the analysis of the example problem for the same load case as given in Table 2. The application of a torsional load M_{GY} has produced a twist about the vertical axis of the pile group that the majority of analysis methods do not consider.

The vertical axis twist in this example, see Figure 2(a), is small because of the considerable stiffness associated with this mode of deformation of the pile group. The long aspect ratio of the pile group provides a large torsional moment resistance with very small lateral loads associated with the piles at the extremity of the group.

Figure 2 (b) schematically illustrates the variation of pile settlement with position in the group as a column chart.

Since the load case is a structural Ultimate Limit State then it would be useful to compare every pair of bending moment and axial load combinations, in both the x and z directions, for every depth along every pile in the pile group with the capacity envelope derived from consideration of the pile cross section properties.

This has been plotted in Figure 2 (c). The piles are seen to be well within their capacity and if there were sufficient confidence in the geotechnical model some refinements in the group could be made to increase its efficiency.

Figure 2 (d) presents the same data in more detail and shows that it is bending about the z axis that is predominant, but that bending about the x axis is also present. Each pile may be investigated separately for any particular depth based upon the behaviour evident in this graph and critical piles would be detected. Changing pile geometry could then be considered if warranted by the results.

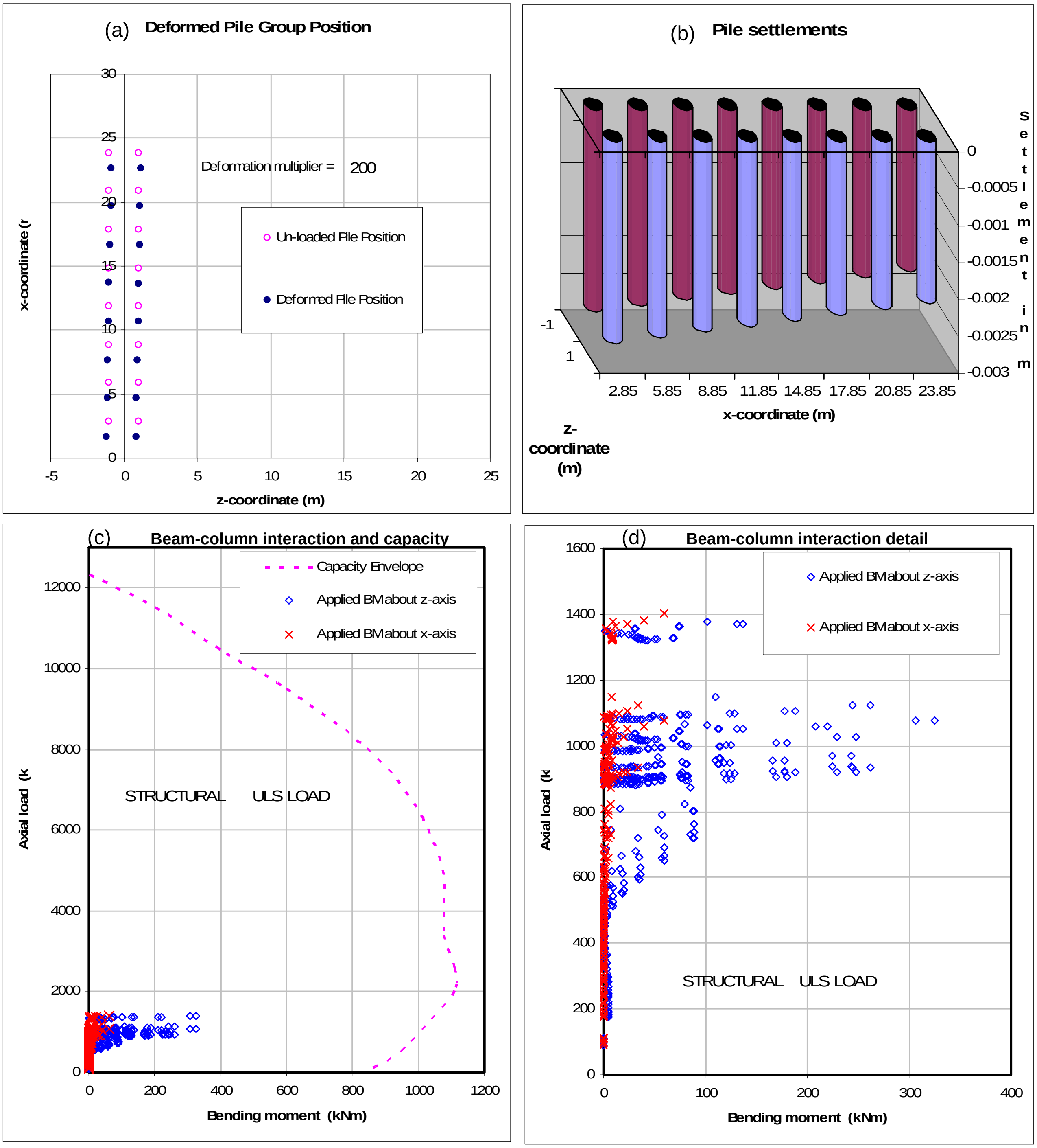


Figure 2 (a) Plan view of the pile group showing the deformed position, (b) 3-D column view of the settlement distribution across the group, (c) Pile beam-column capacity chart showing data along all piles and (d) Detailed view of data for axial load and bending moments.

5 ADVANTAGES OF METHOD

A consistent approach to the analysis of a pile group is one of the key features of the adopted elastic continuum soil model. The analysis of a single pile or a pile group will require the same input and therefore provide a basis for comparison of different foundation arrangements. This may be especially beneficial if load requirements vary at a site and a number of different group configurations are considered to be necessary to achieve similar foundation performance across the site.

Output data for, and along, all piles is available for checking against section capacity to investigate the possibility of economies from varying pile type in terms of section capacity or even adopting more than one pile diameter and/or length in the group. Other solution methods cannot easily provide this ability.

The possibility of modelling a large number of loading cases relatively quickly means improved confidence that the most critical load case for design has been considered.

Similar to other commonly adopted pile group analysis techniques this method incorporates the axial pile stiffness in the group response. Although commonly assumed to be of minor significance, an effect from vertical twisting loads applied to the pile group is incorporated. But, the means by which interaction is modelled in the basic analyses also accounts for the stiffening of the soil by the presence of all piles in the group, not just the interaction based upon the extension of a two-pile analysis employing interaction factors which has been criticised as over-estimating interaction.

6 CONCLUSIONS

The group analysis presented here has been carried out for a linear elastic soil model but extension to consider non-linear soil effects is possible. The analyses are relatively quick to perform, especially in comparison to any attempt at modelling the group in 3-D finite element formulations, which will suffer from poor discretisation of the piles in order to make the analyses tractable.

Results can be presented in axial load versus bending moment interaction diagrams at any depth and for all piles in the group for cases considered to be critical. Piles that develop large skin friction or high lateral bearing pressures or piles with excessive displacements or rotations can be identified. The model may then be relatively quickly re-analysed with appropriate adjustments to the design. This step is considerably more onerous in any 3-D finite element modelling of a pile group.

The overall pile cap stiffness, which is a result of the analysis, may be used to assess the assumptions made in, say, the structural analysis from which the load cases for the pile group were derived. The reliability of the device of modelling of the structure foundation with a ‘rigid stiffness’ of the foundation may then be investigated. This aspect of design is too often ignored in many instances and only relatively few major pile design projects have been considered as requiring advanced modelling techniques incorporating this structure-soil feedback. Structural (and some geotechnical) engineers view this information as being less important for design but as yet we have no convincing proof that this is the case.

The mechanics of the analysis presented here could be incorporated into a single analysis and result in one program with consistent input of the soil model and output of resultants for each individual pile. As experience with using the approach increases it would be possible to develop the application to economically consider smaller pile groups than the group considered here and to incorporate soil-structure interaction.

The issue of whether a pile group analysis has incorporated sufficient complexity of soil behaviour and employed a consistent model of soil response to allow the pile structural design to make the most efficient solution may always be one of personal preference. However, it is hoped that the analysis presented here may be of some use in alerting engineers to the benefits of using one of the analysis options currently available.

7 ACKNOWLEDGEMENTS

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