PERFORMANCE ASSESSMENT OF CUT OFF WALLS SUBJECT TO TIDAL VARIATION

Antony Orton
Geotechnical Engineer, Coffey Geotechnics
Level 19, Tower B – Citadel Tower 799 Pacific Highway, Chatswood, NSW, 2067,
ph: +61 2 9406 1023   antony_orton@coffey.com

ABSTRACT

Cut off walls constructed in coastal settings are subject to external tidal variations in water pressure. A method to exploit this tidal variability to check the performance of cut-off wall construction is presented supported by analytical and numerical modelling results. The method described allows progressive assessment of performance of the wall during its construction. The approach is illustrated with reference to two case studies: one for a major building project and the other for an environmental control measure.

1 INTRODUCTION

Cut off walls provide low permeability barriers which reduce groundwater flow. They are used to prevent seepage from contaminated zones or to facilitate dewatering and excavation. In tidal areas, cut off walls influence the response of groundwater to the tides. In this paper, tidal response in groundwater is discussed using analytical and numerical techniques with emphasis on the effect of cut off walls in limiting tidal response in groundwater.

Initially, a one dimensional solution to the groundwater flow equation with a sinusoidal boundary condition is presented, showing the reduction of tidal response as well as phase lag as a function of distance away from the tidal source. This solution is improved by incorporating a fixed boundary which limits the decline in tidal response. The analytical theory may be found in numerous references including Edelman (1972), Todd (1959) and from a more mathematical point of view, Farlow (1982).

In the second part of the paper, the staged construction of a cut off wall is simulated using finite elements. The solution shows the reduction in tidal response caused by partially constructed cut off walls.

In the third part of the paper, a coupled linear elastic / pore pressure model is used to investigate pressure transfer with negligible flow of groundwater through a completed cut off wall. This model provides an explanation of tidal response observable on the non-tidal side of a completed cut off wall.

In the final part of the paper, groundwater monitoring results are presented from two large projects in Sydney and compared against analytical and numerical predictions from the preceding parts of the paper.

2 THE INFLUENCE OF TIDES ON GROUNDWATER

In tidal areas, a proportion of the tidal range is observable in groundwater levels. The proportion observable declines with distance away from the coast. Figure 1 presents a schematic of groundwater levels in a tidal region and the effects of a cut off wall on the tidal response of groundwater.

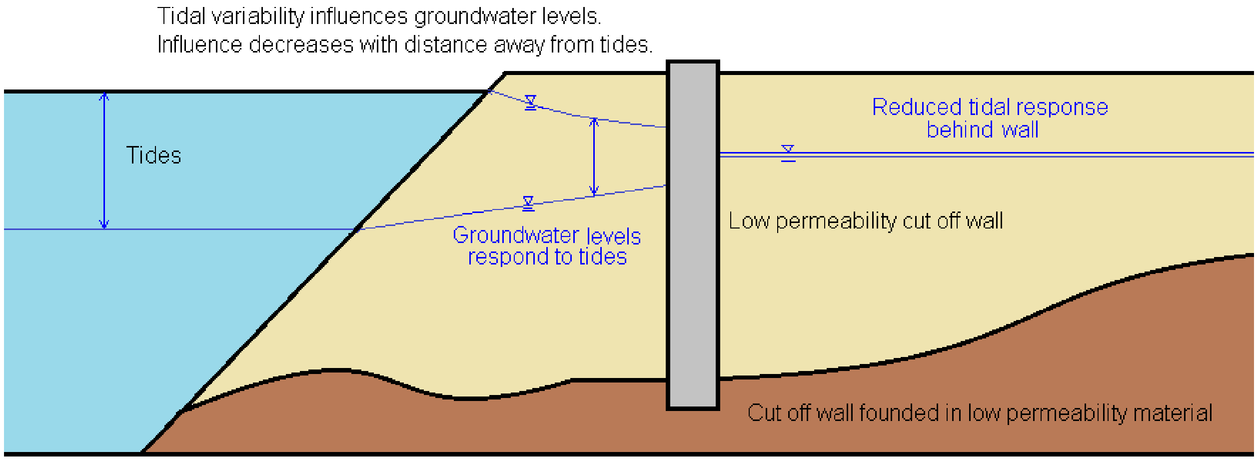

Figure 1: Tidal influence on groundwater levels

3 ONE DIMENSIONAL TIDAL RESPONSE

It is a consequence of Darcy's law, which states that groundwater flow is proportional to difference in head, that groundwater flow is governed by the diffusion or heat equation. The groundwater form of the diffusion equation for a one dimensional aquifer with uniform and constant transmissivity is given by:

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} \qquad (1)$$

Where:

- h = Hydraulic head (m)
- S = Storativity (dimensionless)
- T = Transmissivity (m^2/s)
- x = Distance from the left boundary (m)
- t = Time (s)

We also need to specify an initial hydraulic head at t=0 and lateral boundary conditions at x=0 and x=L.

3.1 INFINITE AQUIFER

Formulating our boundary conditions to include tidal forcing at x=0 and assuming for the moment that the aquifer extends an infinite distance from the tidal source, our boundary conditions without specifying an initial hydraulic head are:

$$\begin{aligned} h(0,t) &= h_0 \sin(\omega t) && (boundary\ condition\ 1) \\ h(\infty,t) &= 0 && (boundary\ condition\ 2) \\ \omega &= \frac{2\pi}{tidal\ period} \end{aligned}$$

The solution to this problem is well known (see for example Edelman, 1972 or Todd, 1959) as:

$$h(x,t) = h_o\ e^{ax} \sin(ax - \omega t) \qquad (2)$$

This is readily seen to conform with the governing equation if:

$$a = \sqrt{\frac{\omega S}{2T}} \qquad (3)$$

Observe that (2) defines a sinusoidal function with an amplitude decreasing exponentially with distance away from the tidal source.

We also see from (2) that the phase lag is given by:

$$t = \frac{ax}{\omega} \qquad (4)$$

The solution (2) therefore allows us to predict the response of groundwater affected by tidal conditions where the flow can be considered one dimensional and where the aquifer extends a large distance from the tidal source. Edelman (1972) provides several solutions for extensive aquifers, covering phreatic, confined and leaky aquifers.

3.2 FINITE AQUIFER

A more realistic situation is an aquifer whose extents are finite, with an impermeable or no flow boundary a distance L (m) away from the tidal source. In this case the boundary and initial conditions become:

$$
\begin{aligned}
h(0,t) &= h_0 \ \sin(\omega t) && (boundary\ \ condition\ \ 1) \\
h_x(L,t) &= 0 && (boundary\ \ condition\ \ 2) \\
h(x,0) &= 0 && (initial\ \ condition)
\end{aligned}
$$

The solution of (2) with the above boundary and initial conditions is achieved by separating variables and obtaining solutions of the form (see for example, Farlow, 1982):

$$h(x,t) \ = \ \sum_{n=0}^{\infty} X_n(x) \ T_n(t)$$

This method provides a solution for a finite aquifer as:

$$\frac{h(x,t)}{h_0} \ = \ \sin(\omega t) \ + \ \sum_{n=0}^{\infty} b_n \ \sin(\lambda_n x) \ \cos(\omega t) \ + \ \sum_{n=0}^{\infty} c_n \ \sin(\lambda_n x) \ \sin(\omega t) \qquad (5)$$

Where:

$$\lambda_n \ = \ \frac{(2n+1)\pi}{2L}, \qquad b_n \ = \ \frac{-4a^2\lambda_n}{L \ (\lambda_n^{\ 4} + \ 4a^4)}, \qquad c_n \ = \ \frac{-8a^4}{\lambda_n \ L \ (\lambda_n^{\ 4} + \ 4a^4)}, \qquad a \ = \sqrt{\frac{\omega S}{2T}}$$

Note importantly that the coefficients b_n and c_n are dimensionless. The solution is best evaluated using a computer. However, one can not overstate its usefulness in providing a check on the numerical solutions (presented in the following sections) and vice versa with the numerical solutions being used to provide a check on (5).

A comparison of the solutions (2) and (5) showing tidal response is presented in Figure 2 for a transmissivity of $1\text{x}10^{-4}$ m^2/s, storativity of $1\text{x}10^{-3}$ and L = 50 m. Tidal response is defined as the pore pressure range at a point divided by the range of the tidal source.

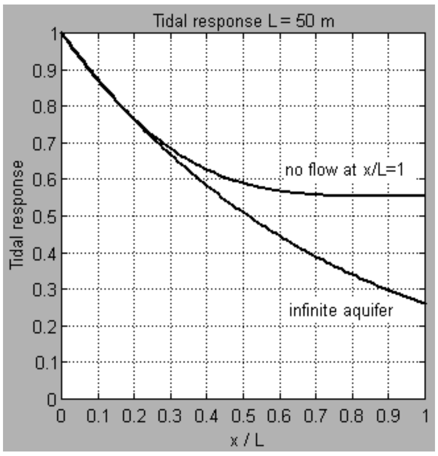

Figure 2: Comparison of one dimensional solutions

The two solutions are seen to be almost equal for small x.

In Figure 3, the reduction in tidal response is charted for a range of L values using the dimensionless variables **ax** and **aL**, with **a** as defined in (3).

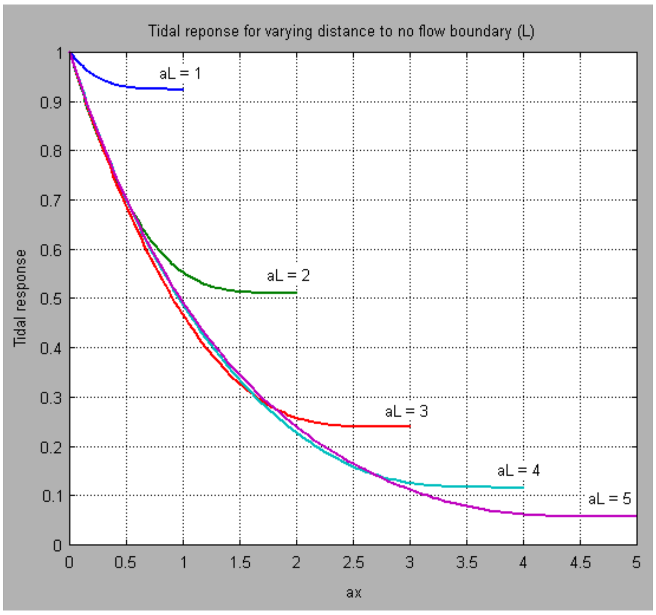

Figure 3: Tidal response for varying distance to no flow boundary

Observe the rapid reduction in tidal response as the dimensionless variable **ax** increases. For **ax > 4** the tidal response is less than 5%. For any given transmissivity, storativity and distance to a no flow boundary, it is possible to form the variables **ax** and **aL** and thus estimate the tidal response from Figure 3.

It is interesting to note that the tidal response is increased significantly by the proximity of a no flow boundary compared with the response for an extensive aquifer.

4 TWO DIMENSIONAL SOLUTIONS

Now we aim to obtain a solution in two dimensions, in order to analyse in plan view the effects on tidal response due to a partially constructed cut off wall. This is a practical problem as cut off walls are often used in coastal areas to provide barriers to contamination or to aid with deep excavations.

In two dimensions, the governing equation (1) becomes:

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \qquad (6)$$

Boundary and initial conditions are also necessary to completely formulate the problem. Analytical solutions of (6) are harder to find than those for (1) and it is often practical to solve the problem numerically. A finite element program was developed to aid with solution of (6). The program uses the open source programming tool FreeMat (Basu, 2011). A two dimensional mesh consisting of linear triangular elements was generated using Delaunay triangulation. Details on the use of finite element methods to solve (6) are presented by Lewis, Nithiarasu and Seetharamu (2004).

The program was used to look for solutions with a tidal source of range 2 m being specified on the line x = 0. No flow boundaries were specified on each of the other three boundaries. The model domain and mesh are shown in Figure 4. The model domain represents an area in plan view with the region between x = 5 and x = 6 being used to model a low permeability zone simulating a cut off wall.

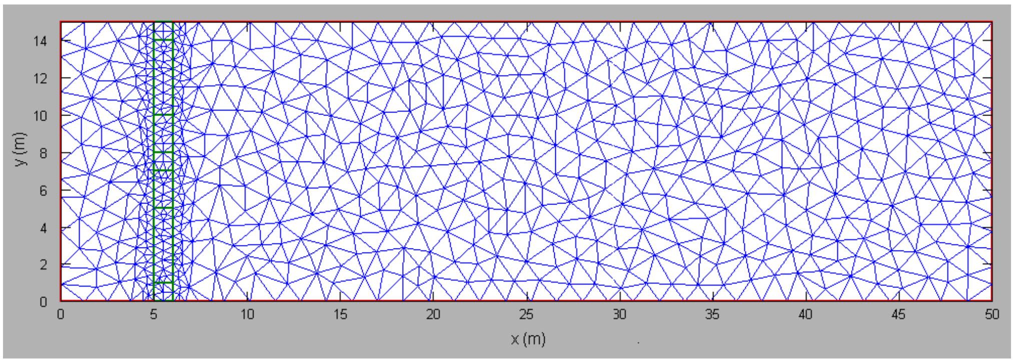

Figure 4: Finite element model domain and mesh

The model was compared to the analytical solutions presented earlier to ensure that it gave matching results for one dimensional flow for a range of transmissivity and no flow boundary distances.

4.1 STAGED CONSTRUCTION

A staged construction was simulated, with the aim of assessing the proportion of tidal response remaining at any point and how this changed as wall construction progressed. The final tidal response at a point was divided by the initial tidal response at the same point to show the proportion remaining. Note that this is not a proportion of the tidal forcing amplitude at $x = 0$, it is a proportion of the amplitude at the point in question prior to wall construction.

The values of the constants were:

- Transmissivity = 1×10^{-3} m^2/s (everywhere except between $x = 5$ m and $x = 6$ m).
- Transmissivity = 1×10^{-9} m^2/s (between $x = 5$ m and $x = 6$ m).
- Storativity = 1×10^{-2}
- A no flow boundary at 50 m.

A transient solution was obtained for five stages of cut off wall construction. The stages were for 3 m, 7 m, 10 m, 14 m and 14.75 m of wall constructed out of a possible 15 m. The case for 0 m constructed is equivalent to (5) and the case for 15 m constructed was not plotted as virtually no effects (in a groundwater flow only model) will penetrate the low permeability zone of $5 \text{ m} < x < 6 \text{ m}$. The results are presented in Figure 5.

The no flow boundaries at $y = 0$ m and $y = 15$ m represent impermeable boundaries or axes of symmetry.

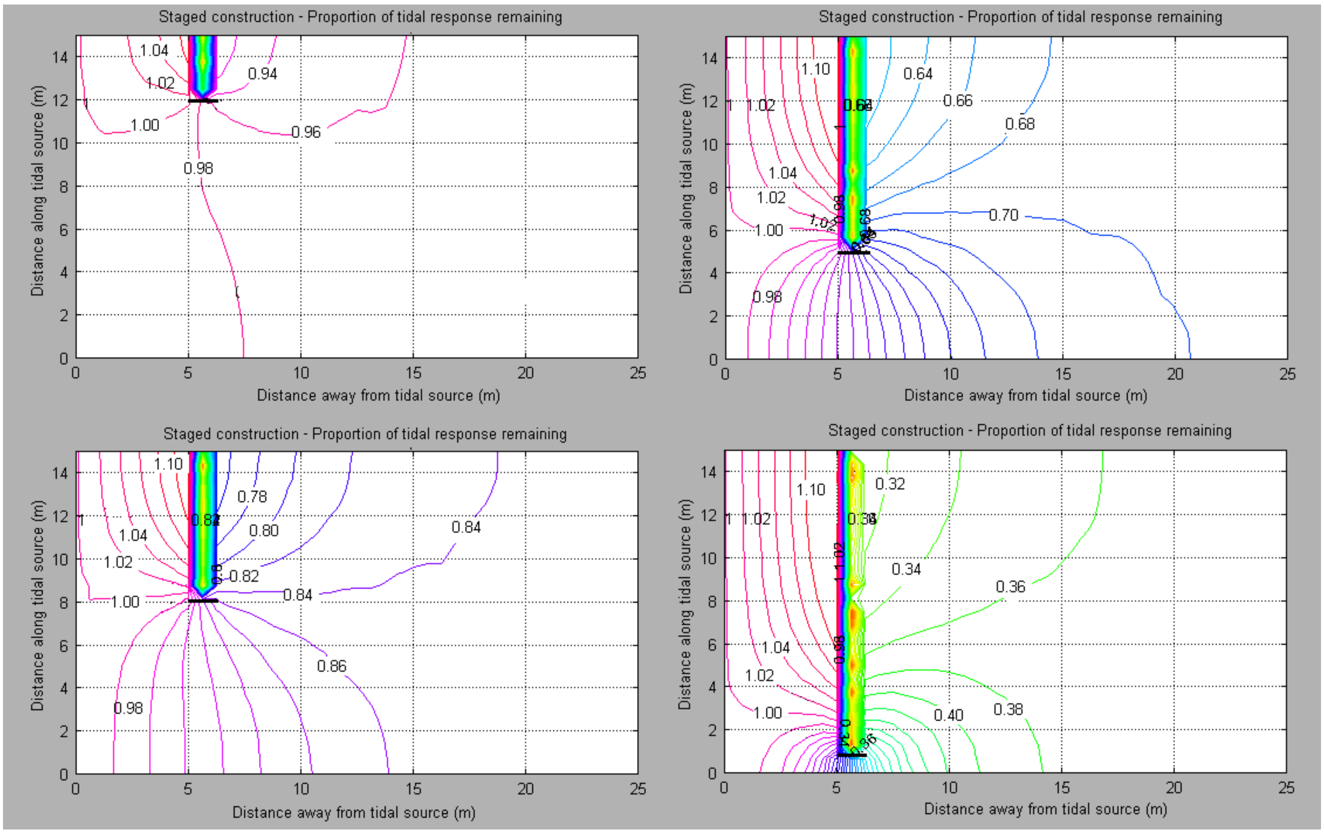

Figure 5: Proportion of tidal response remaining during cut off wall construction

Observe that on the tidal side of the wall, the tidal response is increased by 10 %, even when only a small amount of wall has been constructed. On the non tidal side of the wall, the tidal response continues to be reduced as the wall is constructed. With 14 m constructed out of 15 m (93 %), the tidal response is still more than 32 % of its original value, showing that the influence of the tide does not diminish linearly with wall construction and that small holes may be easily detectable from groundwater monitoring near the wall. This is made clear in Figure 6. Approximately 18% of the original tidal response remains when a gap of just 0.25 m out of 15 m remains open.

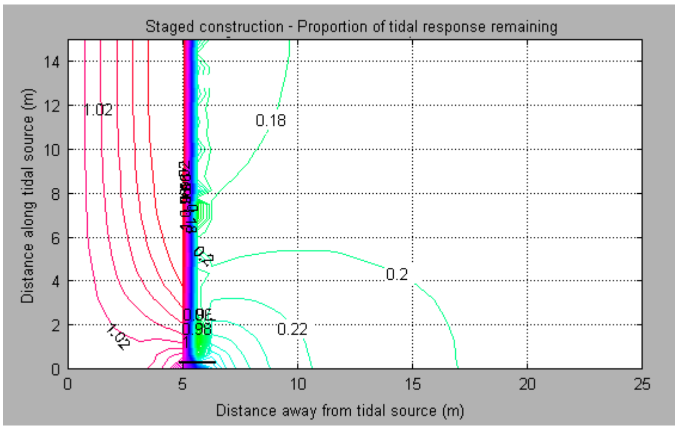

Figure 6: Proportion of tidal response remaining with a 0.25 m gap in wall

A consideration of the results in Figure 5 and Figure 6 shows that if we let y = proportion of tidal response remaining and x = meters of wall constructed, the results lie approximately on the logarithmic curve:

$$y = \frac{\ln(16-x)}{\ln(16)} \qquad (7)$$

This result is independent of the distance away from tidal source. Figure 7 shows the finite element results in relation to a logarithmic curve. This relationship is perhaps not entirely unexpected noting the exponential form of the one dimensional analytical solution (2).

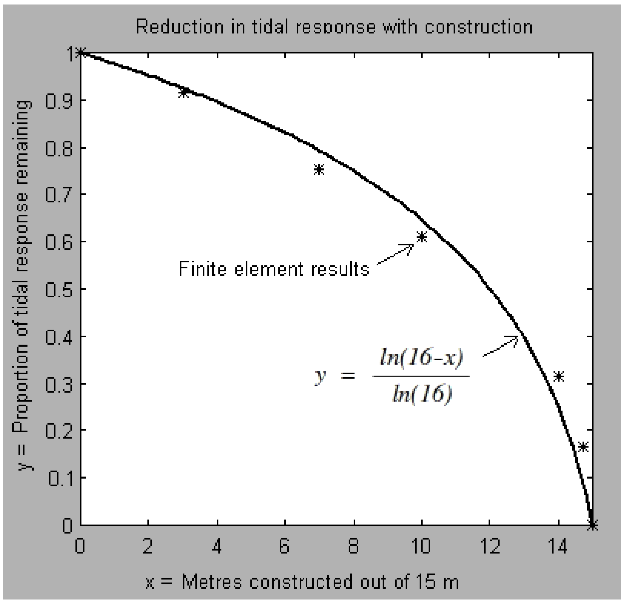

Figure 7: Tidal response reduction during staged construction

It is important to note that the results presented in Figure 5, Figure 6 and Figure 7 were for a specific choice of aquifer constants. Further work is required to provide chart/s, similar to Figure 3, which provide solutions for other combinations of aquifer constants.

5 TWO DIMENSIONAL COUPLED SOLUTION

In the real world, it has been observed that after a cut off wall has been fully constructed, there remains an observable tidal response on the non tidal side of the wall. An evaluation of (5) makes it clear that with a pore pressure flow only model, there will be a negligible tidal response observable on the far side of a zone of sufficiently low permeability.

The observed phenomenon is explained by relating the effects of elasticity with pore pressure generation, invoking the principal of effective stress. An outline of how these two effects are combined is provided by Potts and Zdravkovic (1999).

The coupling process requires the introduction of two elastic constants:

- E (kPa) = the drained elastic modulus.
- The Poissons ratio.

We note here, importantly, that that E is related to the storativity by the relationship:

$$S = \gamma_w \ b \ m_v \qquad (8)$$

$$\begin{aligned} &where: \\ &m_v = constrained\ modulus = \frac{(1+\nu)(1-2\nu)}{E\ (1-\nu)} \\ &\quad b = vertical\ aquifer\ thickness\ (m) \\ &\quad \gamma_w = unit\ weight\ of\ water\ \ (\frac{kN}{m^3}) \\ &\qquad \nu = Poissons\ ratio \end{aligned}$$

The above relationship is derived using the principal of effective stress and appropriate elastic stress strain relationships.

A coupled linear elastic stress strain and pore pressure finite element program was implemented using FreeMat. The coupled model was used to determine the tidal response on the non tidal side of a fully constructed 1 m wide cut off wall (transmissivity of 1x10^{-9} m^2/s).

For this analysis, the model was considered in section view as opposed to plan view for the non-coupled analysis above. The model assumes a confined aquifer and does not model the phreatic surface. The Poissons ratio was set to 0.33. Figure 8 presents the model domain including the stress strain and pore pressure boundary conditions on each of the four specified boundaries.

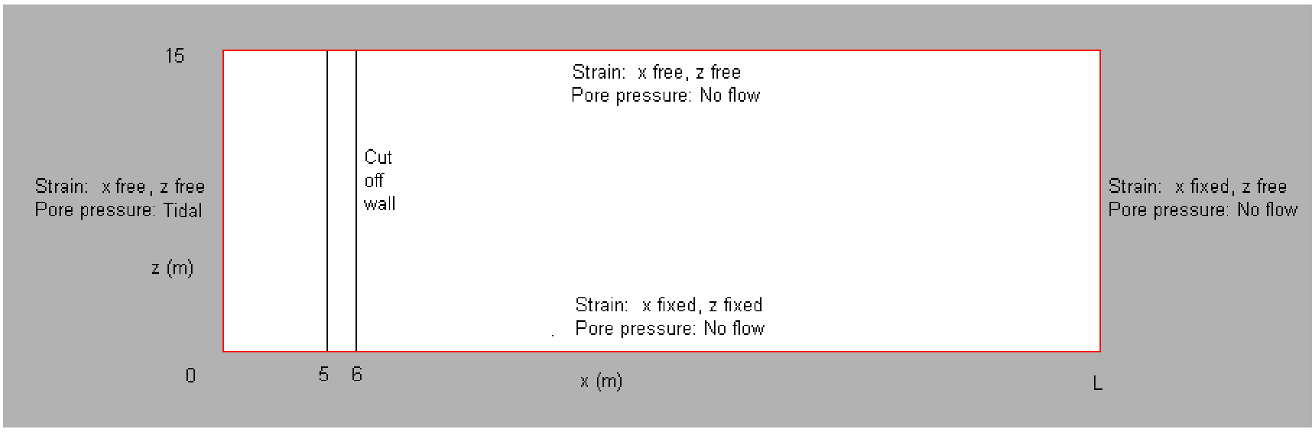

Figure 8: Model domain and boundary conditions for coupled solutions

Modelling was conducted for two alternative no flow boundary distances and for a range of k and E values. The results are presented in Table 1 and Table 2. As can be seen, tidal responses behind the wall are all less than 3%, except for the case of highly compressible and rather impermeable soils. This indicates that a measure of quality assurance for cut off walls is achievable during construction. In most cases, a perfectly constructed wall will produce a 95% or greater reduction in tidal response in groundwater levels near the wall.

The effects of the vertical depth to an impermeable boundary were not fully investigated, in the following table and figures the depth assumed was 15 m. An analysis for a 4 m depth to no flow boundary showed a slight reduction in tidal response compared to the corresponding values shown in Table 1 and Table 2, for a transmissivity of 1.5x10^{-6} m/s.

Table 1: Maximum tidal response on non tidal side of wall, no flow boundary at L = 20 m

Transmissivity	E = 10 MPa	E = 100 MPa	E = 1000 MPa
15 m x 1x10^{-4} m/s	1.3 %	1.4 %	1.1 %
15 m x 1x10^{-5} m/s	1.9 %	1.4 %	1.1 %
15 m x 1x10^{-6} m/s	4.5 %	1.9 %	1.1 %

Table 2: Maximum response on non tidal side of wall, no flow boundary at L = 50 m

Transmissivity	E = 10 MPa	E = 100 MPa	E = 1000 MPa
15 m x 1x10^{-4} m/s	0.9 %	0.7 %	0.5 %
15 m x 1x10^{-5} m/s	2.7 %	1.0 %	0.5 %
15 m x 1x10^{-6} m/s	6.3 %	2.8 %	0.6 %

Figure 9 presents a section from x = 0 m to x = L, approximately half way vertically up the wall.

The values of the constants used were:

- Transmissivity = 1.5x10^{-4} m^2/s (everywhere except between x = 5 m and x = 6 m).
- Transmissivity = 1x10^{-9} m^2/s (between x = 5 m and x = 6 m).
- Elastic modulus = 30 MPa (storativity = 1.5x10^{-2} /m).

- Poissons ratio = 0.33
- A no flow boundary at 50 m from the tidal source.
- A no flow boundary at 15 m depth.

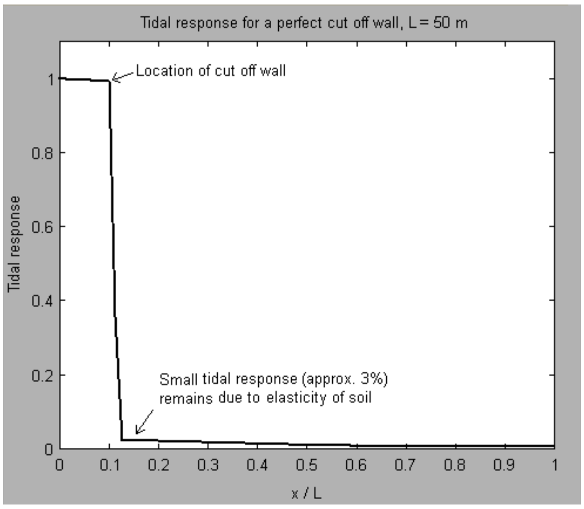

Figure 9: Section from x = 0 m to x = L

6 CASE STUDIES

6.1 BARANGAROO SOUTH, STAGE 1A

Part of a large scale commercial development at Barangaroo in the Sydney CBD involved the construction of a cut off wall to isolate the construction site from nearby harbour waters. This enabled excavation to -4.5 m AHD and the construction of a multi level basement area for three large buildings. The cut off wall was constructed using diaphragm wall construction methods as a series of 6 m by 1 m panels. Each panel was penetrated into the underlying sandstone bedrock, which was typically between 8 m and 20 m below the surface. The material above the bedrock comprised up to 8 m of highly permeable fill overlying variable amounts of alluvium and residual soil.

Prior to wall construction, 16 monitoring wells were installed with data loggers to provide quality assurance for the wall during construction. The monitoring wells were positioned on the non tidal side of the wall and spaced at approximately equal distances apart. The wells were typically 10 m away from the wall. Figure 11 shows the location of the site and the location of two selected monitoring wells close to the harbour waters, DW4 and DW5.

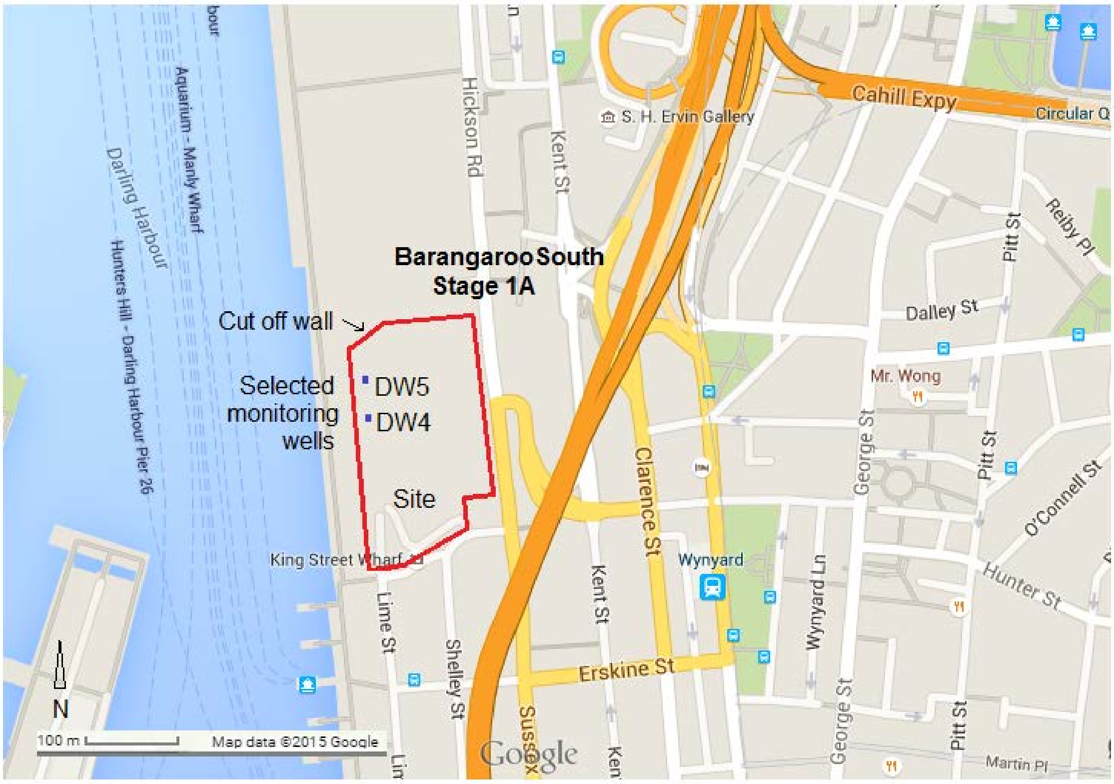

Figure 11: Barangaroo South – Stage 1A

Construction of the cut off wall at DW4 and DW5 occurred between February and April 2012. The wall was not constructed in a sequential fashion. Instead, isolated panels would be constructed and the gaps filled with the remaining panels at a later stage. Monitoring well data was processed to provide a chart indicating the (tidal) range of water levels in each monitoring well. The data is presented in Figure 12.

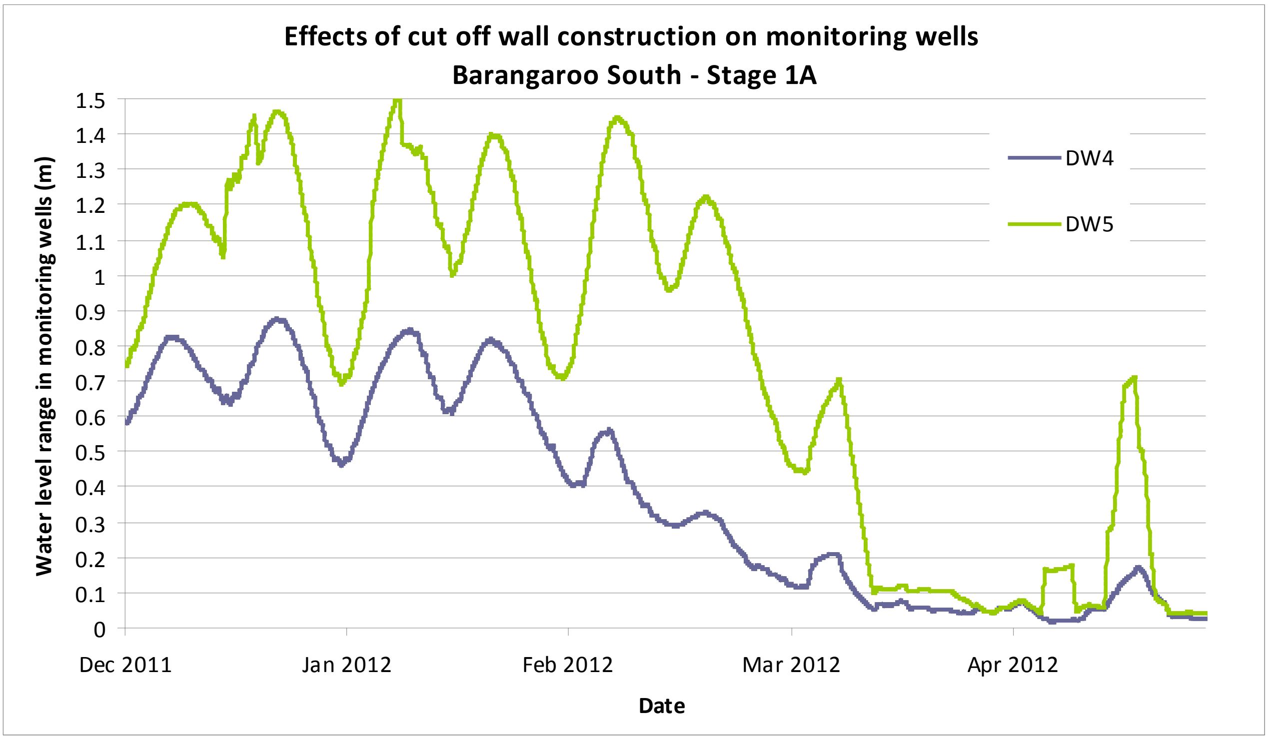

Figure 12: Monitoring well observations, Barangaroo South

The effects of wall construction during February and March are clear from the chart. King tides and neap tides are visible, especially prior to wall construction. In addition, rainfall also influenced the groundwater levels as can be seen by a sharp spike in levels in mid April. The average tidal range in Sydney Harbour is approximately equal to the range in DW5 between December and February and may be taken to equal 1. Using this range for the tide, we see that during April, after the cut off wall had been completed, the tidal responses in DW4 and DW5 were both of the order of 3% to 5%. From the discussions in the previous sections and the chart shown in Figure 9, we see that the cut off wall near DW4 and DW5 appears to be functioning near perfectly as an impermeable barrier and the responses in the monitoring wells appear to be due to the effects of elastic compression only.

6.2 TEMPE LANDS REMEDIATION

In 2004 a cut off wall and leachate collection/treatment system was constructed as part of a remediation program at the former Tempe Landfill in order to limit the migration of contaminants into the nearby Alexandria Canal (tidal). The wall was a soil / bentonite barrier constructed to below the level of stiff to hard residual clays. These clays were at a depth of 5 m to 10 m and were overlain by variable amounts of fill and silty clays. The wall was designed to provide a cut off equivalent to 0.5 m of soil at a hydraulic conductivity of $1x10^{-9}$ m/s and was constructed progressively from west to east.

19 piezometer monitoring wells with data loggers were installed on the non tidal side of the cut off wall, 5 m away from the wall. Additionally, 11 monitoring wells with data loggers were installed on the tidal side of the wall. Figure 13 presents a plan of the site showing the canal and the locations of three select monitoring wells, namely MPI 6, MPI 10 and MPI 14. A chart showing the tidal response ranges observed in MPI 6, MPI 10 and MPI 14 is presented in Figure 14.

Figure 13: Tempe Lands Remediation

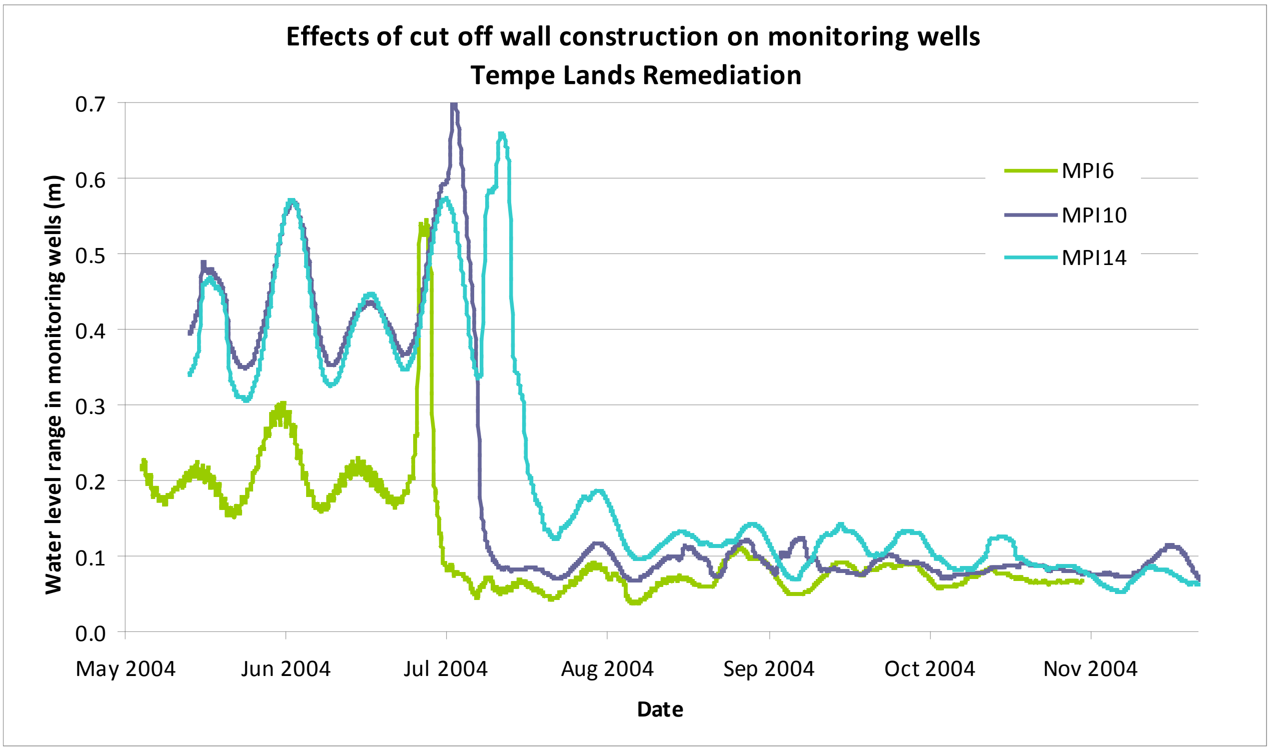

Figure 14: Monitoring well observations, Tempe Lands Remediation

In Figure 14, wall construction is observable from the small but noticeable aberration in monitoring well water levels. It is possible to note the progression of wall construction, first at MPI 6 and then at MPI 10, followed by MPI 14. This shows the rate of wall construction.

In November, following the completion of the wall, we see that the tidal responses of the three monitoring wells were of the order of 5% to 8%. While larger than 3%, these values may still be within the range for effects caused by purely elastic compression, indicating a nearly impermeable cut off wall.

7 CONCLUDING REMARKS

An investigation of groundwater response in tidal areas has been presented. The results were derived from first principals and provide an understanding of the mechanics of tidal response of groundwater. An analytical solution showing the rate of increase of tidal response on the tidal side of a no flow barrier was presented.

Finite element solutions were presented to model the effect on tidal response during construction of a low permeability cut off wall. Small imperfections in wall construction were shown to have a marked response which would be observable on the non tidal side of the wall. A coupled model showed that a certain amount of tidal response will be transferred to the non tidal side of a perfectly constructed wall. This explains observations presented from two case studies in Sydney.

Further work is required to investigate the effects of different (vertical) aquifer thicknesses on the coupled solutions. An investigation of the effects of anisotropic soil conditions on tidal response may be investigated. It may also be possible to obtain (rather complicated) analytical solutions to some of the results presented using finite elements which may further aid our understanding of the problem.

8 ACKNOWLEDGEMENTS

The author gratefully acknowledges the support of Lend Lease and of Marrickville Council for allowing publication of groundwater monitoring results at Barangaroo South and Tempe Lands respectively.

This paper is a result of a study the author undertook as part of the Specialist Technical Apprentice Scheme at Coffey. The author wishes to thank Coffey Geotechnics for the material on the Barangaroo and Tempe projects. Particular thanks are due to Ross Best for providing guidance and for reviewing the paper and for helping to discover a few important issues which had completely escaped the author’s attention.

9 REFERENCES

Basu S., 2011, *FreeMat v4.1 Documentation*

Edelman J. H., 1972, *Groundwater Hydraulics of Extensive Aquifers*, International Institute for Land Reclamation and Improvement, Wageningen

Farlow S. J., 1982, *Partial Differential Equations for Scientists and Engineers*, Wiley, New York.

Lewis R. W., Nithiarasu P. and Seetharamu K. N., 2004, *Fundamentals of the Finite Element Method for Heat and Fluid Flow*, John Wiley and Sons Ltd, Chichester

Potts D. M. and Zdravkovic L., 1999, *Finite Element Analysis in Geotechnical Engineering*, Thomas Telford Ltd, London.

Todd D. K., 1959, *Groundwater Hydrology*, John Wiley and Sons Inc., New York and London